ENGINEERS'

SURVEYING INSTRUMENTS,

THEIR CONSTRUCTION, ADJUSTMENT, AND USE.

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PREFACE.

THIS volume was prepared for the author's own students, and has been used in his classes for twelve years past, in the form of a blue-print manuscript text-book. Two series of extracts were published in two engineering journals and afterwards reprinted in book form. As one of these little books had the same title as this volume, the latter is gratuitously called a second edition.

The object of this volume is to acquaint the student with the construction, adjustment, and use of surveying instruments. In no degree is it intended for a treatise on surveying, since there is no lack of good books on the various branches of that subject. It has not, however, always been possible to draw a sharp line between instruction in the use of the instruments, and methods of surveying; but, as a rule, only such of the latter have been given as are common to all surveys made with the particular instrument under consideration. In a few cases, methods of surveying not given in the common text-books and manuals are briefly discussed.

The author began the preparation of this volume after becoming convinced, by his own observation as well as by that of others, that students, as well as many practicing engineers, would be benefited by a more thorough study of the instruments. The knowledge of the instruments and the practice in their use gained in the study of subjects in which the instruments have

but subordinate consideration, are not sufficient to secure either economy of time and effort, or maximum accuracy. The experience of the author, the testimony of his students after leaving college, and the commendations the first edition received from engineers in practice, have confirmed his opinion of the value of a detailed study of the instruments themselves, the sources of error in using them, the methods of eliminating the errors, and the degree of accuracy attainable.

In Appendix IV will be found the problems which the author assigned in connection with a study of the text. These problems are designed to familiarize the student with the methods of using the instruments, to acquaint him with the qualities of good instruments, and also to teach him the degree of accuracy attainable. Some of the problems are solved several times with different instruments and under different conditions as to distance, weather, experience, etc.

In the preparation of this volume great care has been taken to present the subject clearly and concisely, and to make the book convenient for daily use or ready reference. The volume is divided into chapters and articles, and it may be helpful to the reader to notice that successive subdivisions of the latter are indicated by capital black-face side-heads, by lower-case black-face side-heads, by italic side-heads, and by simply the serial section number. In some cases the major subdivisions of the sections are indicated by small numerals. The running title at the head of the pages will be of assistance in finding the different parts of the book. An index at the end of the volume makes everything in the book easy of access.

CHAMPAIGN, ILL., Nov. 19, 1892.

CONTENTS.

CHAPTER IV. SOLAR COMPASS.

CHAPTER V. VERNIERS.

CHAPTER VI. OPTICAL PARTS OF THE TELESCOPE.

CHAPTER VII. THE TRANSIT.

CHAPTER XI. SPIRIT LEVELS.

CHAPTER XII. BAROMETERS.

APPENDIX I. ELIMINATION OF LOCAL ATTRACTION IN SURVEYING WITH THE MAGNETIC COMPASS.

APPENDIX II. AREA BY TRAVERSING WIT TRANSIT.

APPENDIX III. PROBABLE ERROR.

APPENDIX IV. PROBLEMS.

ENGINEERS' SURVEYING INSTRUMENTS.

INTRODUCTION.

THE importance to the engineer of a knowledge of the best forms of construction of his instruments and of a thorough understanding of the principles which should govern their adjustments, is self-evident; and it is no less evident that he should be expert in handling his instruments. The general plan of this discussion will be (1) to call attention to the principal points in the common forms of construction of each instrument, noting the advantages and disadvantages of each; (2) to consider certain relations which the parts of a perfect instrument should bear to each other, and which are supposed to be adjusted by the maker once for all, explaining how the engineer may test them for himself although he may not be able to correct them; (3) to explain the method of making the several adjustments; (4) to describe the method of using the instrument which will secure the most accurate results in the shortest and easiest way, and to note the various sources of error to which the work is liable, and also to give data to show the degree of accuracy attainable; and (5) to add a few hints on the proper care of the instrument. It will be unnecessary to describe minutely the different

parts of the several instruments and the objects of each, except as is requisite in carrying out the above outline; for more can be learned by a moment's inspection of the instrument than from any printed description. No attempt will be made to describe any of the special devices proposed by the different instrument-makers, the desire being to state the general principles which will enable the reader to judge of the merits of any modification of the ordinary forms.

CHAPTER I.

CHAIN AND TAPE.

ART. 1. CONSTRUCTION.

1. COMMON CHAIN. The ordinary chain consists of one hundred pieces of wire, called links, bent into rings at the ends and connected together by two (sometimes three) rings. In using the chain a "link" includes a ring at each end. Iron chains are made of two sizes of wire, Nos. 8 and 10, the former being about five thirty-seconds of an inch in diameter and the latter nearly one eighth of an inch. Steel chains are made of No. 10 or 12 wire, the latter being about seven sixty-fourths of an inch in diameter. The best chains are made of No. 12 tempered-steel wire. All joints in the rings and links should be brazed to prevent their opening, and the consequent lengthening of the chain. The chain is divided decimally by brass tags numbered from each end to the middle.

The surveyor's chain (Gunter's chain) is 66 feet long, each "link" being 7.92 inches. It is used only in finding the area of land where the acre is the unit of measure, and is much less frequently used for this purpose now than formerly. The 66-foot chain is used on all the U. S. public-land surveys; and in all deeds of conveyance and other documents, when the word *chain* is used, it is Gunter's chain that is meant.

An engineer's chain is 100 feet long, each "link" being 1 foot. This chain is used in surveying railroads, canals, and where extensive line surveys are being conducted. It is not infrequently employed in finding areas in acres. It is preferred to the surveyor's chain on account of its greater length, which enables one to work more rapidly and more accurately.

When the chain is folded up the links should not be parallel to each other, but should be crossed in such a manner as to touch each other in the middle, thus preventing the bending of the links in tying up the chain. See Fig. 1.

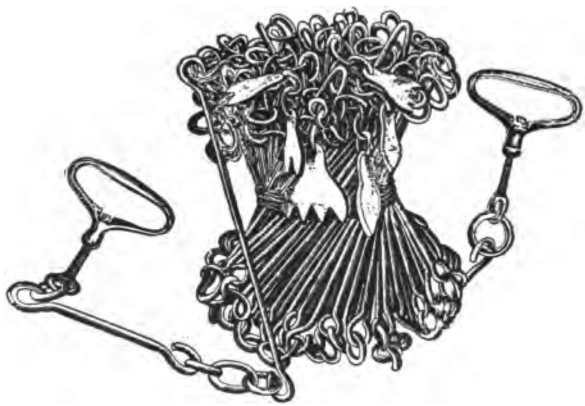

FIG. 1.

2. Advantages and Defects of the Common Chain. The principal advantages of the chain are its flexibility and the ease with which its subdivisions are distinguished.

The ordinary chain is defective on account of (1) unavoidable wearing of the numerous points of contact, (2) opening of the rings, (3) lengthening by stretching and by flattening of the rings, (4) shortening by mud, ice, or grass getting into the joints, (5) varying in length with

temperature, and (6) kinking. Some of these defects are so small as to be appreciable only in very careful work; and some exist even in the most elaborate apparatus that can be devised. If each of the six hundred points of contact of the common 100-link chain wears only one hundredth of an inch, the length of the chain is increased 6 inches. Mud and ice in the joints have a still greater effect in the opposite direction.

3. Steel Tapes. Steel ribbons are now made of any length up to 1,000 feet without joint or splice from end to end. In width these ribbons vary from an eighth of an inch to a half inch, and in thickness from one hundredth to four hundredths of an inch. For general work a tape about a quarter of an inch wide and two hundredths of an inch thick, blued and polished or nickel-plated, is generally preferred. The wider and thinner tapes are nearly useless in general field-work, owing to the ease with which they are broken.

The divisions of the tape are marked in several ways, viz.: (1) the numbers and graduation marks are stamped in a piece of brass which is soldered on the tape; (2) the marks are countersunk in a lump of solder attached to the tape for that purpose; (3) the foot divisions are marked by single rivets and the 10-foot divisions by a brass burr riveted on, the 10, 20, etc., marks being distinguished either by numbers countersunk in the brass burr or by 1, 2, etc., small rivets; or (4) the face of the tape is etched with acid so that the division marks and figures stand out in relief while the etched surface appears dull. No method of indicating the graduation is entirely satisfactory, but the first and second are probably best. The first is probably more durable than the second, although the graduation marks are liable to wear or tear off, particularly on gravel or stony ground. In either case there should be two series of numbers, one counting from each end. It is not desirable to have

the graduation indicated by rivets, since the tape is weakened by the rivet-hole, and also by the accumulation of moisture under the rivet head. One advantage of this method is that the graduation can be read from either side of the tape. The first and second methods are sometimes employed upon the same tape. The last method is employed only for pocket tapes graduated to feet and inches, owing to the liability of weak places from over-etching.

Steel tapes graduated by any of the first three methods mentioned above are frequently called band chains and sometimes chain tapes, to distinguish them from tapes graduated by the fourth method, *i.e.*, graduated fully throughout.

Steel tapes can be had graduated to feet and inches, to feet and tenths, to links, or metrically. The wide thin tapes graduated to feet and inches by etching are very convenient in city surveying, or where accurate measurement of irregular distances is frequently required. The tape ordinarily employed in general field-work is a narrow ribbon graduated to feet, with each end foot to tenths. This form of tape would be greatly improved if, instead of subdividing the first and last foot of the hundred, an additional foot were added at each end, and so divided. When the end foot of the hundred is subdivided, if one desires to lay off, say, 28.3 feet, one chain-man must hold the tape at 29 feet and the other at 7 tenths from the end of the graduation, which operation produces mental confusion and is apt to cause error, since neither chain-man holds the tape at the number to be laid off. With an extra foot at the end of the tape, one chain-man holds the tape at the whole number of feet to be laid off, while the other chain-man holds at the number corresponding to the fraction to be laid off.

Frequently the end graduation of a steel tape is

simply a line across the face of the tape or a rivet in the middle of its width, in which case it is not possible to stick the pin quickly or exactly opposite the mark. The end graduation mark should be indicated by the square shoulder of a piece of brass soldered or riveted to the tape for that purpose. These shoulders, if put on at all, generally face opposite ends of the tape. They should both face the same end (preferably the rear end) of the tape, as shown in Fig. 2; in which case the

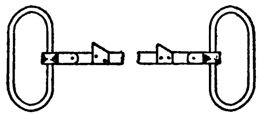

FIG. 2.

same side of the pin can be used by both the fore and hind chain-man.

4. Reels. For convenience of transportation steel tapes are wound up either on an open reel or in a tight case, the narrower thicker ones on the former and the wider thinner ones in the latter. In one respect the open reel is better than the tight case, since with the former the tape has a better chance to dry off. It is desirable that the reel should be strong, durable, and convenient, and at the same time be light and of such a form as to be carried in the pocket when the tape is in use. No reel that the writer has seen fulfills this condition fairly well. Generally the axis about which the tape is wound is so small as to break the tape near the end.

Most of the reels require that one handle be removed before beginning to wind up the tape, which necessitates the handles being easily detached without liability to come off when in use. Most of the handles on the market are deficient in one or the other of these

respects. On the whole, it is probably best to wind the tape up in the hands in the form of a figure eight (8) about two feet long, and tie with a string at the center.

5. Advantages and Disadvantages of Steel Tapes. Steel tapes have superseded chains for nearly all kinds of work for the following reasons: (1) Tapes are lighter than chains of equal strength. (2) Tapes being smooth and having no projections are easier to drag. (3) Tapes do not alter their length by wear.

The disadvantages are: (1) The steel tape will not stand as rough handling as the chain. (2) It can not be repaired in the field when broken, while the chain can lose one or more of its links and still be of service. (3) The graduation is not as legible as that of the chain, and becomes obliterated with use.

Notwithstanding its defects, a steel tape is vastly better than a chain for nearly all kinds of work.

6. Wood Rods vs. Steel Tapes. Before the introduction of steel tapes short wood rods were employed when great accuracy was necessary. They were free from some of the imperfections of the common chain, but were defective in other and generally more important respects. Short rods should be used only with the accurate alignment and elaborate means of securing delicacy of contact employed in geodetic measurements; and recent experience* seems to show that in measuring geodetic base lines, work can be done with greater speed and accuracy with steel tapes than with the most elaborate and expensive base apparatus yet devised.

* See Annual Report of the Missouri River Commission for 1886,—Executive Document No. 28, 49th Congress, 2d Session,—pp. 31–35. For method of accurate measurement with two steel tapes or wires used as a metallic thermometer, see Annual Report of the Chief of Engineers, U. S. A., for 1890, pp. 1838–45.

7. Steel Wires. A steel wire makes a good substitute for a chain. Fig. 3 shows a method of indicating the end of the distance to be laid off. *a b* is a small rod with a hole through it, into which the wire is fastened; *c* is a V-shaped groove around the rod. The pin may be set in the groove *c* without any danger of the back pin's being displaced by the jerking of the wire by the fore chain-man in getting it into line; or the distance may end at *a* or *b*, in which case it can be more accurately marked with a knife in a board. The ends *a*

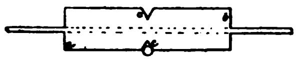

FIG. 3.

and *b* should be exactly at right angles to the direction of the length of *a b*. A handle can be formed by passing the wire through a piece of wood to protect the hand, and then bending the wire into a loop. Unfortunately fractional parts of the unit are not easily obtained; but in many cases only units are required.

8. LINEN TAPES. Although accurate work can not be done with linen tapes, they are useful in many kinds of work, as, for example, in cross-sectioning for railroad earthwork. A linen tape contracts in wet weather and expands in dry, and it can easily be permanently elongated by overstraining.

9. Metallic Tapes are a species of linen tapes having fine brass wires woven through their entire length. Metallic tapes do not vary in length with changes in the hygrometric state of the atmosphere as much as linen tapes, and are not so easily elongated by overstraining.

ART. 2. TESTING THE CHAIN AND TAPE.

10. STANDARDS. Each instrument-maker claims that the chains and tapes which he sends out are true U. S. standard; but it is certain that chains and tapes from different makers, purporting to be of the same standard, differ in length. Even if correct in the beginning, chains wear and tapes break; and therefore neither are likely to be of the same length after being used some time. Hence every engineer should have some means of testing the length of his chain or tape. The simplest way to get a reliable standard is to send a new tape to the Superintendent of the U. S. Coast and Geodetic Survey, Washington, D. C., who will compare it with the standard and place the government stamp on it to show its degree of accuracy. For a degree of accuracy sufficient for ordinary engineering a fee of 50 cents is charged; but when extreme accuracy is desired the fee is higher.

The tape so compared should be reserved for testing other tapes, or the standard distance should be carefully laid off on the floor of a building, or on a stone water-table, or on two stone or iron posts firmly set in the ground for that purpose, or in any way that shall permanently preserve the exact distance. When laying off the standard distance the tape should be supported throughout its entire length; should be perfectly straight, both horizontally and vertically, and should be stretched with the same tension as that employed in testing it at Washington. In laying off this distance the temperature of the tape should be carefully noted, and if it is not the same as that at which the tape is a standard, a correction (see § 19, paragraph *e*) should be applied before making the permanent mark.

11. It is sometimes held that since the chain when in use is seldom if ever stretched perfectly straight, it

should be made a little longer than the standard so that the full length of the standard may be laid off each time. The instructions issued by the U. S. Land Office to Surveyors-General states * that "the 66-foot chain shall be 66.06 ft.," for the above reason. The French have, or at least had,† a similar practice, the addition being from 1 in 1,000 to 1 in 2,000. In all the arts depending in any way upon accuracy of measurements there is great confusion on account of differences between so-called standards. In some cases, particularly that of iron work, this diversity arose in a manner similar to that referred to above. The chain should be *exactly* as long as the standard.

12. Notice that in many cases the standard by which the engineer is to test his chain or tape is not an absolute one. For example, the law under which the U. S. public lands were surveyed says "all the corners marked in the surveys returned by the Surveyor-General shall be established as the proper corners," etc., and "*the length of such lines as returned shall be held and considered as the true length thereof.*" This law establishes a standard of measure between every pair of adjacent corners of the government survey, and this standard is the only one that can legally be used in measuring that line. Assume, for example, that a surveyor being called upon to establish the corner at the middle of the west side of sec. 1, measures the west side of that section and finds it to be 79.26 chains by his chain, while the recorded distance is 79.83 chains. Then, since he must consider the recorded distance as the true distance, the true length of his chain is $79.83 \div 79.26 = 1.007$ times the standard. Since the surveyor is to establish a corner 40 chains north of the south-west corner of sec. 1, the dis-

* In the Instructions for 1880, for the first time.

† Gillespie's Land Surveying, page 18, foot-note.

tance as measured by his chain is $40 \div 1.007 = 39.72$. A similar principle applies in city surveying when the land is described as being a certain lot in a particular block of a recorded plat.

If the land to be surveyed is described by metes and bounds, then it is important that the surveyor shall lay off true standard distances.

13. Testing the Tape. In comparing a tape with the standard distance laid off as above, the tape should be under the same conditions as to temperature, tension, etc., that it is to have when in use. The men who are to use the tape should test it that they may better understand the proper tension required.

14. Testing the Chain. The same precautions are to be observed in testing the chain as for the tape. The length of the chain should be considered as the distance from the inside of one handle to the outside of the other (see § 16). In any case, the points considered as the ends of the chain depend upon the manner of setting the pins.

The custom of taking up the wear of the chain by a screw at one end is wrong in principle, although it does not produce much error in practice. The increased length is produced by wear at every joint; taking it up at the ends destroys the equality of the scale of equal parts, and when only a fractional part of the chain is used an error is produced. The better method is to compare frequently the chain with the standard, and apply a correction to the measured distance or computed area.

15. Correcting for Error of Chain. Owing to wear it frequently happens that the chain is not of standard length, and in using a tape in making re-surveys the tape often does not agree with the unit used in laying off the line which is being re-measured. Consequently it is often necessary to correct a measured dis-

tance for error of chain. If in testing the chain by the method of § 10 or § 12 it is found to be four tenths of a foot (or link) too long, then the chain is 1.004 times the true standard; and if the distance as measured is 273.8 ft., the true distance is $273.8 \times 1.004 = 274.9$ ft.

If the area is required, make the computations as though the chain were correct; and then the true area is equal to the computed area multiplied by the square of the length of the chain in terms of the standard. For example, assume that the computed area is 10.875 acres, and that the chain is 1.002 times the standard. Then the true area is $10.875 \times (1.002)^2 = 10.919$ acres. Notice that the length of the chain can be expressed thus: $1 + .002$; and that the square of that quantity can be expressed thus: $(1 + .002)^2 = 1 + 2\,(.002) + (.002)^2$. Since the square of .002 is very small, it may be omitted, and then $(1 + .002)^2 = 1 + 2\,(.002)$ nearly. Hence the correction to the area is equal to the computed area multiplied by twice the correction of the chain. For illustration, the true area for the above example is $10.875 + (10.875 \times 2 \times 0.002) = 10.918$ acres. In this example the results by the two methods differ by less than one thousandth of an acre, and in this class of problems the error will always be inconsiderable. If the chain is, say, .002 too *short*, then its length is expressed by the formula $1 - .002$, and the correction to the area is found as above, except that it must be subtracted.

The student is cautioned to remember that if the chain is too long the distance and the area are too small; and, *vice versa*, if the chain is too short the distance and the area are too large. Very frequently errors are made by applying this correction in the wrong way.

ART. 3. USING THE CHAIN.

16. How to Chain. The following is the general method of procedure in chaining, but is frequently modified as circumstances require. To measure a distance with a chain, two men are required, a fore chain-man and a hind chain-man. The hind chain-man has the more responsible position. They should be provided with eleven marking pins.

Supposing the chain to be tied up, the fore chain-man throws it out in the direction opposite to that in which the chaining is to be done, gives the hind chain-man a pin, takes nine in his left hand and the end of the chain and one pin in his right hand, and draws the chain in the direction of the line. The hind chain-man examines the chain as it passes, to see that there are no kinks or bent links; or, if a tape is used, to see that there are no loops in it.

When the fore chain-man has gone the proper distance, he stops, rests his right elbow on his right knee, and extends his right hand, in which he holds the handle and the pin,* as far as possible from his body, so that the hind chain-man may have an unobstructed view of the pin and the farther end of the line. The fore chain-man is to keep the chain straight and taut, and obey the signals of the hind chain-man.

The hind chain-man places his end of the chain at the point of beginning, and, by placing himself behind the point, with a motion of his arm directs the fore chain-man where to place his pin. For example, if the pin ought to be moved a considerable distance to the

* On a steel tape the handle extends beyond the end graduation, and hence the fore chain-man should grasp the handle of the tape in his *left* hand, rest his left elbow on his left knee, and hold the pin in his right hand, instead of as described above for the chain.

right, the right arm is held far out from that side of his body; if it should be moved only a little, the arm is held nearly vertical. The signal thus indicates both the direction and the amount of motion required. As the pin approaches the proper position, the arm comes more nearly vertical; and when the pin is at the proper place, the hind chain-man calls out "stick." The fore chain-man then brings his left hand to bear on the top of the pin, and forces it *vertically* into the ground.* After the pin is set, he should test it to see that the pin, at the surface of the ground, is just in contact with the front face of the handle. When the position of the pin is satisfactory to the fore chain-man, he calls out "stuck." At this signal the hind chain-man loosens his end of the chain, and both move forward the length of the chain.

The leader should keep his eye steadily on the farther end of the line, so that he may keep near the line. When the follower reaches the pin already set, he calls "halt," and the leader prepares to set a pin. *After* the fore chain-man has placed his pin in line, the follower drops his end of the chain *over* the pin,† at the same time placing his hand on the pin to hold it firm,‡ and calls out "stick." At the reply "stuck," he removes his pin and the work proceeds as before.

When the leader has set his last pin, he calls "tally;" and the hind chain-man comes up, and gives the fore

* When the tape is used, the end of it is held in the left hand and the pin is forced into the ground with the right hand.

† Notice that both men measure to the same side of the pin.

‡ The liability of the back pin's being pulled over in this operation is an objection to this method; but it can be eliminated by having the fore chain-man set the pin on the inside of his handle, while the back chain-man simply brings the outside of his handle against the front side of the pin. However, it is very difficult for the fore chain-man to set the pin on the inside of the handle. Notice that with a properly made steel tape (see last paragraph of § 3) neither of these difficulties will occur.

chain-man the ten pins which he has, both men counting them to be sure that none have been lost. The follower then makes note of the tally, and the work proceeds as before.

When the leader reaches the end of the line, he stops, holds his end of the chain against it and calls "stuck." The follower then comes forward and counts the distance beyond the last pin, being careful to notice on which side of the middle the pin is. Each tally represents ten chains, each pin held by the follower (not including the one in the ground) represents a chain, and the feet just counted make up the total distance. Notice that the pin last set is not counted. It should always remain in the ground until the distance is recorded.

17. Chaining on a Slope. In nearly all cases, it is the horizontal distance which is required. Therefore it is necessary to determine the flattest slope that must be taken into account. The difference between the distance measured on the slope and the true horizontal distance is given very nearly* by the formula

$$d = \frac{p^2}{2b}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

in which d is the difference sought, and p the rise of the slope in any distance b. The quantities d, p, and b all must be in the same unit. For example, if the slope

* To determine the difference between the base and perpendicular of a right-angle triangle, represent the base by b, the hypotenuse by h, and the perpendicular by p. Then we have

$$b = \sqrt{h^2 - p^2} = (h^2 - p^2)^{\frac{1}{2}} = h - \frac{p^2}{2h}, \text{ nearly.}$$

Similarly,

$$h = \sqrt{b^2 + p^2} = (b^2 + p^2)^{\frac{1}{2}} = b + \frac{p^2}{2b}, \text{ nearly.}$$

Therefore we may derive this simple rule: *The difference between the base*

rises 2 feet in 100 feet, then $d = \frac{p^2}{2b} = \frac{4}{200} = 0.02$ ft. Hence, if the slope is 2 in 100, measuring on the slope causes an error of 1 in 5,000. Disregarding a slope of 3 in 100 causes an error of nearly 1 in 2,000, and disregarding a slope of 4 in 100 causes an error of 1 in 1,200.

If the slope is too great to be disregarded, the question then arises as to the best method of eliminating the error. Either of two methods may be employed: (1) one end of the chain may be raised to a level with the other, or (2) the measurement may be made on the slope and a correction applied to the result.

1. To measure by keeping the chain level, the chainmen should be provided with a small plumb and line, so that the end of the chain may be held vertically over the proper point.* If the slope is not very steep, the whole length of the chain can be laid off at once by raising the lower end until the chain is horizontal and transferring the end to the ground with the plumb; but

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and hypotenuse is equal to the square of the perpendicular divided by twice the known side.

The degree of approximation involved in the preceding relation is shown by the following table:

Slope.	Error.
5 vertical to 100 horizontal	1 in 1,000,000
10 " " 100 "	1 " 100,000
20 " " 100 "	1 " 5,000
30 " " 100 "	1 " 1,000
40 " " 100 "	3 " 1,000
50 " " 100 "	6 " 1,000
60 " " 100 "	12 " 1,000
80 " " 100 "	30 " 1,000
100 " " 100 "	57 " 1,000

This approximation is frequently very convenient, and the student should get it well fixed in mind.

* The common practice of dropping a chaining-pin when one end of the chain is elevated only two or three feet, although recommended by many text-books, is objectionable. The error due to the pin's not dropping vertical is probably greater than the error to be eliminated.

when the slope is so steep that the two ends of the chain can not conveniently be brought to the same horizontal line, then only part of the chain can be laid off at a time, in which case some aliquot part should be used. When only part of the chain is used, great care must be taken not to confuse the count.

This method of chaining involves three difficulties: (*a*) keeping the chain horizontal; (*b*) transferring the elevated end of the chain vertically to the ground; and (*c*) making the stretch from the pull equal to the shortening from sag. *a.* It is very difficult to determine a level line, particularly when one is standing at one end of it and looking up or down hill. In chaining up steep slopes it is a great advantage to have a third man, who shall stand at one side of the line and tell when the chain is horizontal. Even then one is liable to be greatly deceived as to the position of a horizontal line. Generally the apparently horizontal line is too nearly parallel with the slope. *b.* It is nearly impossible to hold the plumb-line exactly at the end of the chain and keep the chain both horizontal and sufficiently stretched, and at the same time hold all so steady that the plumb-line will hang still. *c.* The amount of pull required to overcome the shortening from sag can be determined only by trial. To do this, stretch the chain between two points at the same elevation, having it supported its entire length, and remove the supports, noting how strong a pull is required to bring the ends of the chain to the marks again. This should be done by the chain-men themselves, to enable them to judge how hard to pull it when it is off the ground. If only part of the chain is to be laid off at once, this test must be applied to that portion also.

Obviously it is not possible to perform all of the preceding operations with any considerable degree of accuracy; and it is generally more expeditious and also

more accurate to measure on the slope and apply a correction.

2. To compute the difference between the distance on the slope and that on the horizontal requires the determination of the rate of slope. This can be found by estimation, by means of a pocket level or a clinometer, or by estimating the horizontality of a flag-pole and measuring down. The quantity to be subtracted from the distance on the slope is then computed by equation (1), page 16.

Notice that the accuracy of the second method is dependent upon the exactness with which the rate of slope can be determined; but as this is only one of the three difficulties encountered in the first method, we may conclude that the second is the more accurate. It is also the more expeditious.

18. Compensating vs. Cumulative Errors. Before considering the several errors to which chaining is liable, it will be well to notice that in all measuring operations the observer should carefully distinguish between two classes of errors; viz., *compensating* errors, or those which are as likely to be plus as minus, and tend to balance each other; and *cumulative* errors, or those which always have the same sign and affect the final result in the same way. This distinction is very important. The observer should avoid errors which usually occur in a single direction, but he need not always take so great care to avoid errors which are as liable to be negative as positive. An apparently inappreciable but cumulative error may in the course of a series of observations amount to more than a much larger but compensating error. The effect of compensating errors is reduced nearly to zero simply by multiplying the number of observations; but cumulative errors should be avoided entirely, or observations made by which they may be corrected.

The uncertainty in the length of a line due to compensating or accidental errors varies as the square root of the number of units in the line, while the effect of cumulative or constant errors varies directly as the length. The whole amount of the cumulative errors remains uncorrected, while only the square root of the compensating errors is uncompensated. For example, if the chain is 0.1 of a foot too long, a line 25 chains long will be recorded 2.5 feet too short; but if the pin is sometimes set 0.1 of a foot beyond the end of the chain and sometimes the same amount behind, the final error at the end of the line due to this error is probably only $0.1 \sqrt{25}$ or 0.5 foot. If the head chain-man has a fixed habit of setting the pin beyond the end of the chain, then this becomes a cumulative error, and varies as the distance.

This illustrates that in the prosecution of any work it is desirable that the operator should be cognizant of the nature and importance of every source of error. The more thorough and complete his knowledge in this respect, the more readily and accurately will he be able to decide what source of error may be wholly neglected, what may be partially provided against, and what must be carefully avoided or eliminated. This knowledge is conducive both to greater accuracy and to economy of time and effort; for the observer, knowing that what might otherwise have been attended to with considerable care may be neglected, is free to give all his attention to the weakest link in the chain of observations. It enables the observer to correctly proportion his pains to the degree of precision required. A good observer is one who is able to take just care enough to attain the desired accuracy, without wasting time and energy in uselessly perfecting certain parts of the work. He must be able to discover the relative accuracy required in different parts of a complete observation. All this calls

for a clear understanding of the causes of error, and the ability to determine their effect upon the final result.

19. Sources of Error. The sources of error in chaining are (*a*) incorrect length of chain, (*b*) kinking of the chain and bending of the links, (*c*) the chain's not being in a vertical plane, (*d*) unequal tension of the chain, (*e*) expansion and contraction with changes of temperature, (*f*) errors of lining the fore chain-man, (*g*) not placing the pin at the end of the chain, (*h*) drawing the pin by hanging the back handle over it, (*i*) chain not being level, and (*j*) such errors as miscounting tallies or chains, counting from wrong end of chain, making a mistake of 10 in the number of links, reading 18 for 22, 37 for 43, etc. Errors of the last class are much too common, but can be obviated by care and thoughtfulness.

a. An incorrect length of chain is a constant or cumulative error, and may be plus or minus. When one is desirous of attaining the last degree of accuracy, this is the most difficult error to eliminate; and even for the accuracy required in ordinary surveying, it is an important element. Recent experience of the author will illustrate the difference which exists between so-called standards. He had occasion to compare a 100-foot steel tape just from the shop of a reputable manufacturer, which was "guaranteed to be true to U. S. standard," with a 20-foot pole said to have been pronounced correct by the "U. S. A. engineers," and also with a standard derived with inappreciable error from two 2-foot steel rules made by the best tool-makers in the United States. The errors of the intercomparisons were inappreciable. The tape was 0.95 of an inch short by the first, while by the second it was 0.45 of an inch short. A subsequent comparison at the office of the Mississippi River Commission showed the tape to be 0.256 ± 0.07 of an inch short. The above differences were practically independent of temperature correction,

After the chain has once been adjusted to the standard, it should be tested frequently. The common chain has 600 wearing surfaces, and if each wears only 0.01 of an inch, the length is increased 6 inches. A tempered-steel chain with brazed links lengthened half an inch in chaining 70 miles. It is not uncommon to find chains differing 1, 2, or even 3 inches.

A steel tape is liable to have its length changed each time it is repaired, and is also liable to be permanently lengthened by excessive pull in using, and by hammering it to straighten out short bends. Therefore the steel tape, as well as the chain, should be tested occasionally.

For the method of applying a correction to reduce the chain to the true standard, see § 15.

b. Kinking of the chain and bending of the links are sources of plus cumulative errors; *i.e.*, they tend to make the recorded distance too great. Mud and ice in the joints produce an error in the same direction, while the opening of the rings produces an error in the opposite direction. The net result may be either plus or minus. This source of error is easily avoided by substituting a steel tape or wire for the chain.

c. If the chain is not stretched straight, the resulting error is cumulative, and tends to make the recorded distance too great. If the center of a 100-foot chain is 1 foot out of line, the error is 0.02 ft.,* and varies as the square of the error of alignment; that is, if the middle point is half a foot out of line, the error in distance is only 0.005 ft.† The time required to get the chain straight between pins can be greatly lessened by

* See foot-note page 16.

† Let the student show that if a point not the middle of the chain is out a given amount, the error is a little greater than if the center were out a like amount.

the fore chain-man's being careful to walk on the line being measured.

d. The effect of a difference in the pull on the tape or chain is compensating. For the *tape* this error varies directly as the difference of pull, directly as its length, and inversely as its cross-section. To find the elongation of a steel tape, represent its length in inches by L, and its cross-section in square inches by S; then the elongation for a pull of one pound is $\frac{L}{30,000,000S}$. For a 100-foot tape 0.2 inch wide by 0.02 inch thick, this gives an elongation of 0.01 of an inch per pound of pull. Tapes are sometimes made with a spring balance attached at one end, whereby this source of error can be practically eliminated.

The elongation of a *chain* will depend mainly upon the form and size of the rings which connect the several links together, and can be determined only by trial with a spring balance.

e. The error due to expansion and contraction is generally cumulative, and may be either plus or minus. Tapes are usually made standard at 60° Fahr. For a 100-foot tape a change of 30° F. makes a difference of one fourth of an inch.* By remembering this relation it is easy to determine the tape correction at any temperature. For example, if the tape is true at 60°, then at 90° the tape will be 0.02 ft. (one quarter of an inch) too long; *i.e.*, at 90° the tape is 1.0002 times the standard length. At 30° the length of the tape will be 0.9998 (= 1 − .0002) times the standard. In ordinary work it is only the extremes of temperature that will cause the expansion and contraction of the tape to

* The co-efficient of expansion varies with the kind and quality of the steel, but closely approximates 0.0000065 per unit of length per 1° F.

be appreciable; but in accurate work this is one of the chief sources of error, and one very difficult to eliminate. Tapes intended for city surveying and other accurate work sometimes have a thermometer attached, by means of which to determine the corrections for temperature; but even this is not a perfect remedy, since when the sun is shining the temperature of the tape is often quite a good deal higher than that of the atmosphere. There are several other, but minor, difficulties to be overcome in eliminating this error.

f. The error due to not setting the fore pin exactly in line is generally overestimated in proportion to the other errors. An undue amount of time is therefore given to lining in the fore chain-man. With a 100-foot chain, if the pin is 1 foot out of line, the error is 0.005 ft. (one sixteenth of an inch). The error varies directly as the square of the error of alignment and inversely as the length of the chain. Consequently the longer the tape the less the error from this cause, and the greater the speed. The error is cumulative and plus.

Vertical inequalities in the ground produce errors similar to those in aligning the fore chain-man, and unfortunately in many cases this element limits the degree of accuracy attainable. Supporting the tape by hand and plumbing down is of very little advantage. This source of error can be eliminated by suspending the tape between fixed supports. The attempt has frequently been made to introduce such devices for city surveying, but they have not met with general favor. They are complicated, cumbersome, expensive to operate, and contribute little, if any, to accuracy. The chief source of error in all such devices that the writer has seen is in plumbing down from the end of the tape. The effect of the wind upon the sag is

another important source of error. Such devices are appropriate for the measurement of geodetic base lines. For an illustrated account of a simple, but accurate, method of using a suspended tape, see Annual Report of the Missouri Commission for 1886—Executive Document No. 28, 49th Congress, 2d Session,—pp. 31–35.

g. Many chain-men hold the pin, while setting it, in such a manner that the point enters the ground considerably in front of the end of the chain; then when the rear end is brought forward it is laid on the ground, thus introducing considerable error. This error may occur with a tape, but is usually in the opposite direction. The remedy is obvious.

The very general practice of placing the forward handle of the chain against one side of the pin and the back handle against the other side, can not be considered precise. The same side of the pin should be used both times; then the length of the chain is from outside of one handle to the inside of the other. A similar criticism is applicable to the way many steel tapes are used (see the last paragraph of § 3).

h. The error due to pulling the pin over after it is set is usually cumulative, and may be either plus or minus. It may be entirely eliminated by care. The back handle should not be dropped over the pin until the chain is in position preparatory to laying off the distance. With a properly made tape (see the last paragraph of § 3) this particular source of error can be entirely eliminated; but even then the rear chain-man must be careful that he does not push the back pin over when holding the tape against it.

i. The amount of error due to the chain's not being horizontal and the method of reducing the resulting error have already been discussed in § 17.

20. Let us see if we can determine the final error due to an assumed value for each of the above sources

of error. Notice that, under ordinary conditions, all the errors are cumulative except d and g. If the line to be measured is n chains long,

the final error =

$$n(\pm a \pm b + c \pm e + f \pm h \pm i) + \sqrt{n}\,(\pm d \pm g). \quad (1)$$

According to the theory of probability, equation (1) becomes,

the probable final error =

$$\sqrt{n^2(a^2 + b^2 + c^2 + e^2 + f^2 + h^2 + i^2) + n(d^2 + g^2)}. \quad (2)$$

To illustrate the method of using this formula, assume that a line 1,000 feet long is to be measured with a steel tape. Assume that the several partial errors are as follows: $a = 0.01$; $b = 0.0$; $c = 0.01$; $d = 0.02$; $e = 0.01$; $f = 0.005$; $g = 0.01$; $h = 0.0$; $i = 0.01$. Substituting these values in equation (2), and solving,

$$\textit{the probable final error} = 0.22 \text{ ft} \quad . \quad . \quad . \quad (3)$$

The student should carefully consider the degree of care necessary to reduce the several errors to the values assumed above. Are the above values correct relatively? *i.e.*, do they represent equal care in the several operations? Are the errors correctly classified as cumulative and compensating?

Notice that if several measures of a line made with the same tape are to be compared, the error a should not be included in the above equations. Notice also, that if the reduction for grade is computed by the second method of § 17 (page 19), the error i should not be included. Notice again, that if the same men measure the line with the same appliances under the same conditions, the probable error deduced from equation (2)

should be greater than that deduced from the measurements of the distance, for the latter involves only the variation in the errors, while the former depends upon the errors themselves. See Problem No. 1, Appendix IV.

21. Limits of Precision. It is very desirable that each engineer should know the uncertainty of his ordinary work. If this could be definitely stated for each method of measuring and for the different kinds of ground, it would be very instructive; but an engineer can learn only by experience the amount of care and time required to attain any particular degree of accuracy. The labor required increases more rapidly than the degree of precision attained, and the more accurate the work the greater the difference.

A few words are needed as to the difference between real and apparent errors. If a line is twice measured with the same chain by the same men under the same conditions, the difference between the two measurements represents the difference between the accidental errors each time, and does not show the real error in the observed length. Any constant or cumulative error, as an incorrect length of the chain, would be the same in each. This distinction is very important in discussing the errors of any series of observations.

22. The following data are given to assist the student in forming his standard of good work. Unfortunately, there is very little on this subject to be found in engineering literature.

Burt* found in the early surveys of the U. S. public lands, that for common timber-land "with two sets of chainmen instructed alike in the proper manner of keeping their chain level and straight on the line, and of setting the tally pins plumb, as well as holding the

* Key to the Solar Compass, p. 35.

ends of the chain to them, the average difference was 1 in 500, and sometimes 1 in 220, and under the most favorable conditions it was 1 in 1,600."

At present the standard for good chaining in surveying the public lands of Canada is a difference of 1 in 5,300 for two sets of men chaining the same line under the same conditions.* "Experience with very careful men shows that chaining on level prairie is about two links to a mile (1 in 4,000) longer than over hilly and broken prairie, or over windfall and brush in the woods." *

The author's students, in ordinary class work in land surveying, measure lines from 200 to 1,000 feet long, with a 66-foot chain, and repeat with a difference of 1 in 15,000 to 1 in 20,000 for the same men; for different men on different days with different chains (compared, however, with the same standard), the maximum difference is 1 in 800 to 1 in 1,000, the average difference being 1 in 3,000 to 1 in 4,000. The ground is favorable, but the work is done with the ordinary expedition of actual practice.† The same students, after a term's practice, with a 100-foot steel tape measure lines 500 to 1,000 feet long and repeat with a maximum probable error for the surface distance of 1 in 40,000, and an average of 1 in 103,000 for the same men; and for different men on different days with different tapes (compared with the same standard), the maximum error is 1 in 5,000, and the average error 1

* Report of the Proceedings of the Association of Dominion Land Surveyors for 1890, p. 58.

† All values of the degree of precision credited in this volume to the author's students are results attained in the ordinary class work of the first or second term's field practice. Usually the value given is the mean for the whole class. The results are obtained from a specially prepared area the dimensions of which are accurately known. The lines vary from 30 to 800 feet, and the areas from 0.5 to 7 acres, the longer lines and the larger areas occurring more frequently in the problems.

in 14,300. For the purpose of this record, eleven pairs of chainmen on different days measured a line 1,000 feet long with a probable error for a single measurement of the surface distance of 1 in 7,800; omitting one result, the probable error is 1 in 10,800. The difference of elevation was 17 feet, and the reduction for grade was 0.288, as determined from an accurate profile of the line; and the probable error of a single estimate of the correction for level (by equation (1), page 16, the student having had no experience in leveling) was 1 in 11,600. The probable error of a single determination of the horizontal distance was 1 in 6,500, and omitting one result the error was 1 in 7,400.

Seventy miles of the finished track of the Illinois Central R. R. was measured in 1876 with a 100-foot steel chain, and re-measured in 1885 with a 100-foot steel tape, with a difference, after correcting for the difference of standard and applying a correction for wear of the chain, on the average for the different miles of 1 in 2,360.

On the Atchison, Topeka and Santa Fé R. R., through Western Kansas, the difference between the preliminary survey and the government land survey averaged about 1 in 1,000; and the difference between the preliminary survey and the location was about 1 in 2,500.

The U. S. army engineers in re-measuring the Union and Central Pacific railroads with steel chains found the error of their own work to average 1 in 23,600, the maximum being 1 in 10,000, as determined by retracing distances varying from 2,000 to 26,000.*

On the U. S. Lake Survey† base lines for topographical surveys were measured with a chain 20 meters

* House Executive Document, No. 37, 2d Session, 44th Congress, pp. 14, 21, 37.

† Report of Chief of Engineers U. S. A., 1876, part III, p. 9.

long, which differed from the ordinary chains only in being made of heavier wire and having links 20 inches long. In 15 lines, the maximum error was 1 in 5,700, the mean being 1 in 17,500, and the minimum 1 in 45,100.

In connection with the U. S. Lake Survey work, a line 12 miles long was measured on the railroad track with a wire 150 feet long, at the rate of about 10 miles per day, with a difference between two measurements of 1 in 32,000.

The U. S. Coast and Geodetic Survey Report for 1882, page 191, contains an account of the preliminary measurement of two base lines with an iron wire $\frac{1}{8}$ of an inch in diameter, 200 feet long, in which the difference, as compared with the measurement of the geodetic base apparatus, was 1 in 30,000 and 1 in 28,000.

CHAPTER II.

TRIPOD, LEVELING SCREWS, AND PLUMB-BOB.

ART. 1. THE TRIPOD.

23. CONSTRUCTION. The manner of connecting the leg with the head needs attention. The leg should not be placed between two lugs or ears fastened to the plate, for in case the leg wears or shrinks there is no adequate means of making it fit. Drawing the ears together by means of the screws through the top of the leg bends the plate, and even then only partially remedies the evil. An excellent form is that in which the leg is made of two pieces that bear upon opposite sides of a lug cast upon the plate. Sometimes the leg is in one piece with a slot at the top, which is also very good. Another good form is that in which the leg is not open at the top, but bears upon only one side of the lug. In the three forms last mentioned any looseness is taken up by a thumb-nut. Sometimes the leg is inserted in a metal cap, which is then fastened to the tripod head. This is better than putting the leg directly between two metal ears, but is inferior to all the other forms. No instrument can stand firmly if there is any looseness in fitting of legs or shoes.

The best instrument makers construct the tripod legs of two pieces braced together, or of a solid piece cut out in such a way as to lighten it without materially affecting its strength.

Some instrument manufacturers make a tripod with

legs that may be made longer or shorter as circumstances require. In surveying in a mine or tunnel this is a great convenience, if not a necessity.

24. Setting the Tripod. In setting the tripod notice that to alter the position of the plumb-line the legs must be swung on their pivots, but not sidewise; and also, that to get the plate level the legs must be swung sidewise, but not on their pivots. These two points, though seemingly trivial in themselves, are worth remembering as a means of saving time and labor, and also of preventing unnecessary wear and strain on the leveling screws, which last, owing to faulty construction of the leveling appliances, is frequently very important.

Attention to the following principle also will save much time and hard labor in setting the tripod. Let the lines radiating from *c*, Fig. 4, represent lines joining the feet of the tripod legs with the point over which the plumb-bob is to be placed, and *b* the position of the plumb-bob. It is desired to move the plumb-bob from *b* to *c*. Press the leg ~~1~~ 2 into the ground until the bob swings to *a* in the line 3*c* prolonged, and then force leg 3 into the ground until the bob swings to *c*. In general, press one of the legs into the ground until the plumb-bob swings to the side of the point opposite one of the other legs; then press that leg in until the plumb-bob arrives at the center.

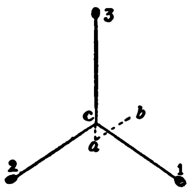

Fig. 4.

The tripod should be set firmly, but a great deal of time and effort is frequently wasted in forcing the legs into the ground needlessly. Setting the tripod is an operation that must be repeated many times, and the beginner should learn to do it quickly and easily.

ART. 2. LEVELING SCREWS.

25. Most instruments are provided with four leveling or foot screws, while others have only three. The instrument can be leveled more quickly with three screws than with four, since by using both hands the instrument can be leveled in both directions at the same time. Three screws are also more sensitive, and are less liable to strain and damage the instrument than four—an important consideration with many instruments, owing to faulty construction of the leveling appliances. Although the best American instruments, and many if not most European ones, have only three foot screws, nearly all American engineering instruments have four. Doubtless this is partly due to custom, and partly to the fact that for mechanical reasons it is a little easier to arrange the shifting plate of the transit (§ 111) with four than with three foot screws.

Whatever the number, there should be no looseness between the screw and the nut. This looseness is particularly objectionable in a leveling instrument or in a transit used to measure vertical angles. To insure steadiness the leveling screws should work in the arms of a solid star-shaped casting, instead of in a thin round plate into which the nuts are simply stuck—as is usual. The end of the arm should be split and provided with a clamp screw by which to adjust for wear.

26. A very serious defect in many instruments with four leveling screws is that the lower ends of the screws are above the center of the ball-and-socket joint which fastens the upper part of the instrument to the tripod (see Fig. 24, page 95). Then when one pair of screws is being used to level the plate, the upper part of the instrument must revolve about a line

parallel to, but below, a line joining the feet of the pair of screws not in use; therefore the instrument can not revolve without causing the screws not in use to bind, and it can turn only by causing the feet of these screws to slip on the lower plate. Besides the annoyance in leveling the instrument, this binding tends to bend the screws and warp the plates; and the slipping defaces the instrument by cutting spherical holes in the lower plate. Placing the feet of the screws in small cups prevents the holes in the upper face of the lower plate, but increases the objections in the other and more important respects. Providing the foot of the leveling screw with a ball-and-socket joint is no improvement over the simple cup, except to prevent the cup (socket) from getting lost.

The above defect may be remedied by bringing the center of the ball-and-socket joint into the plane of the feet of the leveling screws. Since with three leveling screws the instrument can not be attached to the tripod by a ball-and-socket joint, this defect can not exist in that form of construction. With three leveling screws the instrument is fastened to the tripod by a spiral spring.

Since nearly all instruments have the above defect, it is very necessary that they should be leveled approximately by manipulating the tripod legs as already described (§ 24).

27. Some instruments are provided with an arrangement for approximately leveling the instrument very quickly without manipulating the foot screws, of which there are several forms on the market. A quick-leveling device is specially suitable for leveling instruments, but by using a little care in setting the tripod it can be dispensed with easily. All such additions are an advantage in the greater convenience they afford, but a disadvantage in the increased weight, complexity, and cost involved.

ART. 3. PLUMB-BOB.

28. The string employed should be small but strong. The woven ones are best, since they do not twist or untwist. The string should be connected to the instrument by a hook, so as to be easily attached and detached ; but it should be suspended in such a way that the vertical of the plumb-line will always pass through the center of the instrument, however much the lower plate may be out of the horizontal.

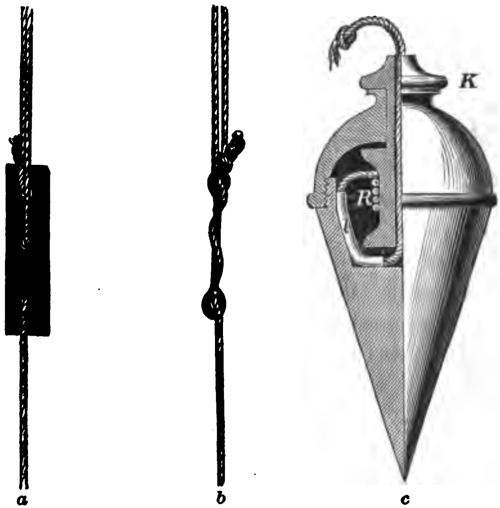

FIG. 5.—METHODS OF ADJUSTING LENGTH OF PLUMB-LINE.

To allow for an adjustment in the length of the plumb-line, the free end of the line may be passed through the attachment on the instrument and then knotted around the suspended portion, the adjustment being obtained by sliding the knot up and down the string. A better method, particularly with a small string, is to pass it through a small strip of wood, leather, or metal, as shown at *a*, Fig. 5; or, the end may be fastened

to a wire as shown at *b*, Fig. 5. The latter is the better, since less surface is exposed to the action of the wind. The adjustable plumb-bob shown at *c*, Fig. 5, is still better but more expensive. This plummet has a concealed reel *R*, around which the string is wound by turning the milled head *K*. The friction of the cord through the guide *l* holds the bob at any desired point on the line. Another form of the last consists of a short spool on top of the plumb-bob, around which the plumb-line is wound. The spool is turned by hand, and held in any position by friction.

The body of the plumb-bob should be a long slender cone, rather than a short thick one, so that the point may be seen without stooping. The tip of the bob should be of steel, for durability. In case the pointed plumb-bob is lost, and only a rough piece of some heavy substance can be had, the instrument may still be plumbed down accurately, by holding a second plumb-line before the eye in such a position that the eye shall be in the same plane with the two lines; then, without moving the eye, have an assistant mark a line under the instrument in this plane. Repeat the operation at 90° from the first position. The intersection of these two lines is the desired point.

Owing to defective casting, the plumb-bob may not hang vertically, although of true form and apparently solid. In very accurate work this might cause error. A rough method of testing this is to hold the string in the hand and twist it a little, and while the string is untwisting, lower the point into a basin of water. If the weight is not truly distributed and consequently the plummet not true, the eccentric motion of the point will scatter the water.

If it is desired to plumb down from a high point, instead of using a plumb-line, which is liable to be disturbed by the wind, it is better to use a transit according to a method to be explained farther on (§ 125).

CHAPTER III.

MAGNETIC COMPASS.

ART. 1. CONSTRUCTION.

29. THE ordinary form of the magnetic compass is shown in Fig. 6.

The value of the magnetic compass as an engineering instrument depends chiefly upon (1) the delicacy of the needle, and (2) the constancy with which it assumes the direction of the magnetic meridian. The first depends upon the ease with which the needle turns on the pivot, and the second upon the intensity of the directive force. To satisfy the first condition, there is attached to the center of the needle an agate cup to receive the point on which it turns; and the pivot is made of the hardest steel ground to a smooth, round, sharp point. The second condition is satisfied by making the needle of shear steel and strongly magnetizing it.

The needle should be of such a length that it shall just clear the graduation, otherwise there will be difficulty in reading. Beyond a certain length (about 5 or 6 inches) no additional power is gained by increasing the length of the needle, owing to the formation of secondary poles in long needles. It should not come to rest too quickly; for if it does, it indicates either that the needle is weakly magnetized or that the friction on the pivot is great. The needle should be so sensitive that when it is drawn to one side by the attraction of a piece

of iron, it will settle to the same reading several times in succession.

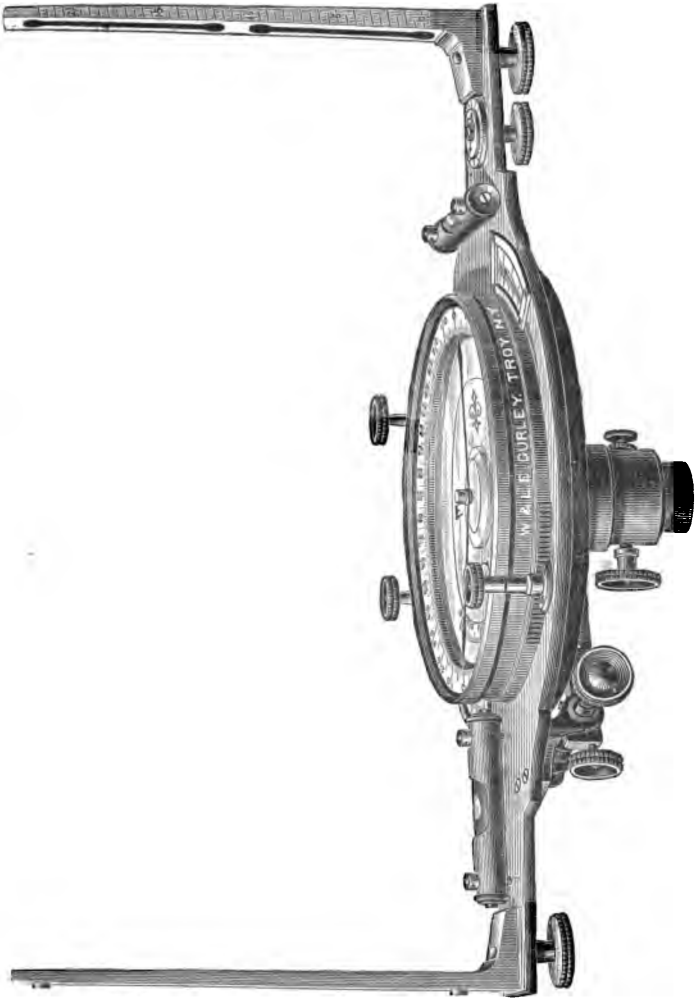

FIG. 6.—MAGNETIC COMPASS.

A compass is usually provided with a vernier (Chapter V) for setting off the magnetic declination. This is

indispensable in re-surveying land originally laid out with reference to the magnetic meridian; but it is not requisite where the land is laid out according to the U. S. public land system.

Ordinarily compasses have upon the edges of the sights graduations such that angles of elevation and depression can be measured. This arrangement is particularly valuable for measuring the angle of slopes in applying the correction to reduce slope measurements to the horizontal distance.

30. Extras. Compasses are sometimes provided with a graduation for reading angles independently of the needle. This is an important addition; but the degree of accuracy attainable is limited by the precision of sighting, which with the open sights is not very great.

Magnetic compasses are sometimes provided with a telescope instead of sights. This reduces the error of sighting, and increases the length of sight; but on the whole the advantage is not very great. The chief merits of the compass are cheapness, portability, and rapidity and facility with which it may be used. It never can be an instrument of great accuracy; and hence the telescope adds cost, weight, and complication, without a compensating accuracy. If greater accuracy is desired than that attainable with the ordinary magnetic compass, use the transit.

Some British instruments have the graduation on a ring attached to the needle and moving with it, the angles being read by a point projecting from the compass box. There is no advantage, but great disadvantage, in this arrangement, owing to increased weight on the pivot.

31. Prismatic Compass. The peculiarity of this instrument is that a triangular glass prism is substituted for one of the sights. In looking through the prism the distant point and the graduation of the compass are

visible at the same time. The graduation is upon a ring attached to the needle and moving with it, and the bearing is read by a point projecting from the compass-box, under the prism. A mirror is attached to the sight in such a way as to reflect into the prismatic eye-piece points above or below the horizontal plane of the instrument. The prism is sometimes worked

FIG. 7.—PRISMATIC COMPASS.

convex on the two faces at right angles to each other, so as to magnify the graduation, and is moved up and down to focus it (see Fig. 7). The instrument is sometimes provided with colored glass shades for observing the sun.

Prismatic compasses vary from 2½ to 6 inches in diameter. They are generally held in the hand, although sometimes mounted upon a Jacob's staff. The prismatic compass is used in preliminary reconnoissances, in clearing out lines, in "filling in" in topographical surveying, etc.

ART. 2. TESTS OF THE COMPASS.

32. THE NEEDLE. *The needle should be strongly magnetized.* If the needle is not strongly magnetized, the directive force will not overcome the friction on the pivot, and hence the needle will not take its proper direction. Needles may be re-magnetized with bar magnets, as described in the text-books on physics, but such methods are tedious, and give unsatisfactory results. A needle may be quickly and thoroughly saturated with magnetism by putting it into the magnetic field of an electric light or electric railway dynamo. In doing this there is a liability of reversing the poles; and therefore, after having magnetized the needle, place it in the compass box or rest it upon a pin held in the hand, and notice if the poles have been reversed. If so, put the needle back into the magnetic field the other end about. A good needle loses its magnetism very slowly if properly cared for (§ 43).

33. *The magnetic axis of the needle should coincide with the axis of figure or line connecting the two ends.* An error in this respect affects only the declination to be set off; therefore it produces no error, provided the declination is determined by observing with the instrument on a true meridian. The amount of this error could be determined by inverting the needle on the cap and reading in both positions. "The magnetic axis is liable to have its position changed by shocks, in using and transporting the compass. This is especially true of freshly magnetized needles." For the above reason, as well as for others which will be discussed in Art. 4, it is best to eliminate the above error by observing the declination with each instrument used.

34. METAL OF COMPASS-BOX. *The metal of the compass-box should contain no magnetic substance.* If the metal is

not pure, the effect on the needle will be different for different positions of the box, and consequently cause error. Iron is liable to get mixed with the brass in casting. To detect impure metal, set the sights upon some well-defined object and read the needle. Move the vernier (Chapter V), say 10°, sight on the object, and note whether the needle has changed the same amount. This operation should be repeated all the way around the circle. If the vernier and the needle change like amounts, the metal of the box is pure.

When reading the needle be sure that nothing which will affect it is carried on the person. Watch-chains, buttons, stiffening-wire in the hat rim, and iron rivets in the frame of the magnifier for reading the vernier, etc., may produce error.

35. Sights. *The line of sight should pass through the center of graduation.* If this condition is not satisfied, the angles will be sighted from one point and measured at another. The error will vary inversely as the length of sight. There is no probability that the error will be sufficient to affect the work done. However, it could be tested by stretching a fine thread through the sights and observing whether it covers divisions on the limb 180° apart.

36. Zero of Vernier. *The zero of the vernier should coincide with the line of sights.* If it does not, the proper declination will not be set off, and an error will be produced in the bearings. To test this condition, set the vernier at zero, stretch a fine thread through the sights, and observe whether it covers the zeros of the graduation. If it does not, it will produce no error, *provided* the declination is determined with the instrument by an observation on a true meridian.

This condition should be satisfied for the compass on a transit, but there is no rigorous way of making the test. Hence it is more important with the transit than

with the simple compass that the declination should be observed with each instrument—compare the first line of Table I, page 50, with the other line.

ART. 3. ADJUSTMENT OF THE COMPASS.

37. The study of the adjustments of engineering instruments is a very important part of our subject, as no one is competent to handle an instrument who is not able to determine when it is in adjustment, to adjust it in every particular, and to discuss the effect of any error of adjustment upon the work in hand. Instruments should be examined frequently, for the adjustments, though properly made, are liable to become deranged.

Nearly all of the adjustments of engineering instruments consist in placing certain parts either perpendicular or parallel to each other. The usual method in making the various adjustments is that of reversions, which doubles all errors and places them on opposite sides, so that if there is no difference after reversal, there is no error. If there is a difference, the mean of the two positions is the true one.

As far as possible, the adjustments should be made in such a manner as to be independent of each other. The student should consider carefully the principles involved, and also determine the effect upon subsequent work of an error in the adjustment. The different methods of construction may modify the manner of making the adjustments of any instrument, but the same general principles apply in all cases; and hence the great importance of understanding the principle involved, instead of performing the adjustments by routine simply.

38. LEVELS. *The axes of the levels should be perpendicular to the vertical axis of the instrument.* The chief reason for

demanding this condition is that the needle may play freely when the bubbles are in the center, whatever the direction of the line of sight.

To make this adjustment, bring the bubble to the middle of the tube (any other point would do equally well, but the center is most convenient) by turning the instrument on the ball-and-socket joint. Turn the instrument half-way round; then, if the bubble does not stand in the middle, correct one half of the difference by means of the screws at the end of the level tube, and the other half by turning the instrument on the ball-and-socket joint. This operation should be repeated until the bubble will remain stationary in the tube during a complete revolution of the instrument. If the levels are much in error, it is best to adjust each approximately before completing the adjustment of either. Of course, before pronouncing an adjustment in error, it should be carefully tested.

39. The above adjustment of the levels is sometimes erroneously called an adjustment "to cause the circle to be horizontal in every position." The adjustment of the levels does not in any way involve the horizontality of the plate. "Leveling the instrument" is really bringing the vertical axis vertical. It is assumed that the plate is perpendicular to the vertical axis, and that consequently when the latter is vertical the former is horizontal. A method of testing the perpendicularity of limb and axis will be given in the chapter on the transit. For the compass the above assumption will produce no appreciable error.

The compass should not be leveled by the needle. The instrument should be leveled by the levels, and then the needle should be balanced by sliding the coil of wire, which is around the south half, in or out.

40. SIGHTS. *The sights should be in the plane of the vertical axis.* This is the rigorous requirement, but for the

compass it is sufficiently exact to put the sights in a vertical plane, *i.e.*, in a plane parallel to the plane of the vertical axis. If the slits are not vertical, sighting through the bottom of one and the top of the other will give a different direction from that obtained by sighting through the bottoms or tops of both.

To make this adjustment, bring the vertical axis vertical by the method of the previous adjustment. Remove one sight, and range two points, on the side of a building, in the plane of the remaining sight. The farther the points are apart the better. Reverse the instrument on the vertical axis and bring the bottom of the sight in range with the lower point; if the upper point is then in range with the top of the sight, the sight is in a vertical plane. If the upper point is not in range, correct half the error by filing off one side of the bottom of the sight, or by putting paper under the other side. Adjust the other sight in the same way. The plane of the sights is now vertical, but it may not coincide with the plane of the vertical axis. The above method of correcting the error is best, but in case of necessity it may be done by holding both sights and bending the plate as needed. With fair use the sights should not get out of adjustment.

41. Needle. *a. The ends of the needle and the point of the pivot should be in the same horizontal plane.* This condition is required so that the ends of the needle may remain stationary even though the needle may roll or quiver on the pivot. To make this adjustment, bend the ends of the needle up until they remain steady even though the middle of the needle may swing.

b. The ends of the needle and its center should be in the same vertical plane. If this condition is not satisfied, the readings of the two ends of the needle will not agree. To test it, read both ends, and revolve the box until the north end reads what the south end did. If there is a

difference between the second reading of the south end and the first reading of the north end, correct half of the difference by bending the needle.

42. Center-pin. *The pivot on which the needle swings should be in the center of the graduated circle.* If the pivot is not in the center, either the needle will strike the graduation and not turn freely, or the end will be so far from the graduation that it can not be read precisely. Furthermore, if this condition is not satisfied, a bearing read from the north end of the needle will not agree with the value obtained from the south end. To test this adjustment read one end of the needle in four positions 90° apart; if the readings of the other end differ by 90°, the pivot is in the center. If the readings of the second end do not differ by 90°, read both ends of the needle again, move the box until one end has passed over 90° and then bend the pivot until the other end shall have passed over 90°. Turn the box about 90° and repeat the last operation. Test the work by trying points half-way between those previously used.

This adjustment can be made, but less elegantly, by using half the length of the needle to measure the distance from the pivot to the graduation.

This adjustment should be examined every time the pivot is taken out. Notice that the adjustment of the needle and of the center-pin are entirely independent, text-books to the contrary notwithstanding.

Art. 4. Using the Compass.

43. Care. It is quite important that the engineer should understand the means by which he can prolong the usefulness of his instrument. In the compass the main thing is to avoid dulling or breaking off the fine point of the pivot; consequently never jolt or carry

the compass without being sure that the needle is lifted off the pin. Remember that the harder and more perfect the point, the more liable it is to injury. In lowering the needle do not let it fall upon the pivot. To prevent unnecessary wear, check the vibrations of the needle on letting it down, by lifting it off the point a little; or, better, turn the needle in about the proper direction before letting it down.

When the pivot becomes dulled it can be sharpened on an oil-stone. Care should be taken to obtain a conical and not a pyramidal point. The sharpness of a needle is easily ascertained by sliding the thumb-nail over the point, at an angle of about 30° to it. If the point sticks and holds the nail, it is sharp; if it glides upon it, it is dull. Unnecessary grinding of the pivot should be avoided, for if it becomes much shortened, the ends of the needle will come below the graduation, thereby producing parallax in reading. This parallax could be avoided by bending up the ends of the needle, but this would destroy the condition that the two ends of the needle and the pivot should be in the same horizontal plane (see *a*, § 41).

Never allow the needle to be played with by attracting it with a knife, piece of iron, etc.; for every passage of a piece of steel or iron removes a portion of the magnetism. When the compass is not in use, it is best to let the needle assume its normal position in the magnetic meridian, for in this position it will longer retain and even increase its polarity. After the needle has assumed this position it should be raised against the glass.

44. Practical Hints. If the needle is sluggish in its movements and settles quickly, either it has lost its magnetic force or it has a blunt pivot; and in either case it is likely to settle considerably out of its true position. The longer a needle is in settling, the more

accurate will be its final position. It can be quickly brought very near to its true position by checking its motion by means of the lifting screw; but in its final settlement it must be left free.

The glass cover may become electrified from friction and attract the needle. This electricity can be discharged by touching the glass with a wet finger, or by breathing upon it.

45. Sources of Error.* The errors of compass work may be classified as (1) local attraction, (2) instrumental errors, and (3) observational errors.

46. Local Attraction. A common method of taking the bearings with the compass is to set it at each corner or station in succession, and take the forward bearing. If there is any local attraction of the needle, *i.e.*, if the needle is deflected from its normal direction by iron, etc., the bearings will be incorrect. A method of eliminating all local attraction is described in Appendix I, and hence this source of error need not be discussed here.

47. Instrumental Errors. Errors of this class are due either to imperfect adjustment of the instrument or to sluggishness of the needle. It is a good rule to adjust an instrument carefully in all particulars, and then use it in such a way as to eliminate any residual errors of adjustment; or, in other words, adjust it carefully and then use it as though it were not in adjustment.

Guard against errors of coincidence of magnetic and geometrical axes of the needle (§ 33), and also against errors in straightness of needle (§ 41), by reading always the same end of the needle.

48. In work involving the magnetic declination, to guard against the possibility (1) that the magnetic axis of the needle may not coincide with its geometric axis,

* See discussion of Cumulative *vs.* Compensating Error, § 18.

(2) that the zero of the vernier may not coincide with the line of sight, and (3) that the needle is not straight, determine the declination by setting the compass upon a true meridian, sighting along it, and then moving the vernier until the needle reads zero. This also provides against errors in charts or tables giving the declination, and also against changes in the declination since the chart was made. This observation should be made about 10 A.M. or 6 P.M., as then the effect of the daily variation is generally nearly zero.

It is very important that the three sources of error just mentioned should be eliminated. For one or the other or all of these reasons, the bearing of a line as read from several instruments at the same time and place, by the same person, will often differ considerably. In one instance,* the magnetic declination as read from five instruments by four men at the same time and place differed 20 minutes. As nearly as can be determined from so few observations, the probable error (see Appendix III) of a single determination of the declination is $5\frac{1}{4}$ minutes. In another instance,* the readings of four men from four instruments read at the same time and place differed 31 minutes, with a probable error for each observation of 9.1 minutes. The following is from a prominent instrument maker:† "I made six needles as near alike as I could. I then set up a compass and directed it toward a certain fixed object. Then I placed these needles, made at the same time of the same kind of material, on the center-pin one after another. Three of these gave the same reading, but the other three varied from 5 to 10 minutes. The result was indeed surprising, for I had taken great pains to have the needles all alike."

* Report of the Ohio Society of Surveyors and Engineers for 1884, p. 71.
† Id., p. 74.

In the course of ordinary class instruction, the author had his class in land-surveying, consisting of eleven members, determine the magnetic declination by observing upon a true meridian with six instruments at the same time, each man observing with each instrument. At the time of making the reading no one knew what the others had read. The instruments were in good adjustment in every particular. The following table shows the mean declination for each instrument, and also the probable error* of observation.

TABLE I.

DIFFERENCE IN MAGNETIC DECLINATIONS OBTAINED WITH DIFFERENT INSTRUMENTS.

No.	Instrument.	Length of Needle.	Mean Declination.	Probable Error of a Single Observation.
1	Black transit.........	5 inch	5° 42′	1′.5
2	Yellow "	5 "	4° 27′	2′.8
3	Mining "	3 "	4° 33′	1′.6
4	Black compass.......	6 "	4° 40′	1′.6
5	Yellow "	6 "	4° 51′	1′.2
6	Old "	5 "	4° 37′	1′.2
	Mean.............	....	4° 48⅛′	1′.7

Since each value of the declination in the above table is the result of eleven observations, the error of observation is practically eliminated, and hence the difference in the declinations is due almost solely to a difference in the instruments. The above results show that the declination determined by a single observation has a probable error (§ 2 of Appendix III) of 1.7 minutes due

* Since the readings were made, as a rule, only to the nearest 5 minutes, it is not strictly correct to compute the probable error of such results. The probable errors are given in this case to show that the difference in the declination was due to the instruments, and not to the reading.

to the observer and 13.4 minutes due to the instrument; or a total probable error of 13.5 minutes. In other words, if both instruments are in good working condition and skilfully used, we may expect a difference in the bearing of the line due to differences in the instruments, of 13.4 minutes (21 feet in a mile), as read by two instruments at the same time and place. Notice that the greatest difference between the declinations is 1° 15′ (120 feet in a mile). This source of error is generally disregarded, and frequently it is impossible to do otherwise; but it should be continually borne in mind as a possible source of error.

49. Observational Errors. These may be divided into (1) errors in sighting and (2) errors in reading. There is a possibility of error owing to the compass or flag-pole's not being set at exactly the right point; but this can only occur on short sights, and with reasonable care will not occur at all.

1. To eliminate any residual errors due to the plate's not being level (§ 38) or to the sights' not being perpendicular to the plate (§ 40), sight through the top or bottom of both sights. To insure the slits' being vertical, give the most care to the bubble perpendicular to the line of sight. Care must be taken that the flag-pole is seen through both slits, and not through one slit and one hole. The flag-pole should be set vertical; but, as a precaution, sight as low on it as possible.

2. After the needle has come to rest it should be tapped gently to destroy the effect of any adhesion to the pivot. In reading the needle beware of magnetic substances on the person as wire in the hat-brim, watch-chains, rivets in the handle of the magnifying-glass, etc., which may affect the needle. The most common errors made in practice are such as reading 28° for 32°, 30½° for 29½°, etc.; and reading N. for S., etc. The remedy is obvious.

The common practice is to read the bearings to the nearest quarter-degree. The accuracy of the work can be appreciably increased by estimating the fraction of a degree to the nearest five minutes. Since compasses are ordinarily graduated to half-degrees, this necessitates the estimation of sixths of a division. With a little thoughtful attention this can be done with considerable precision, and the increased accuracy is well worth the extra time required.

It is sometimes recommended that the vernier be used to read the fractions of a degree; but a trial will show that this is no advantage. The reasons are obvious. It has also been proposed to place a light paper vernier on the south end of the needle. Concerning all such devices remember that if the needle were read without any error at all, the magnetic compass would not be an instrument of any considerable precision.

50. Limits of Precision. With a compass graduated to half-degrees, the angles can be estimated to the nearest 5 minutes, in which case the maximum error of the bearing should not exceed 10 minutes. The average error of a bearing by the author's students, as deduced from the errors of closing the angles around a field (§ 8 of Appendix I), the average length of sight being about 300 feet, is 3 minutes.* The average error for ten selected men, using a highly magnetized needle and a sharp pivot, was 2.5 minutes. The same men read, for the purpose of this record, the bearing of a flag-pole at 100, 200, and 300 paces, with average errors of 4, 5, and 5.2 minutes, respectively, the sun and wind being in the observer's face while sighting.

51. The error of an area found by compass surveying is made up of the errors of chaining, of reading the

* The error per sight is equal to the total angular error in closing a field, divided by the square root of twice the number of sides.

bearings, and of the computations. With proper care, the error of chaining should be much less than the error of the bearings. Since 1 in 57.3 corresponds to 1°, the above maximum error of 10′ in reading an angle corresponds to an error of 1 in 344; and the average error of 3′, as above, corresponds to 1 in 1,146, which is a greater error than that of chaining under the same conditions (see fourth paragraph of § 22). Since the computations are self-checking at nearly every step, there is little probability of any material undetected error in this part of the work; and as the work may be carried to any desired number of decimals, the inaccuracies of the computations can be made much less than those of the field-work. We conclude, therefore, that the accuracy of the area depends mainly upon the accuracy of reading the needle.

The average error in area of eighty problems solved by thirty-two (the best of thirty-six) of the author's students in ordinary class-work * was 1 in 1,530.†

52. Balancing the Latitudes and Departures. An answer can now be given to the following question, which is frequently asked: How great a difference between the sums of the + and – latitudes and departures is admissible?

Let C represent the linear error of closure due to the chaining, P the perimeter of the field, d the distance in which the error of chaining is a unit; then

$$C = \frac{P}{d}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (1)$$

* See foot-note, p. 28.

† For the sake of comparisons, it may be interesting to know that under the same conditions the error of areas obtained with the chain alone was 1 in 1,520; and with a home-made plane table (§ 172) the error was by radiation (§ 182) 1 in 586, by traversing (§ 184) 1 in 826, and by radio-progression (§ 187) 1 in 1,111.

Let A represent the lineal error due to the measurement of the angles; and a the angular error of measuring the angles, *i.e.*, the difference between the last fore-sight and the first back-sight,* in minutes. It can not be known how this error occurred, whether all in one sight, or equally among the sides, or among the sides in proportion to their length; but as the last assumption is most probable, and also as it gives the largest linear error in closing, we will assume the error a to have occurred among the sides in proportion to their lengths. Hence to reduce a to its linear equivalent, we must multiply it by the length of the perimeter ($= P$) and divide it by the distance at which a unit subtends an angle of one minute, or

$$A = a\frac{P}{3,438} = \frac{3aP}{10,000}, \text{ nearly.} \quad . \quad . \quad . \quad (2)$$

Let $E =$ the total error due to chaining and to measuring the angles. Notice that the error E is the hypotenuse of a right-angled triangle of which the differences of the latitudes and departures are the other sides. Hence, if $L =$ the difference of the $+$ and $-$ latitudes, and $D =$ the difference between the $+$ and $-$ departures, $E = \sqrt{D^2 + L^2}$.

By the theory of probabilities we know that

$$E = \sqrt{A^2 + C^2} = \sqrt{\left(\frac{P}{d}\right)^2 + \left(\frac{Pa}{3,438}\right)^2}$$

$$= P\sqrt{\left(\frac{1}{d^2} + \frac{a^2}{12,000,000}\right)}, \text{ nearly.} \quad (3)$$

* See § 8 of Appendix I.

Equating these two values of E, we get

$$D^2 + L^2 = P^2\left(\frac{1}{d^2} + \frac{a^2}{12,000,000}\right). \quad . \quad . \quad . \quad (4)$$

As a rule, equation (4) is the one to be used in practice; but we may simplify the matter a little further by finding a relation between D and L. Assume that the sum of the latitude $= n$ times the sum of the departures. n is easily determined from the computations, or it can be estimated in the field or from the plat with sufficient accuracy. Then, from the theory of probabilities, $L = D\sqrt{n}$. Equation (4) then becomes

$$(1 + n)\,D^2 = P^2\left(\frac{1}{d^2} + \frac{a^2}{12,000,000}\right). \quad . \quad . \quad (5)$$

53. To illustrate the method of applying the preceding formulas, let us assume that in surveying a field whose perimeter is 10 chains the difference between the first back-sight and the last fore-sight was 10 minutes. We will also assume that the conditions were such that we might expect an error in chaining of 1 in 2,000. Equation (4) then becomes

$$D^2 + L^2 = 10^2\left[\left(\frac{1}{2,000}\right)^2 + \frac{10^2}{12,000,000}\right]$$

$$= \frac{1}{40,000} + \frac{1}{1,200} = .00086. \quad . \quad . \quad (6)$$

If the field is twice as long north and south as east and west, then $n = 2$. Substituting in equation (6) and reducing, we get $3\,D^2 = 0.00086$ chains; therefore, $D = 0.017$ chains $= 1.7$ links. This shows that the

error in the departures should be about 1.7 links. The error in the latitude should be $1.7\sqrt{n} = 1.7\sqrt{2} = 2.4$ links. Results much greater than these show an error in the work other than the usual inaccuracy.

Finally, knowing the error of closing the angles and the errors in balancing the latitudes and departures, we may reverse the problem and compute the error of chaining. To do this, notice that all quantities in equation (4) would then be known except d, which could then be computed.

CHAPTER IV.

SOLAR COMPASS.*

54. THE solar compass† is the result of an attempt to overcome the defects of the magnetic compass due to the variation in the magnetic declination. The solar compass determines lines with reference to the sun; and hence the bearings are independent of any changes of the magnetic needle. The solar compass consists of an ordinary magnetic compass, to which is attached an apparatus for sighting at the sun, briefly called the solar apparatus.

55. PRINCIPLE OF THE SOLAR APPARATUS. In its most elementary form the solar apparatus consists of a right line parallel to the axis of the earth, to which is pivoted another right line which is free to move only in the plane of the first line. For convenience of explanation, call the first line the polar axis, and the second the solar sight-line.

If the polar axis is parallel to the earth's axis, and the solar sight-line is directed towards the sun, then, as the solar sight-line is revolved about the polar axis, it will trace on the sky the diurnal path of the sun. But, if the polar axis is not parallel to the axis of the earth and the angle between the axis and the sight line remains the same as before, it will not be possible to

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* For a discussion of the Solar Transit, see Chapter VIII.

† Invented by Wm. A. Burt, a U. S. public-land surveyor of Michigan, in 1836.

make the sight line point to the sun.* In other words, if the polar axis of the solar apparatus is parallel to the axis of the earth, and if the sight line makes an angle with the perpendicular to the polar axis equal to the declination of the sun, then the sight line can be brought to bear upon the sun *only when the plane of the two lines coincides with the true meridian.* This is the fundamental principle of all solar apparatuses, however much they may differ in detail.

The solar apparatus, then, consists (1) of a device for making the polar axis parallel to the axis of the earth, *i.e.*, for giving the polar axis an angle of elevation equal to the latitude of the place of observation; (2) of a device for setting off an angle, between the perpendicular to the polar axis and the solar sight-line, equal to the declination of the sun; and (3) of some device for revolving the solar sight-line about the polar axis. If the solar apparatus is attached to a common compass or transit in such a manner that the plane of the polar axis is parallel to the plane of the terrestrial sight-line of the instrument, then when the latitude of the place of observation and the declination of the sun are correctly set off, and the solar sight-line is directed to the sun, the terrestrial sight-line will indicate a true meridian.

ART. 1. CONSTRUCTION OF THE SOLAR COMPASS.

56. The usual form of the solar compass is shown in Fig. 8. It consists essentially of three arcs of circles by which the latitude of a place, the declination of the sun, and the hour of the day may be set off. These

* Strictly, if these conditions are not fulfilled the solar sight-line can be made to point toward the sun but once, or at most only twice, during the twenty-four hours, and then only for an instant.

arcs are designated in the cut by the letters *a*, *b*, and *c*, respectively.

The latitude arc *a* has its center of motion in two

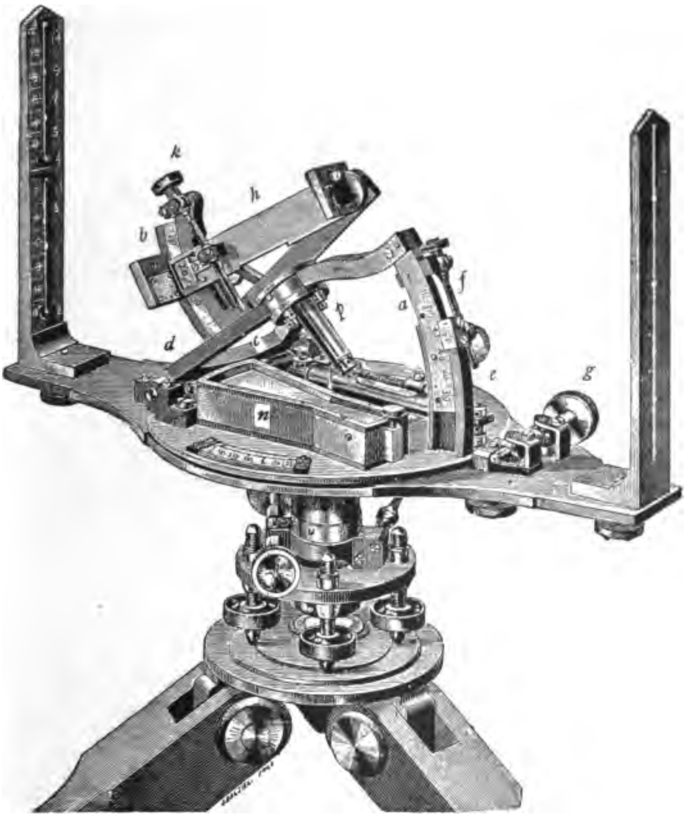

FIG. 8.—SOLAR COMPASS.

pivots, one of which is shown in Fig. 8, at *d*. The latitude arc is moved either up or down within a hollow arc, seen in the cut, by a tangent screw at *f*, and may be fastened in any position by a clamp screw *e*. The

vernier of the latitude arc usually reads to single minutes.

The declination arc *b* has a range of about 25°, and is read by a vernier to minutes. The arm carrying the vernier is moved over the surface of the declination arc, and its vernier set to any reading, by turning the head of the tangent screw *k*. The vernier is clamped in any position by a screw, not seen in Fig. 8.

At each end of the collimation arm *h* is a rectangular block of brass. In the upper of these blocks is set a small convex lens, having its focus on the surface of a little silver or ivory plate fastened to the inside of the opposite block. On the surface of the plate are two pairs of lines intersecting each other at right angles. The interval between the lines of each pair is just sufficient to include the circular image of the sun as formed by the solar lens.

Each end of the arm *h* has both a lens and a silver or ivory plate. As shown in Fig. 8 the compass is set for south declination; and when the sun has north declination, the declination arc and the solar line of sight are turned 180° on the polar axis, and the observation is made with the lens and plate on the end of the arm *h* opposite, respectively, to those used for a south declination of the sun. Thus there are really two solar sight-lines—one for north and one for south declination.

57. When the instrument is leveled, and the co-latitude is set off on the arc *a*, and the declination of the sun* is set off on the arc *b*, if the whole instrument is revolved about its polar axis, *p*, until the image of the sun formed by the lens in the upper end of the collimation arm, *h*, falls in the middle of the lines on the plate on the lower end of the collimation arm, then *the terrestrial line of sight of the instrument will be on a true meridian.*

* After being corrected for refraction (§ 159).

The angle between any course and the true meridian may be read by the graduation on the horizontal plate of the instrument.

58. As solar work can be performed only during clear weather, the instrument is provided with an ordinary magnetic needle—shown at *n* in Fig. 8—by which to run lines in cloudy weather. The needle may be used also to determine the magnetic declination. Instead of sights the solar compass is sometimes provided with a telescope; but this addition is of little, if any, advantage.

ART. 2. ADJUSTMENT OF THE SOLAR COMPASS.

59. The following adjustments of the solar compass must be carefully attended to: 1. The plane of the plate bubbles should be perpendicular to the vertical axis. 2. The two solar sight-lines should be parallel. 3. The declination arc should read zero when the solar sight-line is perpendicular to the polar axis. 4. The latitude arc should read the latitude when the polar axis is parallel to the axis of the earth.* 5. The terrestrial line of sight and the solar line of sight should lie in the same vertical plane.

The method of making these adjustments will not be described here, for the following reasons: 1. It will not be difficult for any one familiar with the methods of adjusting engineers' field-instruments to devise ways of making these adjustments. 2. These adjustments are similar to those of the solar transit to be described presently. 3. On account of the defects of the solar compass it is better to use either the magnetic compass (Chapter III) or the solar transit (Chapter VIII).

* If, however, the value of the latitude used with the instrument be that obtained by it, then no attention need be paid to this adjustment. It is important only when the true latitude is used in finding the meridian, or when the true latitude of the place is to be found by the instrument.

ART. 3. USING THE SOLAR COMPASS.

60. Merits and Defects of the Solar Compass. Merits. 1. The chief merit claimed for the solar compass is that lines run by it are not affected by variations of the magnetic needle; but the common compass can be used (Appendix I) so as to be absolutely independent of variations of the needle, and hence this claim has no force. 2. A second merit claimed is that the solar compass enables the surveyor to obtain a true meridian very easily; but as there are several other ways of finding a true meridian, this advantage is not very great. 3. It is also claimed that the solar compass is peculiarly valuable in the U. S. public-land surveys in running parallels of latitude; but as the transit is more accurate and more convenient for this purpose, this point is not well taken.

61. Defects. The solar compass has the following defects: 1. It can be used only when the sun is shining. At other times it can be no more accurate than the magnetic compass, while it is much heavier and more complicated. 2. Owing to its many adjustments and the difficulty of making and keeping them, the solar compass is not an exact instrument even in clear weather. "A state boundary-line run with the solar compass and intended to be straight was found to deviate ten minutes when re-run with a transit." 3. To use the solar compass, the engineer must know his latitude, the declination of the sun for each hour of the day, the time of day, and the refraction for all altitudes; and the inevitable inaccuracies, not to consider large errors, in these data render work done with the solar compass unreliable. For example, an error of one minute in the declination may cause an error of twelve minutes in the direction of the supposed meridian.

62. History. The solar compass was invented by Wm. A. Burt, a deputy U. S. land-surveyor in the Lake Superior mineral region, in 1836—about the time of the introduction of the American transit. Previously the magnetic compass and the heavy and inconvenient English theodolite were the only angle instruments in general use in this country. The solar compass was much lighter and more convenient than the theodolite, and more accurate than the magnetic compass as then made and used. This probably accounts for the favor with which the solar compass has been received in the past. In popular estimation it is an instrument of great precision, while in reality it is little, if any, more accurate than the magnetic compass. It is certainly neither so accurate nor capable of so great a variety of work as the engineer's transit, which is properly employed where formerly the solar compass was used.

The solar compass is certainly a very ingenious instrument, and reflects more credit upon the inventor than upon the modern user. The form shown in Fig. 8 is the one in use at the present time, and is essentially the form originally given to it by the inventor.

We will not further discuss this instrument, but refer the reader to Burt's "Key to the Solar Compass," in which the inventor fully describes the adjustment and use of his instrument.

CHAPTER V.

VERNIERS.

63. A VERNIER* is a short scale movable by the side of a longer scale, by which subdivisions of the longer scale may be measured. The scale to be subdivided is called a limb. A division of the vernier is a little shorter or a little longer than a division of the main scale or limb. This small difference is the unit of the space measured by the vernier.

64. PRINCIPLES. A vernier may be constructed by taking a length equal to any number of parts of the limb, and dividing it into a number of equal parts, one more or one less than the number into which the same length on the limb is divided. For example, the limb shown in Fig. 9 is a scale of inches divided into tenths, and the divisions of the vernier are of such a length that ten spaces on the vernier are equal to nine on the limb or main scale. Therefore each space on the vernier is equal to 0.1 of 0.9, or 0.09, of an inch; that is, each space on the vernier is 0.01 of an inch shorter than a space on the main scale. Line 1 on the vernier falls short of a line on the limb by 0.01 of an inch, line 2 falls short by 0.02 of an inch, and so on. If the vernier

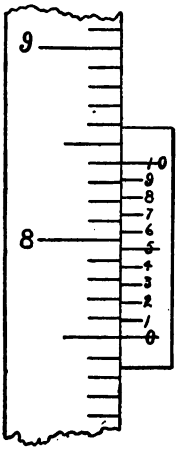

FIG. 9.

* Invented about 1631 by Pierre Vernier of Burgundy.

be moved slowly forward, the successive coincidences of a line on the vernier with one on the limb will indicate successive advances of the vernier, each equal to 0.01 of an inch. If, then, the lines of the vernier are numbered as in Fig. 9, the number *on the vernier* of the line which coincides will indicate the distance that the zero of the vernier has passed a division of the limb.

65. Direct and Retrograde Verniers. In the above illustration the spaces on the vernier were shorter than those on the limb, the supposed motion was in the direction of the graduation of the limb, the successive lines of the vernier came into coincidence in the direction of the graduation, and the numbering on the limb, and also on the vernier, increased in the same direction. A vernier fulfilling these conditions is called a *direct* vernier.

If the spaces on the vernier are larger than those on the limb, and the vernier is moved in the direction of the graduation of the limb, the successive coincidence will occur in the direction opposite to the motion, and also opposite to the direction of the graduation on the limb; and therefore the lines on the vernier should be numbered in an opposite direction to those on the limb. A vernier fulfilling these conditions is called a *retrograde* vernier. For an example, see Fig. 11, page 67.

66. Least Count. The least count of a vernier is the difference in length between a space on the limb and one on the vernier. To find the least count of a vernier, *i.e.*, to determine how small a distance it can measure, let l = the length of a division on the limb, v = a division on the vernier, and n = the total number of spaces on the vernier. Then by the principle of the vernier, $n\,l = n\,v \pm l$, solving which gives $l - v = \frac{l}{n}$ = the least count. For example, in Fig. 9, $l = 0.1$, and $n = 10$; hence the least count equals $\frac{l}{n} = \frac{0.1}{10} = 0.01$,

The preceding formula expresses a very important relation, since it is the key to reading all verniers, whether direct or retrograde. Notice that the least count of the vernier is equal to the smallest division on the limb divided by the number of spaces on the vernier. In practice, it is not necessary to count n: it is indicated by the numbering on the vernier itself. For example, if the limb is divided to half-degrees and there are thirty spaces on the vernier (which would be indicated by the end line being numbered thirty), the vernier reads to one thirtieth of a half-degree or to minutes.

67. To Read a Vernier. Look at the zero line of the vernier. If it coincides with a division of the limb, the number of that line on the limb is the correct reading, the vernier divisions not being required; but if, as usually happens, the zero of the vernier comes between two divisions of the scale, note the nearest division on the limb next less, and then look along the vernier till a line is found which exactly coincides in direction with some line on the limb. The number of this line *on the vernier* is the distance between the zero of the vernier and the next lower division of the limb, and must be added to the reading taken from the limb. The particular division *on the limb* that may be in coincidence is of no consequence.

68. Examples. A number of examples will now be given to further illustrate these general principles. The student should draw the limb and the scale on separate slips of paper or card-board, and move one beside the other until he can read them in any position.

The vernier shown in Fig. 10 is the one used on the New York leveling rod. The main scale is divided to feet, tenths, and hundredths, and ten divisions of the vernier are equal to nine of the limb; therefore the vernier reads to tenths of a hundredth, or to thou-

sandths of a foot. The vernier as drawn reads 3 feet, 0 tenths, 3 hundredths, and 6 thousandths, or 3.036 feet.

Fig. 11 is a retrograde vernier which reads to hundredths of an inch, the limb being divided to inches

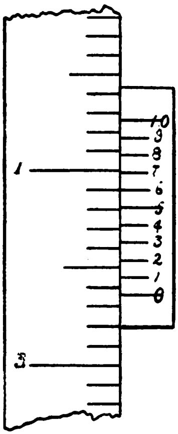

FIG. 10.

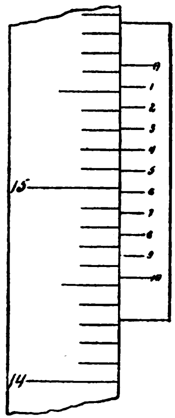

FIG. 11.

and tenths. It is sometimes applied to barometers. The reading is 15.64 inches.

Fig. 12 shows part of a circle graduated to degrees and half-degrees. The vernier has thirty parts, and therefore it reads to minutes. The reading is $212^\circ\ 30' + 11' = 212^\circ\ 41'$. This is a very common vernier for engineering instruments.

The graduation of transits usually has two rows of numbers, one increasing to the right and the other to the left, in which case there are two verniers—one for each series of numbers. Such an arrangement is shown in Fig. 13, and is called a double vernier. The reading of the left-hand vernier and the inside row of numbers is $182^\circ\ 40'$, while that of the right-hand vernier and the outer row of numbers is $177^\circ\ 20'$.

With a double vernier there is sometimes a doubt as to which vernier should be read. To settle this, notice whether the vernier divisions are larger or smaller than those on the limb. If the spaces on the

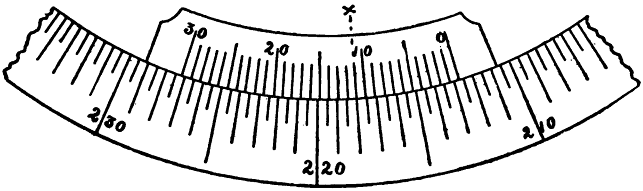

FIG. 12.

vernier are the smaller, it is a *direct* vernier, and that vernier should be read the numbers of which increase in the *same* direction as those on the limb. If the spaces on the vernier are the larger, it is a *retrograde*

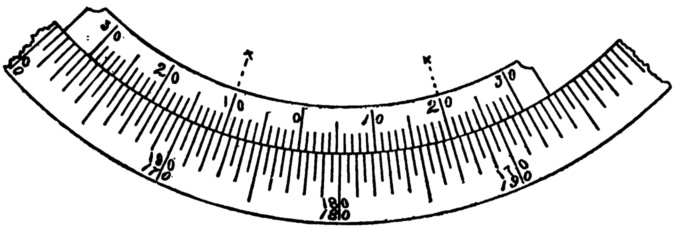

FIG. 13.

vernier, and that vernier should be read the numbers of which increase in the *opposite* direction to those on the limb. Or the vernier to be read can always be determined as follows: Move the vernier, until, say, the ten line of one vernier and the twenty of the other each coincide with a line on the limb; look at the zero, estimate the reading, and then read the vernier that agrees most nearly with the estimated reading. In Fig. 13 the numbers on the vernier are inclined in the same direction as the numbers on the limb to which the

vernier belongs. Instruments are not generally made in this way, but such an innovation would be a great improvement.

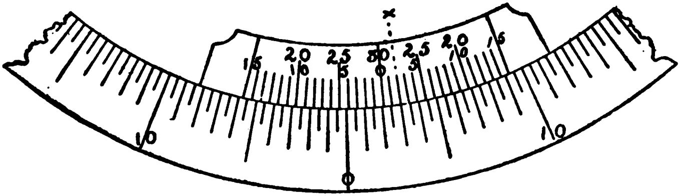

FIG. 14.

Fig. 14 is another form of double vernier. It is called a double folded-vernier, and is often applied to the compass, to be used in setting off the declination. Notice that this form is only half as long as the double vernier of Fig. 13. It is used where there is not space enough for the longer one. The lower numbers on one side of the zero and the upper ones on the other side constitute a vernier. The proper vernier to be read in any given case, can be determined by either of the rules of the preceding paragraph. The vernier as drawn reads 2° 2′ to the right. Notice that the reading of the vernier is 2′, and not 28′.

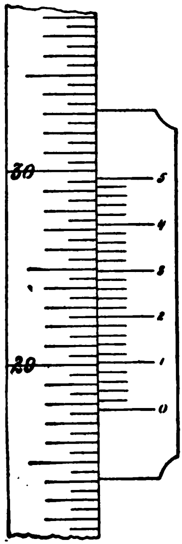

FIG. 15.

Fig. 15 represents the scale and vernier of the mountain barometer. The limb is divided into inches, tenths, and half-tenths or five-hundredths. There are 25 spaces on the vernier, and therefore it reads to ($.05 \div 25 = .002$) two thousandths of an inch. The reading is $28.75 + 0.020 = 28.770$ inches. Notice

the method of numbering the vernier. The numbers on it indicate hundredths. This form is much more convenient to read than if the numbers ran from 0 to 25.

Fig. 16 shows the usual scale and vernier of the better sextants. The limb is graduated to 10 minutes, the vernier has 60 spaces, and therefore it reads to sixtieths of 10 minutes, or to 10 seconds. The numbers on the

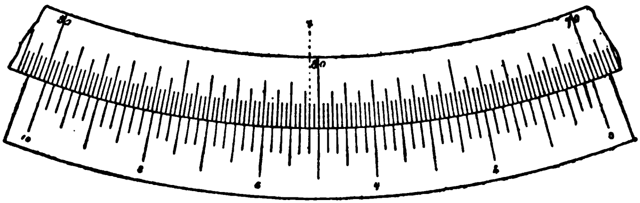

FIG. 16.

vernier indicate minutes. The reading is 70° 00′ on the limb, plus 5′ 10″ on the vernier, or 70° 5′ 10″.

69. PRACTICAL HINTS. The most frequent error in reading a vernier is to omit part of the reading of the limb; as, for example, in Fig. 12, forgetting to record the half-degree from the limb. With circular arcs having two rows of figures there is great danger of reading the wrong row.

To determine whether a particular line on the vernier coincides exactly with one on the limb, notice the next line on either side, and see whether both fall short (or overreach) equal amounts. When several lines of the vernier appear to coincide equally with several lines of the limb, take the middle line. When none of the lines exactly coincide, but one line on the vernier is on one side of a line on the limb and the next line on the vernier is as far on the other side of a line on the limb, the true reading is midway between the readings indicated by these two lines. If the graduation is very

accurate and the lines fine, it is possible by this method to estimate the reading to a half, or even to a third, of the least count (§ 66).

It frequently happens that the instrument is to be used to lay off a number of equal angles, as, for example, 1°. The vernier is then to be set each time at some particular line. If it is a double vernier, set the zero line to coincide each time, and notice whether the next lines on either side fall short equal amounts; if it is a single vernier, set, say, the fifteen line to coincide, and note the agreement on both sides. This method is considerably more accurate than reading the vernier by looking at only one line.

70. Transits are sometimes made with two verniers, one of which reads to decimals of a degree and the other to minutes, as usual. A decimal vernier is very convenient in laying out railroad curves, besides having some minor advantages in other kinds of work.

71. MICROMETER. When the highest degree of accuracy is desired, a micrometer is used. A micrometer consists of a fine screw, having a graduated head, which moves a frame on which is a fine line or spider's thread. The movement of the line or thread is observed through a microscope. The distance is determined by the revolutions and fractions of a revolution of the screw.

Micrometers are not used on ordinary engineering field-instruments.

CHAPTER VI.

OPTICAL PARTS OF THE TELESCOPE.

ART. 1. CONSTRUCTION.

72. A TELESCOPE consists, optically, of certain lenses which assist the eye in seeing distant objects; and, mechanically, of certain parts which facilitate the use of the optical parts. The mechanical parts can best be discussed in connection with the instrument to which the telescope is applied; and hence in the present chapter only the optical parts will be considered. No attempt at an elaborate discussion of the theory of the optical workings of the telescope will be made; but attention will be confined to such points as need to be understood by the engineer for the intelligent use of his instruments.

73. KINDS OF TELESCOPE. There are two forms of the simple refracting telescope,—usually known as the Galilean, and the astronomical. The last term is not a happy one, and *measuring* is suggested as being more appropriate, as will appear farther on.

The Galilean telescope consists of a double convex lens, called the object-glass or objective, placed next to the object, and a double concave lens, called the eye-piece or ocular, placed near the eye. The chief purpose of the objective is to increase the amount of light

which reaches the eye from the object viewed. The sole object of the eye-piece is to magnify the thing looked at. The Galilean telescope assists the eye by its magnifying and light-gathering power; but such a telescope would be useless for making precise measurements, since there is no means of indicating the exact point at which the telescope is sighted. It would not be inappropriate to call it a *seeing telescope.* The first telescope was of this form, and took its name from the inventor, Galileo. It shows objects erect. An opera-glass is a good example.

The *measuring* telescope consists of three essential parts: 1, a convex objective, which collects the rays of light and forms a bright inverted image of the object; 2, a convex eye-piece, which is essentially a microscope, for viewing the image formed by the objective; and 3, two fine wires or spider threads, placed in the plane of the image, the intersection of which indicates the precise point sighted at. The objective collects the light, the eye-piece magnifies, and the cross hairs indicate the point at which the telescope is directed. If such a telescope neither magnified nor increased the illumination, it would still be of great advantage in making measurements. This form shows the object inverted. Nearly all telescopes belong to this class.

Both of the above skeleton forms of telescope are very imperfect. The methods of improving the optical qualities of these elementary forms will now be considered.

74. The Objective. A single lens used as an objective has the following defects: 1. The rays of light which traverse a spherical lens near the edge, are refracted to a point nearer the lens than the rays which pass through the central portion; consequently the image is blurred. This deviation of the rays from the focus is called *spherical aberration.* 2. Rays of white

light, in passing a single lens, are resolved into the colors of the prismatic spectrum; consequently the image will be disfigured by colored light. This deviation of the different colored rays is called *chromatic aberration.*

The objective of a telescope is rendered almost free from these defects by substituting for the single lens, a compound one composed of a double-convex crown-glass, and a concavo-convex flint-glass, lens. The two components have different refractive and dispersive powers, and by giving the four surfaces proper curvatures the above defects may be nearly eliminated.

75. The Eye-piece. A single lens used as an eye-piece possesses the same defects—spherical and chromatic aberration—as when used as an objective, but in a less degree. A single lens as an eye-piece has also the following defects: 1. The image of a flat object formed by a lens does not lie in a plane, but is concave towards the lens. This deviation of the image from a plane is termed *aberration of sphericity*, which is wholly separate and distinct from spherical aberrations (§ 74). The objective also possesses this defect, but in so slight a degree as to be inappreciable; while in an eye-piece of a single lens it is very serious. 2. A telescope with a single lens for an eye-piece has a very limited field of view.

In both forms of telescope, the chromatic and spherical aberration, and also the aberration of sphericity of the eye-piece, are nearly eliminated by substituting two plano-convex lenses for the single lens, which also increases the field of view.

76. Huyghen or Negative Eye-piece. This is a modification of the concave eye-piece of the Galilean telescope. It consists of two plano-convex lenses with their convex sides turned towards the objective. The relations of the eye lenses to each other, and to the objective, are shown in Fig. 17.

P is the object; *O* is the objective; and *a* and *b* constitute the eye-piece, *a* being known as the field lens and *b* as the eye lens. Notice that the eye-piece is

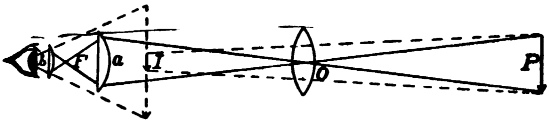

FIG. 17.—HUYGHEN OR NEGATIVE EYE-PIECE.

placed between the objective and its principal focus, *F*. The large arrow at *I* represents the object as it appears through the telescope; the small one represents it as it would appear without the instrument. The angle which the large arrow at *I* subtends at the eye, divided by the angle which the object subtends at the eye, is equal to the magnifying power; that is, the magnifying power is equal to the ratio of the larger arrow at *I* to the small one.

77. Ramsden or Positive Eye-piece. This is a modification of the convex eye-piece of the measuring telescope. It consists of two plano-convex lenses with their convex sides towards each other. The relations of the eye lenses to the objective are shown in Fig. 18.

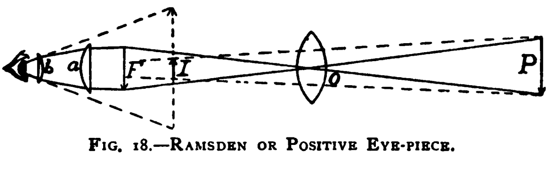

FIG. 18.—RAMSDEN OR POSITIVE EYE-PIECE.

The nomenclature is as before. Notice that the eye lenses are farther from the objective than its principal focus. The objective forms an image at *F* which is viewed with the eye-piece. The magnifying power is the ratio of the two arrows at *I*, as before.

Notice that the essential difference between the positive and negative eye-pieces is that only with the former is a real image of the object formed, and hence it is the only eye-piece with which cross hairs can be used. Spider lines are sometimes placed in negative eye-pieces, for example in sextants, simply to indicate the middle of the field of view; but they are only indirectly concerned in the accuracy of the observations. The negative eye-piece is better for simply seeing, while the positive is absolutely necessary in making precise measurements. The positive eye-piece is always used in transits, levels, etc.

Modifications of Ramsden's eye-piece, consisting in the substitution of compound lenses instead of the single lenses shown in Fig. 18, are frequently employed. The principal ones are Kellner's, which consists of an achromatic combination instead of lens *a* of Fig. 18, and Steinheil's, which consists of two achromatic combinations instead of the single lenses *a* and *b* of Fig. 18. These eye-pieces are better than Ramsden's in that they are free from color and have a perfectly flat field. Of the two, Kellner's permits the larger field.

78. Erecting Eye-piece. The erecting or terrestrial eye-piece, in its most elementary form, consists of a

FIG. 19.—ERECTING EYE-PIECE.

convex lens placed between the eye and eye-piece of the measuring telescope, which inverts the image formed by the objective and shows the object erect; but in its common form it consists of a pair of plano-convex lenses instead of the single lens. Fig. 19 shows the usual arrangement of the lenses and diaphragms of an erecting eye-piece. The pair of lenses next to the

eye is called the erecting piece; the pair next to the objective is the ordinary positive eye-piece. For a Kellner's erecting eye-piece, see Fig. 27 (page 100) or Fig. 62 (page 222).

The erecting eye-piece is inferior to the inverting eye-piece, owing to the loss of light occasioned by the two extra lenses. The inconvenience of the inversion of the object is easily overcome with a little practice. Furthermore, other things being equal, the telescope is shorter with the inverting eye-piece—which is quite an important advantage in the transit. Most American engineering instruments have an erecting eye-piece; but it would be a great improvement if all were provided with the inverting eye-piece. The latter is much more common in Europe than in America.

79. Telescope Slides. To assist in focusing the objective for different distances, the object-glass is fastened in a tube which slides, with a rack and pinion, into the end of the main tube of the telescope. In instruments provided with an inverting eye-piece the objective is fixed, and the cross hairs and eye-piece move together, to and from the objective. It is immaterial which form is used; the principle is the same in both, it being only important that the slide shall be straight and fit snugly.

To facilitate the focusing of the eye-piece upon the cross hairs, the ocular is provided with a similar slide. In some instruments the ocular is moved by a rack and pinion; but this is unnecessary and even worse than useless, for, having once adjusted the ocular for distinct vision of the cross hairs, it needs no change except for different observers, and it is better that it should not be easily moved. If no rack and pinion is provided, the ocular is moved in and out by hand, with a screw-like motion.

80. Cross Hairs.* These are usually two spider threads, one vertical and the other horizontal, fastened to a ring which is adjusted in the tube by four screws. Fig. 20 shows a front view of the ring in place. *BB*, Figs. 27 (page 100) and 62 (page 222) show side views of the ring. Lines ruled or etched upon a piece of thin glass are sometimes used. The cross hairs are sometimes made of very fine platinum wire. Platinum can be drawn to the required fineness only by being previously surrounded by silver, which is removed by acid after the wire is drawn. Both spider threads and platinum wires have their advantages and disadvantages, and as a consequence their advocates and opponents.

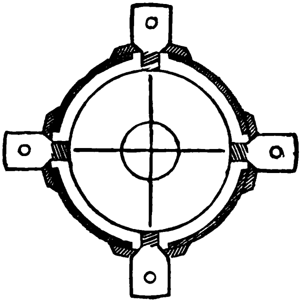

Fig. 20.

For the platinum wires, it is claimed that they are best because they are opaque; and this is a desirable property when the cross hairs must be illuminated, as in astronomical work and mine surveying. Spider lines, particularly dark-colored ones, can be illuminated fairly well. It is claimed also that wires are unaffected by the humidity of the atmosphere, and hence the line of collimation is not liable to change from this cause—an advantage which does not exist if the spider threads are properly stretched. On the other hand, it is claimed that platinum wires lose their elasticity and sag, and also that they oxidize or corrode. Spider threads, on account of their fineness and cheapness, and the facility with which they may be applied, will con-

* Cross hairs were first used by Picard, in 1669.

tinue in the future, as in the past, to be used almost universally for cross hairs in engineering instruments.

81. Stretching the Spider Webs. It does not require much time or skill to replace spider webs, the belief of many to the contrary notwithstanding. The ring which carries the cross hairs can be taken out (with some telescopes only after having taken out the eye-piece) by removing two opposite screws and inserting a soft wooden stick of suitable size into one of the holes thus left open in the ring (which last is turned sideways for that purpose), and then removing the other screws. Two scratches, at right angles to each other, will be found upon the face of the ring, into which the hairs are to be fastened.

The best spider lines are those of which the spider makes its nest. These nests are yellowish-brown balls which may be found hanging on the shrubs, etc., in the late fall or early winter. When found, the nest should be torn open and the eggs thrown out ; if this is not done, the young spiders when hatched will eat the threads. The fibers next to the eggs are to be preferred on account of their darker color.

Draw out a single fiber and attach each end of it to as heavy a weight as experiment shows it will support. Dampen the thread by breathing upon it, by holding it in a current of steam, or, better, by dipping it in clean hot water. Support the ring in such a way that when the thread is laid across it, the weights may hang freely down the sides, thus stretching the thread. The thread may be moved easily with a pin, and when in the proper position it can be fastened with wax, gum, varnish, or the like, shellac varnish being the best for this purpose. The main point is to stretch the web thoroughly and fasten it tightly. If the work is well done, the threads will remain straight when the reticule is placed in a current of steam or even in hot water.

Art. 2. Testing the Telescope.

82. In buying an instrument it is very desirable to test the optical qualities of the telescope ; and the following directions are sufficient for such examination.

83. Chromatic Aberration. To test for chromatic aberration (1, § 74), focus the telescope upon some bright object, either a celestial body or a white disk, and then move the object-glass slowly in and out. If in the first instance a light yellow ring is seen at the edge of the object, and in the second a ring of purple light, the object-glass may be considered perfect, as this proves that the most intense colors of the prismatic spectrum (orange and blue) are corrected.

84. Spherical Aberration. To test for spherical aberration (2, § 74), cover the object-glass with a ring of black paper, so as to reduce the area of the aperture about one half, and focus on some small and well-defined object for distinct vision. Then remove the ring of the paper and cover that part of the objective previously left open, and notice how much the object-glass must be moved in or out for distinct vision. The amount of shift measures the spherical aberration of the objective. Very little, if any, motion should be required to obtain a distinct view. Another test is to focus sharply upon some object, when the least motion of the objective should render the object indistinct. This last is not as good a test as the former, for the eye will change focus slightly to accommodate itself to the change of distance.

85. Definition. The definition of a telescope depends upon the accuracy of the curvature of the surfaces of the several lenses, and upon the centering, *i.e.*, the coincidence of the axes of the component lenses of

the objective and ocular. The reader should distinguish between illumination and definition. The lack of the former causes the image to be faint; the lack of the latter causes the image to be indefinite. Indistinctness may be caused by spherical aberration; and therefore the test for that should precede the test for definition.

To test a telescope for definition, focus upon some small, well-defined object—small, clear print is the best. If the print appears clear and well defined and fully as legible at 40 or 50 feet as if viewed with the naked eye at 8 or 10 inches (the best distance for distinct vision), the surfaces of the lenses are correct and well finished.

To determine whether the lenses are well centered, fix a white-paper disk having a diameter of, say, an eighth of an inch and a *sharp* outline, in the middle of a piece of black paper or cloth. Place this in a good light 30 or 40 feet from the telescope; then if the image of the disk, when a little out of focus, is equally surrounded on all sides by a uniform haze, the centering is good.

Other things being the same, the lower the magnifying power the better the definition.

86. Flatness of Field. The flatness of the field depends mainly upon the aberration of sphericity of the eye-piece (§ 75). To test a telescope in this respect, draw with India ink a heavy-lined square, 6 or 8 inches on a side, on a sheet of white paper, and fasten the paper to a flat board. Place this object at such a distance that the square shall nearly fill the field. Focus the telescope on it; then, if the sides appear perfectly straight, the telescope is perfect in respect to flatness of field.

A telescope which distorts the image perceptibly causes no error in common use, but is decidedly objectionable for stadia measurements, where two points of the field are used at the same time.

87. Size of the Field. By the field of view is meant all those points which are visible at the same time through the telescope. In Fig. 21 O is the objective, E the ocular, $DABC$ the object, ab the image of AB. The two extreme rays from A and B just strike the edge of the ocular, and rays from C and D do not enter the ocular; therefore the angle AOB is the field

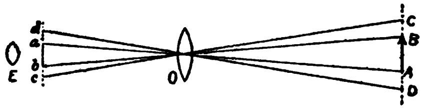

FIG. 21.

of view. Notice that it is independent of the size of the object-glass, and varies (1) inversely as the distance between the objective and the ocular, and (2) directly as the size of the eye-piece. The greater the magnifying power the less the field of view. The wider the field of view the better, since width of field facilitates rapid working. One of the advantages an eye-piece composed of two plano-convex lenses has over a single lens is the larger field of view.

To determine the angular width of field, sight upon any object, and mark the two opposite sides of the field. The distance between these points multiplied by 57.3 and divided by the distance from the object to the objective, is the angular width of field, in degrees. Since the field varies with the distance (particularly if it be short), a considerable distance, say 200 or 300 feet, should be employed in making this test.

The field of view varies with the maker, but for the telescopes on ordinary engineering instruments it is about as follows: for a magnifying power of 20, $1^\circ\ 30'$; 25, $1^\circ\ 15'$; 30, 1°; 35, $50'$.

88. Aperture of Objective. By the aperture of the objective is meant the effective diameter of the object-glass, *i.e.*, that part of the objective which transmits

light that finally reaches the eye. Usually, diaphragms are placed in the tube of the telescope with a view to improve the optical qualities of the lenses. If the diaphragm is placed near the object-glass, it will cut off those rays which pass through the objective near the edge, thus improving the definition—without diminishing the field but with a loss of illumination;—but if the diaphragm is placed in or near the eye-piece, it may diminish both the illumination and the field of view.

To find the real aperture, direct the telescope towards the sky, and place a pointer in contact with the objective so that the pointer may be seen in the small illuminated circle which will be noticed at the small opening of the eye end when the head is drawn back a short distance from the telescope. Then move the pointer over the face of the object-glass until the point just disappears, and measure the distance from the pointer to the edge of the object-glass. The real or clear aperture of the objective is equal to the diameter of the object-glass minus twice the above distance. In making this observation a hand magnifier is of great assistance in viewing the image of the objective.

A better method of finding the real aperture is as follows: Fix, with water or thin gum, several small pieces of opaque paper, say 0.05, 0.10, and 0.15 of an inch square, in a row around the very outer edge of the objective. Then turn the instrument up to the sky, and observe as before. If all the pieces can be seen, then the aperture is equal to the entire objective; but if any piece is invisible, the margin of the glass is cut off to this extent.

As it is more difficult to get the outer portion of the objective to correct curvature, it is not uncommon to cut off this portion with a diaphragm, which renders the objective equal only to a smaller glass.

Even an apparently small margin cut off makes a considerable difference in the light-collecting power. For example, a 1¼-inch objective will collect 56 per cent more light than a 1-inch glass. Not infrequently objectives are reduced in as great a proportion as in this example.

The aperture of the telescopes on ordinary engineer's transits is 1 to 1⅜ inches, and on levels 1¼ to 1½ inches.

89. Magnification. The power of a telescope, or degree of magnification, depends upon the relative focal lengths of the objective and of the eye-piece. Mathematically, any power can be given to any telescope; but, in practice, it is limited by the effects of loss of light, size of field, and imperfection of the lenses. For rapid work, the exact focusing necessary with high powers is a drawback, since a small change in distance requires a corresponding change in focus. The magnifying power varies slightly with the distance to the object; but, fortunately, the exact magnifying power of the telescopes on engineering instruments is not required. There are several methods of measuring the magnifying power, of which the two following are the simplest.

90. First Method. If the telescope be directed toward the open sky in the day-time and the eye be held 8 or 10 inches back from the eye end, a small bright illumined circle will be seen, which is nothing more than the image of the objective opening of the telescope. Measure the diameter of this circle with a finely graduated scale of equal parts. Also measure the real aperture of the objective (§ 88). Then the magnifying power is equal to the quotient arising from dividing the diameter of the aperture of the object-glass by the diameter of the illuminated circle. The chief difficulty in this method lies in the exact measurement of the diameter of the small illuminated circle.

91. Second Method. Let a staff which is very boldly divided into equal parts by heavy lines be placed vertically at any convenient distance from the telescope—for example, 150 feet. While one eye is directed towards the staff through the telescope, the other may observe the staff by looking along the outside of the tube. One division of the staff will be seen by the eye at the eye-piece to be magnified so as to cover a number of divisions of the staff, and this number, which is the magnifying power required, may be observed by looking with the other eye along the outside of the tube. A little difficulty may be experienced on the first trial of this method, but after a few attempts it becomes very easy.

The magnifying power of the telescopes on ordinary engineer's transits varies from fifteen to thirty, twenty to twenty-five being the more common on good instruments; and for levels it varies from twenty to fifty, twenty-five to thirty being the more common. The higher the power the greater the minimum distance at which an object can be seen through the telescope. It is frequently desirable, particularly with a transit, to focus on a point only 4 to 6 feet from the instrument. This can usually be accomplished with a magnifying power of twenty.

92. Illumination. The brightness of an object seen through a telescope depends upon (1) the size of the objective, (2) the polish and transparency of the lenses, (3) the magnifying power of the telescope, and (4) the size of the pupil of the eye.

1. Obviously, the quantity of light entering a telescope is dependent upon the size of the aperture of the object-glass, for the larger its area the more rays of light proceeding from any point of the object will be intercepted and transmitted.

2. In every telescope light is lost by reflection from

the surfaces of the lenses, and by imperfect transparency. It is probable that no engineering instrument transmits over 85 per cent* of the light striking the objective, and many of them transmit much less.

3. The object-glass transmits a certain amount of light, which the eye-piece distributes over a larger or smaller area, according to the magnifying power of the telescope; therefore the brightness of the image varies inversely as the square of the magnifying power. Since brightness of view is an indispensable requisite of a good telescope, the magnification should never be excessively large, as it frequently happens that the telescope is used in viewing objects only faintly illumined.

4. In order that all the light passing through the telescope may be received by the eye, the beam that emerges from the eye end must not exceed the diameter of the pupil of the eye; for if the emergent beam is larger, part of it can not enter the eye, and consequently part of the light intercepted by the objective will be lost. The average size of the pupil is about one tenth of an inch; and hence the diameter of the emergent beam should not be more. The best possible effect will be when the beam is of exactly the same diameter as the pupil, for the brightness of the object will then be the same (except for the losses referred to in paragraph 2, above) as when viewed with the naked eye. If the emergent beam is smaller than this, the brightness will be less than when viewed with the naked eye.

93. Since there can be no unit by which to measure the illuminating power of a telescope, we can find only the relative illumination. If two telescopes are to be compared as to light, they should stand side by side and

* Chauvenet's Practical Astronomy, vol. ii, p. 17.

be looked through at the same time, so as to be under the same atmospheric conditions.

If two telescopes have the same aperture, and also the same magnifying power, they may be compared by placing them side by side, and, as night approaches, observing the same object through each. The one through which the object is longest visible has the better illumination. Of course, both observations must be made by the same person. Instead of waiting for the approach of night, observe the distance at which fine print can be read with each, or the distance at which the time can be read from the second hand of a watch. Notice that the last method involves the definition (§ 85) and spherical aberration (§ 84) of the telescope, as well as the illumination.

If the two telescopes do not have equal apertures nor equal magnifying powers, a numerical expression for the illuminating power may be obtained as follows: By observation determine the maximum distance at which time can be told from the face of a watch, through each telescope. Then, if the lenses are equally transparent, these distances should be to each other directly as the clear apertures, and inversely as the magnifying powers. That is, if d_1 and d_2 represent the distances, a_1 and a_2 the apertures, and m_1 and m_2 the magnifying power, then

$$d_1 : d_2 :: \frac{a_1}{m_1} : \frac{a_2}{m_2}. \quad . \quad . \quad . \quad . \quad . \quad (1)$$

The error of observation by this method is considerable, owing to the inability of the eye to judge accurately of equal illuminations, and because the method also involves the defining powers of the two instruments.

Art. 3. Using the Telescope.

94. Adjustment for Parallax. Parallax is an apparent movement of the cross hairs in reference to the object sighted, caused by a real movement of the eye of the observer. It shows that the image and cross hairs are not in the same plane. Of course, if the object changes its position for different positions of the eye, it will be impossible to do accurate work with the telescope. Therefore a telescope should be accurately adjusted for parallax before being used in precise measurements. All measuring telescopes require this adjustment. It consists in bringing the cross hairs and the image exactly into the same plane. In making this adjustment two steps are required: (1) focusing the cross hairs, and (2) focusing the object.

1. To focus the cross hairs, direct the telescope toward the sky, or throw it out of focus so that no object can be distinguished in the field, and move the ocular in or out until the cross hairs can be seen very distinctly. When the cross hairs are properly focused, little specks of dust may be seen on them. No object should be in the field of view while this adjustment is being made, for the eye is continually changing its focus to accommodate itself to the distance of the object viewed. Before pronouncing the adjustment correct, close the eye for a moment, and then sight again. Having adjusted the eye-piece, it need not be changed except for the change of focus of the eye with advancing age, or for different observers. The rack and pinion frequently provided for focusing the cross hairs is worse than useless.

2. To focus the objective, direct the telescope to the object, keeping the attention fixed upon the cross hairs so that the eye shall not change focus to accommodate

itself to the position of the object, and move the objective in or out until the object appears very sharply defined. Then move the eye back and forth sidewise, and note whether the cross hairs and object alter their relative position. If the cross hairs appear to move with the eye, they are farther from the eye than the image, and therefore the objective should be moved nearer the object; for remember, first, that the farther object appears to move with the eye, and, second, that the farther the object from the objective the nearer is the image. This test is more easily made than described. When the objective is properly focused there should be absolutely no movement of the cross hairs with reference to the object.

Many of the errors of instrument work are due to parallax; and this source of error is more serious as the work becomes more accurate. The higher the power the more difficult it is to eliminate parallax.

95. Care of the Telescope. If the objective or the lens next to the eye becomes dusty, brush it with a fine camel-hair brush, or rub it with a piece of soft, clean chamois-skin or a piece of old linen or silk, taking care to use a clean spot for each rub. Unnecessary rubbing of the lenses should be avoided, since it will destroy the fine polish upon which depends the sharpness and brilliancy of the image. Dust upon the glasses is not as objectionable as a thin, or even almost imperceptible, film of grease; therefore the lenses should never be touched with the fingers. When the lenses become very dirty, wash them with alcohol.

The interior face of the objective and the interior lenses of the eye-piece will seldom need cleaning, unless water should find access to the inside of the tube. Having removed the cell containing the objective, care should be taken to screw it back exactly to its former position, else the adjustment of the line of sight may be de-

stroyed. The component lenses of the objective should never be taken apart nor removed from the cell containing them, since they may not be returned to their former position, thus disturbing their adjustments; but if they should accidentally become separated, be sure to replace them with the double-convex crown-glass outward. Dust should be carefully kept from the inside of the telescope tube; if not, it will get on the lenses and cross hairs. When not in use the eye-piece and object-glass should be covered by their caps.

If the telescope slide should get to fretting or cutting, take it out and smooth the rough places. The blade of a penknife forms a very good instrument for this purpose. Scrape with the edge, slightly inclining it, and burnish with the back of the knife. Wipe out the inside of the tube, and if possible burnish and scrape that smooth. Grease the slide slightly, and wipe off the grease before restoring the slide to its place. Too much grease, however, causes dust to adhere. If this does not remove the trouble, a little grinding with finely powdered pumice-stone and oil may help the difficulty; but great care should be taken to wipe off all abrading material. Emery in any form should never be used. If a slide once commences to fret, it rapidly grows worse, and may get beyond repair.

CHAPTER VII.

THE TRANSIT.

ART. 1. CONSTRUCTION.

97. THE instrument in common use among American engineers for measuring horizontal and vertical angles is usually called a transit. Fig. 22 shows the general form. It is sometimes, but incorrectly, called a theodolite. The theodolite is the name given by British engineers to their favorite portable instrument (see Fig. 23, page 93), which is capable of performing the same work as the transit.

The essential difference between the transit and the theodolite is that in the former the telescope can transit or turn completely over, while in the latter it can not. The telescope of the theodolite can be reversed only by lifting it out of its supports and replacing it end for end, which is a very imperfect substitute for the revolution of the telescope of the transit. The transit is sometimes called an engineer's transit and sometimes a railroad transit, to distinguish it from an astronomical transit.

The transit was invented and first made by a Philadelphia firm in 1831, previous to which time the English theodolite and the magnetic compass—sometimes provided with a full circle graduation, by which angles could be read independently of the needle—were the common angle instruments.

98. A modern engineer's transit has more than 350 distinct pieces. Although it appears quite complicated, this impression disappears when each part is examined

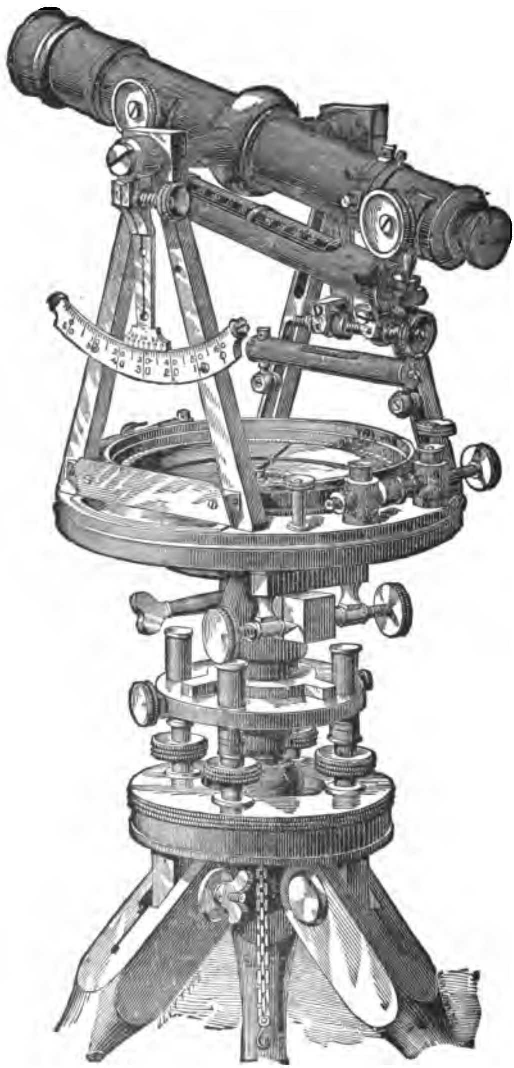

FIG. 22.—AMERICAN TRANSIT.

in turn, and its uses and relations to the rest carefully studied. The great value of the transit as an instrument of precision is due to the telescope, by which great precision in sighting is attained, and to the graduated circle with its vernier, by which angles can be read with ease and accuracy. All other parts are to facilitate the use of these two.

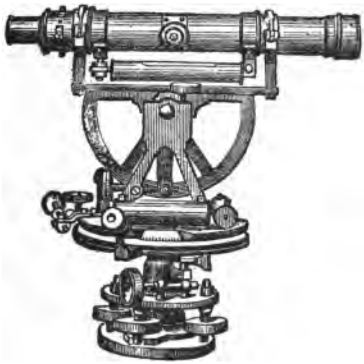

FIG. 23.—ENGLISH THEODOLITE.

The tripod, leveling screws, telescope, and vernier—all very important parts of the transit—have been discussed in preceding chapters.

99. THE GRADUATION. The lines of the graduation should be uniform, and as small as is consistent with legibility. In the best instruments the graduation is upon solid silver.

The most common graduation for the horizontal limb of transits is a circle about 6 inches in diameter divided to half-degrees having a vernier reading to minutes. The better transits read to 30″, and some to 20″. The degrees are usually numbered in two rows, one like the compass and another from 0° to 360°. The field work is most simple and least liable to error, if only the latter

numbering is used. Or, if there are two rows, they should be like those shown in Fig. 13 (page 68).

The vertical circle should be numbered from 0° to 90°; for then angles of either elevation or depression may be read with the telescope both direct and inverted, and the mean of the two will be independent of errors of adjustment of the vertical circle.

100. The Verniers on the horizontal circle are sometimes placed 90° from the line of sight, in which case the observer must change his place between sighting the telescope and reading the vernier. Sometimes the verniers are placed immediately under the telescope, in which case the observer can read the vernier and sight the telescope without changing his position, although the telescope must be revolved before the vernier can be read. The latter position is preferable, especially in confined places, as in underground surveying, etc. An intermediate position would be still better. For convenience of reference, the verniers should have some distinguishing mark, say *A*, *B*, *C*, etc., upon their faces.

The vernier and limb should be exactly in the same plane, to avoid parallax in reading; and the space between them should always have the appearance of a uniform, fine, black line. The verniers should be provided with ground-glass or ivory shades for illuminating the graduation.

Verniers reading 20 seconds should have the reading-glasses permanently attached in such a manner that they can be moved radially along the entire length of the vernier. The tube in which these reading-glasses are mounted should also have, at the end nearest the graduation, a fine pointer which will just reach the end of the lines. This pointer, being in the center of the field, will serve as a guide in moving the reading-glass exactly opposite the coinciding line. The center of the lens is thus used in reading, and parallax is avoided.

101. Centers, or Vertical Axes. Usually there are two concentric vertical axes, the verniers and telescope turning about the inner, and the graduated circle revolving about the outer one. Fig. 24 shows the arrangement of these axes. Ordinarily, the outer axis is useful only in enabling the observer to shift the graduation so as to begin each time at zero. The inner axis is

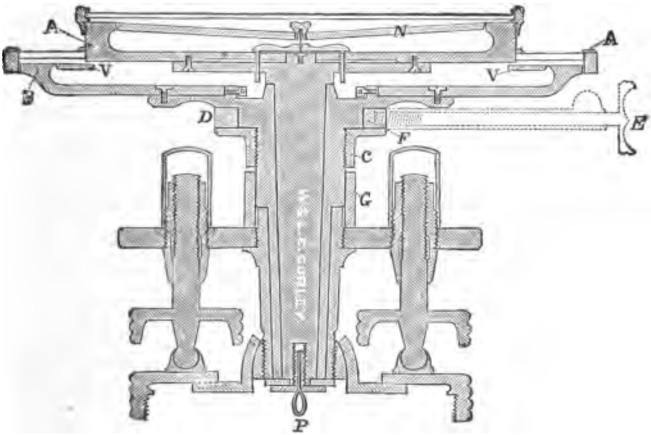

FIG. 24.—SECTION OF THE TRANSIT.

made more carefully than the outer. These two axes are often called "centers" and sometimes "compound centers." For work not requiring great accuracy, but demanding a light portable instrument, these axes are made quite short, and are called "flat centers." The more accurate instruments have "long centers." The instrument should turn on either axis without the slightest play, and yet with very little friction.

102. Levels. Since a transit is to be used in measuring horizontal and vertical angles, it is necessary that levels be attached to the instrument to determine the plane of these angles. Two level vials, perpendicular

to each other, are sufficient to bring the vertical axis vertical, which then serves as a datum to which to adjust the other parts of the instrument. If the instrument is to be used mainly for measuring horizontal angles, the level perpendicular to the telescope should be the more sensitive. In ordinary practice vertical angles are seldom required, and when they are less precision is demanded than in horizontal ones; hence it is very common to attach only two short levels to the plate, in which case the level parallel to the telescope should be the more sensitive. If the instrument is to be used for the precise measurement of vertical angles, a sensitive level should be attached to the telescope to indicate a horizontal line—the zero for vertical angles. With a level under the telescope, a transit can be used as a leveling instrument, but is not capable of very reliable work, owing to a lack of stability.

103. Clamp and Tangent Screw.* With the unaided hand, the telescope can not easily be made to cover or bisect the exact point sighted at; and to assist in thus directing the telescope, the instrument is provided with a clamp, and a tangent or slow-motion screw.

Clamps are made in a variety of ways, but all consist essentially of a contrivance by which a piece may be connected with the axis or rim of the graduation by tightening a screw. The clamp is connected with the vernier plate or the tripod, as the case may be, by a screw which is always nearly tangent to the direction of motion. When the screw is turned, the two parts of the instrument rotate slowly with reference to each other. No description can give an adequate understanding of these parts; they must be seen and examined to be comprehended.

It is common to clamp the vernier and the graduated

* Invented by Helvetius, a celebrated astronomer of Dantzic, about 1650.

plates at their circumferences; but as this is liable to bend the plates, it is far better to clamp the axis of the plate.

104. A perfect tangent screw should have a uniform motion, and be free from lost motion or "back-lash," so as to respond quickly to the touch and yet hold the plate and vernier exactly as set. Lost motion—the common defect of tangent screws—is a source of great annoyance to the engineer, and often causes serious errors in the field.

The tangent screw should have great durability, or at least an even wear, so that the screw will never have lost motion in one part and move hard in another. The wear is greatly lessened by covering the thread with a dust-guard, as is now quite common. The tangent screw should be so made as to work at least fairly well if it should get bent. Several forms of tangent screws will now be described.

105. English Tangent Movement. The oldest and most imperfect of all is that known as the English or stiff tangent screw, in which the screw works through a post in the clamp and against a collar in a post attached to the plate, each post being free to turn about a vertical axis. Fig. 25 shows the English tangent movement

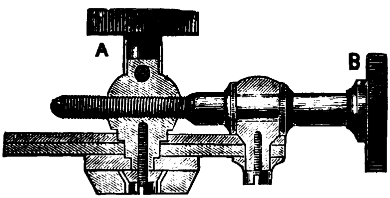

FIG. 25.—ENGLISH TANGENT MOVEMENT.

as applied to the horizontal circle of a transit. *A* is the head of the clamp screw and *B* that of the tangent screw. This form is defective in three particulars: 1,

Since the posts are movable only about a vertical axis, if the screw is not perfectly straight and true it will bind during one part of the revolution. 2. The nut and collars should be the same height above the plate; but to allow for errors of workmanship in satisfying this condition, the hole in the nut is often made too large, and the nut is then tightened until it fits, thus causing it to touch the screw along only two lines, and it therefore soon wears loose. 3. If the posts are tightened up so as to prevent looseness, the friction is so great as to interfere with the freedom of their motion.

106. Gambey Tangent Movement. One of the best forms of tangent screws is the Gambey movement, first made by the celebrated instrument-maker, Gambey, of Paris. The nut is a split sphere which may be confined like any ball-joint. Instead of the collar, as in the English form, another split spherical nut which works in a series of grooves is used. This form of tangent movement is shown at *g*, Fig. 8, page 59. As shown in Fig. 8, the pieces which clamp the balls are too heavy. They should be made light enough to have a little flexibility up and down, thus permitting a free movement of the balls. This construction provides for all necessary motions and all probable imperfections of the screw.

The objections to this form are: 1. The balls are exposed to flying dust and sand, which tends to roughen their surfaces and to cause an uneven motion. Confining the balls between springs remedies this in part. 2. As ordinarily made, the nut is much shorter than the screw, and consequently the latter wears mainly in the middle, and after a little use the screw can be turned only a few revolutions without having lost motion in one part or undue friction in another. This defect is removed by making the thread in the nut nearly as long as the thread on the screw, the long nut being con-

fined as the ball of the ordinary form. There are several slightly different ways of making this modification, all of which are very satisfactory. For one form see Fig. 22, page 92.

107. Spring and Abutting Screw. In a very common, and one of the best, forms of tangent movement a screw abuts against the clamp, and a spring on the opposite side keeps the clamp in contact with the screw. To prevent excessive local wear the nut through which the tangent screw works should be long; and to compensate for wear the nut should be made adjustable and have a little elasticity. An objection urged against this form is that the spring causes a strain between the parts, which may change with changes of temperature, and which is liable to move the plates or derange the levels. This objection has but little weight, particularly if there is some means of tightening the spring. For an example of this form of tangent movement see Fig. 28 (page 101) and also Fig. 32 (page 131).

108. Two Abutting Screws. Sometimes two opposing or abutting screws are used to get rid of lost motion; but these are objectionable, since two hands must be employed in using them. For several reasons they are not at all suitable for the upper motion, but on account of their steadiness are often used for the lower motion. If a stiff flat spring were attached to the side of the lug against which the tangent screw bears, the setting could be finished with one screw. Such a spring is occasionally so added.

109. Object-glass Slide. The object-glass should move in a right line perpendicular to the horizontal axis of the telescope. Some instruments have an adjustment to alter the direction of this motion. This is accomplished by causing the inner end of the slide to work through a ring which is adjustable like the ring carrying the cross hairs (see *CC*, Fig. 27). This device

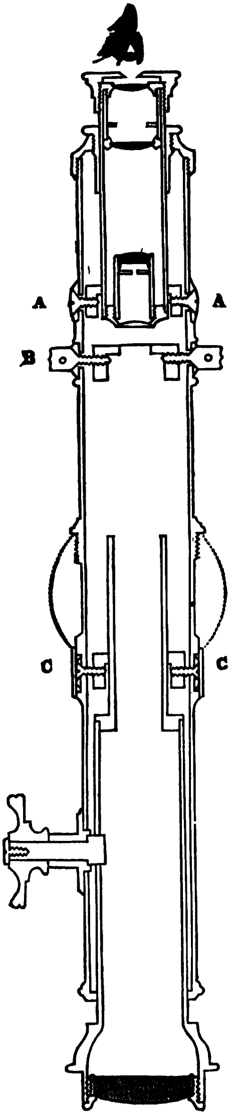

FIG. 27.—SECTION OF TELESCOPE OF TRANSIT.

is quite objectionable, since the ring is liable to get loose, besides wearing too large. Furthermore, the adjustment is quite difficult to make, and no reason can be assigned why the ring should not be fastened permanently when once in the proper position. A better way is to make the whole length of the slide fit the inside of the telescope tube. The latter method requires more care and greater skill in the manufacture. In either case the slide should be perfectly straight.

110. GRADIENTER. This is a simple device which adds very much to the convenience and usefulness of the transit. It is nothing more nor less than an accurate tangent screw with a micrometer head working beside a scale from which the number of complete revolutions is ascertained.* It can be applied to the vertical or horizontal limb, and may be used in establishing grades, determining horizontal distances (see Art. 2, Chap. X), and measuring small angles very accurately without an arc or a vernier.

* Invented by Professor Stampfer, of Vienna, in 1873.

Fig. 28 shows the gradienter as applied to the vertical circle of a transit. *A* is a leg attached to one of the standards which support the horizontal axis of the telescope. The little scale immediately above the graduated head is to register the complete revolutions of the screw, the fractions of a revolution being read from the graduated head. The screw is so cut that one revolution of it moves the telescope through an angle whose tangent at 100 feet from the instrument is 1 foot or 0.5 foot. In the former case the head is divided

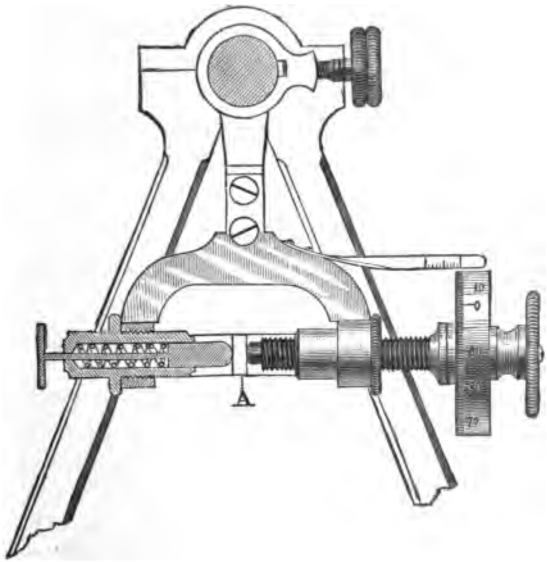

FIG. 28.—GRADIENTER.

into one hundred parts, and in the latter into fifty; and hence in either case a unit of the graduation corresponds to 0.01 of a foot on a rod 100 feet from the instrument. For hints on the use of the gradienter see §§ 237–47.

111. Shifting Plates. This is a device for bringing the plummet exactly over the point, after having brought it approximately into position by manipulating the tripod legs (§ 24). The usual arrangement allows the whole instrument to be shifted laterally on the tripod head about an inch. This is a very great convenience.

112. Compass. It frequently happens that it is absolutely necessary to run lines by the magnetic compass, even though a transit is at hand; and therefore a needle and compass graduation are usually added to a transit. The compass is also valuable as a check against gross errors in reading the vernier of the transit.

The tests and adjustments of the compass on the transit are essentially the same as for the simple compass (see Chapter III, particularly § 36).

113. Various Extras. There are a few additions and modifications of the transit which, though not common, are sometimes made, and which increase the range and convenience of the instrument for certain kinds of work. A few of the most important of these extras will now be described briefly.

When it is desired to take greater vertical angles than is possible with the ordinary eye-piece, the little cap on the eye end of the telescope is unscrewed and replaced by another containing a small prism which reflects the beam at right angles and brings it out to the eye of the observer. When the telescope is used to look at the sun, a colored shade must be placed over the object-glass or be interposed between the eye and the telescope. A piece of smoked window-glass, held in the hand, will serve the purpose; but caps containing colored shades are made by the various instrument-manufacturers, which are much more convenient, and comparatively inexpensive.

When the telescope is used in a mine, or to look at a star, unless some diffused light enters the telescope, the cross hairs will not be visible. Caps are made which support an annulus in front of the objective, at an angle of 45° with the telescope; and the cross hairs are then illumined by holding a lamp or candle at the side of the objective. The same thing may be accomplished, though less easily, with a piece of paper held in the hand or fastened to a board.

In mine surveying it is often necessary to sight vertically up or down a shaft. For this purpose an extra telescope is sometimes attached to the end of the horizontal axis; or sometimes the standards are inclined, so that the ordinary telescope may sight vertically downward past the plates. See § 168 for still another method.

ART. 2. TESTING THE TRANSIT.

114. GRADUATION. The graduation may have two classes of errors—accidental and periodic. Accidental errors are those which follow no regular law, and are equally liable to occur at any given division. There is a variety of causes which may produce them. Periodic errors are those which follow some law, and are probably caused by some peculiarity of the graduating engine.

Since the accuracy of the graduation is a vital point, it is very unfortunate that there is no easy or simple method of testing it. An imperfect test is to notice whether the extreme lines of the vernier span the same number of divisions in all parts of the circle. But with the present graduating engines it is very poor graduation indeed whose errors may be detected in this way.

With astronomical and geodetic instruments very elaborate methods are employed for detecting errors of

graduation; but these methods are too complicated to be explained here, and, moreover, they are not applicable to common engineering instruments.

115. Eccentricity. The center of graduation should lie in the axis of rotation. To test this, read two verniers 180° apart (or any number of equidistant verniers), then move the circle preferably 90° and read again. If the verniers have changed the same amount, the circle is well centered. If the two have not changed the same amount, the mean of the two differences is the actual angle through which the instrument has been moved. Eccentricity is, therefore, wholly eliminated by reading two opposite verniers each time, and taking the mean. The horizontal circle of the common engineer's transit seldom has more than one minute of eccentricity. There is usually but one vernier on the vertical limb, and hence there is no way of testing or eliminating its eccentricity.

It is frequently said that if the readings of the two verniers differ by 180°, the graduation and centering are perfect; but this is no criterion. The difference between the verniers depends upon the accuracy of the graduation, the eccentricity, and the angular distance between verniers. The readings might differ by 180°, and still the graduation and centering be very much in error.

Assuming the graduation to be perfect, the difference between the two verniers can be expressed by $180 \pm e$. If e remains constant, the centers coincide, but the verniers are not opposite; if e varies, the verniers are opposite, but the centers do not coincide.

116. Magnifying Power vs. Vernier. The magnifying power of the telescope and the least count of the vernier should be so proportioned that the least perceptible movement of the vernier will cause sufficient movement of the cross hairs on the object to be easily

noticed through the telescope; and, *vice versa*, the least noticeable motion of the cross hairs on the object should cause a barely perceptible change of the vernier. Since the horizontal circle is usually (and properly) the more accurate, this condition applies more especially to the horizontal circle. A higher power, or a smaller count of the vernier, than that required by this condition is detrimental, the former causing an unnecessary loss of light and the latter a waste of time in reading the vernier.

117. Magnifying Power vs. Level under Telescope. If the transit is to be used as a leveling instrument, the magnifying power of the telescope and the delicacy of the level under the telescope should be so proportioned that a barely perceptible movement of the cross hairs on the object will cause a just perceptible movement of the bubble.

118. Magnifying Power vs. Plate Levels. The relation which should exist between the magnifying power of the telescope and the sensitiveness of the plate levels depends wholly upon the kind of work to be done. The following method of testing this relation suffices for all kinds of work. Bring the bubbles of both plate levels to the middle, and sight upon a well-defined point at as great an angular elevation as the instrument will permit; then slightly derange the levels by manipulating the foot screws, and carefully level up again. If, on again looking through the telescope, the cross hairs cover the same point as before, the sensitiveness of the levels is proportional to the magnifying power of the telescope. If the cross hairs are out horizontally, it shows that the level perpendicular to the telescope is not sensitive enough. If the cross hairs are out vertically, the level parallel to the telescope is too sluggish. This relation is much less important than the two preceding ones.

119. Parallelism of Vertical Axes. The vertical axes should not only be parallel to each other, but should also be concentric. To test the first condition, adjust the most sensitive level about the instrument perpendicular to one axis (§ 38), clamp that axis, and revolve the instrument about the other; then if the levels are in adjustment about the second axis also, the axes are parallel. If the axes are not parallel, no error will be produced except when the instrument is used to measure angles by repetition (§ 132), provided the plate levels are adjusted perpendicular to the inner axis.

120. Limb Perpendicular to Axis. The plane of the graduation should be perpendicular to the axis about which it revolves. If the limb is not horizontal, angles measured on one part of it will be too great, and angles 90° therefrom will be as much too small. However, the error would almost certainly be inappreciable with common engineering instruments.

If desired, this condition can be tested as follows: Bring the vertical axis vertical (§ 38), place a block level on the horizontal limb, read the position of the bubble and reverse the limb; then reverse the block level to eliminate its error, and read the position of the bubble again. If the bubble reads alike both times, the horizontal limb is perpendicular to the vertical axis.

121. Object-glass Slide. The optical center* of the objective should be projected in the line of collimation.†

* The optical center is a point so situated that any ray of light passing through it will undergo equal and opposite refractions on entering and leaving the lens. It will, therefore, be found where a line joining the extremities of two parallel radii of the opposite surfaces intersects the optical axis of the lens. For a double-convex lens, it is always within the surface of the lens. For a plano-convex or a plano-concave lens, the optical center will be at the intersection of the axis with the curved surface.

A lens has been defined as a mathematical point which allows a great deal of light to go through it. The better the lens the more nearly is this condition realized. This "point" is the optical center.

† The line joining the intersection of the cross hairs and the optical center.

If it is not so projected, and the instrument be collimated for one distance (§ 123), it will be out of collimation for every other distance. This is equivalent to requiring that the slide shall be straight and move in the plane of the horizontal axis and perpendicular to it. If the objective is fixed and the cross hairs are movable, the same conditions should be satisfied, and the method of testing is the same in both cases. To make this test, *collimate the instrument* (§ 123, or § 278, or § 285) and test (1) for a deviation from a vertical plane, (2) for a deviation from a plane at right angles to the vertical plane, (3) for a deviation from the perpendicular to the horizontal axis, and (4) for a deviation from the plane of the horizontal axis.

1. *To test for a deviation from a vertical plane.* Select smooth ground and lay off any number of points,—say five or six, 100 feet apart,—having one as near the instrument as possible, and line the points carefully with the telescope. Reverse in altitude and azimuth, sight upon the first point, and clamp the instrument; then locate points in line with the first one, near each of the others. Measure the distances between the several pairs of points. If these distances vary as the distance from the first point, the slide is probably straight and the optical center is projected in a vertical plane but not perpendicular to the horizontal axis. To cause the optical center to move in a plane perpendicular to the horizontal axis, sight upon the first point and clamp the vertical axis; then direct the telescope to the second point, and move the back end of the slide to correct half the error. Moving the back end of the objective slide may have destroyed the adjustment for collimation; therefore recollimate the instrument, and again test for straightness of slide. If the instrument can not be adjusted to bisect a row of points before and after reversal, the slide is not straight, and no good work

can be done with the instrument. Of course, due consideration must be given to the errors of observation. An error may be introduced by the slide's being loose; but this may be avoided by finishing every setting of the object-glass by motion always in the same direction.

2. *To test for a deviation from a plane at right angles to a vertical plane.* The method is similar to the preceding. Drive several stakes into the ground at equal intervals, the first being near the instrument. Read a leveling rod upon each, reverse in altitude and azimuth, and bring the line of sight to the previous reading upon the first stake; then read the rod upon all the other stakes. If the differences of readings vary as the distance from the first station, the object-glass is projected in a plane perpendicular to the vertical plane. Since it is projected in two planes at right angles to each other, it must be in their intersection; therefore the optical center is projected in a straight line, and the slide is straight.

3. *To test whether the motion is perpendicular to the horizontal axis.* Collimate the vertical hair (1 § 123) for a near distance; then, if it is in collimation for a greater distance, the optical center of the objective is projected in a line perpendicular to the horizontal axis of the telescope. If it is not in collimation for the farther point, move the back end of the object-glass slide until it is; but if there is no means of adjusting the slide, send the instrument to the maker. See § 109.

4. The rigorous condition is that *the line of motion of the optical center should pass through the horizontal axis.* It would be impossible to certainly secure this relation; but for any cases likely to occur in practice, including those requiring astronomical accuracy, it is sufficient to require that the line of collimation be in the line of motion of the optical center. Therefore it is only necessary to place the horizontal hair in the line of

motion of the optical center, which is an adjustment, and will be discussed in Art. 3 of this chapter.

ART. 3. ADJUSTMENTS OF THE TRANSIT.*

122. LEVELS. The levels should be perpendicular to the vertical axis. This is the same adjustment as that described for the compass (§§ 38 and 39, which see). As in the compass, this condition is important only in certain kinds of work, as will be discussed in § 128.

123. CROSS HAIRS. This adjustment is ordinarily known as the adjustment of the line of collimation.† For obvious reasons, it is better to call it the adjustment of the cross hairs. It is made in two steps, (1) the adjustment of the vertical hair, and (2) that of the horizontal hair.

1. Vertical Hair. The line of collimation should be perpendicular to the horizontal axis of the telescope. Notice that if this condition is satisfied, the line of collimation will describe a plane during the revolution of the telescope; but if it is not, it will describe the surface of a cone. To make this adjustment, level the instrument, and sight upon some well-defined point; then reverse the instrument in altitude, and fix a point in line at an equal distance from the instrument. (If the standards have not been adjusted, or if the horizontal axis is not level, these points must be in the same horizontal line, or nearly so.) Reverse the instrument in azimuth and sight at the first point; then reverse in altitude, and mark a point in line at the same distance as before. If the two points do not agree, move the vertical hair *one fourth* of the difference, and repeat the whole operation to test the accuracy of the adjustment.

* For general remarks upon adjustments, see § 37.

† The line joining the intersection of the cross hairs and the optical center of the objective is the line of collimation.

If there is any back-lash in the tangent screw, the instrument must be manipulated very carefully, to prevent any change of position on the vertical axis. There are several methods of making this adjustment, but this is the most simple, as well as the most accurate. Notice that this method is independent of all instrumental errors, with the possible exception that the line of collimation does not lie in the plane of the vertical axis; but any possible error in this respect will be inapppreciable except at very short distances.

Next proceed to make the vertical hair vertical. This adjustment is useful only in sighting at a flag-pole, to tell whether it is vertical. The simplest way to make this adjustment is to level the horizontal axis of the telescope (§ 126), and see whether the hairs will coincide with the corner of some building. Or, move rigorously, having the axis of the telescope horizontal, sight upon some well-defined point, and elevate or depress the telescope. If the hair is vertical, the point will seem to travel up and down the hair; if it does not, loosen the screws a trifle and turn the diaphragm slightly.

Before pronouncing upon any adjustment, repeat the test to discover the error of observation. The greater the accuracy of the instrument, the more important this precaution.

2. **Horizontal Hair.** The horizontal hair should be in the plane of motion of the optical center of the objective. If this condition is not satisfied, the line of collimation will change with every change of the object-glass, rendering the instrument useless for leveling, or for measuring vertical angles. In discussions of the adjustments of the transit no mention is ever made of an adjustment of the horizontal hair; although such an adjustment is necessary if the instrument is to be

used for anything except the measurement of horizontal angles.

To make this adjustment, drive a stake near the instrument and read a level-rod upon it; then, without moving the telescope in altitude, read a rod upon a second stake 200 or 300 feet distant. Reverse in altitude and azimuth, and bring the telescope to the former reading upon the first point; then read upon the other stake. If this reading is not the same as before, correct *half* the difference by moving the horizontal hair. After having adjusted the horizontal hair, test the vertical hair, for moving one may have changed the other.

It would be a great improvement if telescopes were so made as not to require an adjustment of the horizontal hair of the transits or vertical hair of levels. This improvement could be most easily applied to inverting telescopes, and is another reason for using such telescopes. In case the hair is non-adjustable, the engineer should test the adjustment for himself.

124. Centering the Eye-piece. After having adjusted the cross hairs, it may be that their intersection will not be in the middle of the field of view. This does not affect the accuracy of the work, but does affect the seeing power of the telescope. Some instruments are provided with two pairs of screws (*A A*, Fig. 27, page 100), like the screws which move the cross-hair ring, by which the inner end of the eye-piece may be moved so that the cross hairs shall appear in the center of the field. In inverting telescopes, no means is provided for making this adjustment.

125. Standards. The line of collimation should revolve in a vertical plane when the vertical axis is vertical; or, in other words, the horizontal axis should be horizontal when the instrument is leveled up, *i.e.*, the standards should be of the same length.

To make this adjustment, *having adjusted the cross hairs* (§ 123), level the instrument very carefully, direct the telescope to some high and well-defined point, and clamp the vertical axis. Establish a low point in the line of the telescope; reverse in altitude and azimuth, direct the telescope to the high point, and clamp the vertical axis. If the telescope, when turned down, does not cover the low point, correct *one half* the difference by lowering the end of the axis towards which the line of sight diverges.

In making this adjustment, it is only necessary to level the bubble perpendicular to the telescope. It is sometimes desirable to throw the instrument into a position that will command a large vertical angle, when the level parallel to the telescope can not be used. If the line of collimation has not been adjusted previously, the points used must be equally above and below the instrument, and at the same distance from it; for it must be borne in mind that if the line of collimation is not perpendicular to the axis of the telescope, it describes the surface of a cone.

Another method of making this adjustment is to cause the line of sight to follow a plumb-line; or, what is in effect the same thing, require it to cover a high point and its reflection as seen in a basin of mercury or water. Of these two, the second is the better, owing to the vibrations of the plumb-line; but both are really tests of the accuracy of the adjustment of the level as well as of the standards, and therefore neither of them is as good as the one first described.

126. Level under Telescope. The tangent of the level should be parallel to the line of collimation when the latter is horizontal. To make this adjustment, bring the bubble to the middle (any point will do equally well, but the middle is most convenient), and clamp the telescope. Read a leveling-rod on a point,

say 200 feet from the instrument; reverse in azimuth, bring the bubble to its former position by moving the tangent screw of the vertical circle, and establish a second point at the same distance from the instrument as the first one, and read the rod upon it. A line joining the two positions of the target is a horizontal line, and the difference of the readings is the *true* difference of level, however much the instrument may be out of adjustment. Move the instrument very near to one point—say, 10 feet beyond it,—and read a rod upon the first point; then, without changing the altitude of the telescope, read upon the second. If the difference of these two readings is the same as the difference of level, the line of collimation is horizontal. If the observed difference is not equal to the difference of level, correct a little more than all the error (for the exact amount see the next paragraph) by moving the farther target.

Let A and B, Fig. 29, be the two points. When the instrument is at C, midway between A and B, the tar-

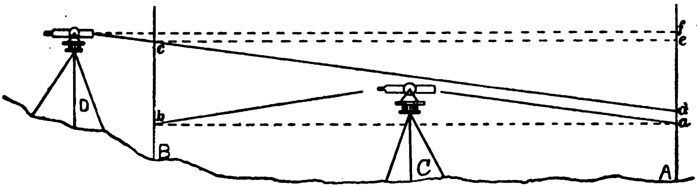

FIG. 29.

gets are at a and b, and ab is a level line; and the true difference of level between A and $B = Aa - Bb$. When the instrument is at D, the targets are at c and d; and the apparent difference of level is $de = (Bc - Bb) - (Ad - Aa)$. If the line of sight were horizontal, the target would be at f; and therefore df is the correction.

$$df = de\frac{AD}{AB} = de\left(\frac{2AC + BD}{2AC}\right).^{*} \quad . \quad . \quad . \quad (1)$$

* This formula is not strictly correct, since (1) it was assumed that a line which is horizontal at C, *i.e.*, perpendicular to the radius of the earth at C, is

If D is between A and B, the quantity BD must be subtracted.

In making this adjustment, take two or three pairs of observations each time for a check, and be careful to see that the instrument is exactly level at the moment of sighting.

When the line of sight is brought horizontal, bring the bubble to the middle by raising one end of the level tube, and the adjustment is complete. In moving the level tube, be very careful not to alter the inclination of the telescope. Notice that it will not do to move the horizontal cross hair to bring the line of collimation parallel to the level, as is sometimes recommended, for moving the cross hair will destroy its adjustment for collimation (2, § 123), and may destroy that of the vertical hair.

127. Zero of the Vertical Circle. The vertical circle should read zero when the vertical axis is vertical and the line of sight is horizontal. If the instrument has a level under the telescope, first adjust it (§ 126); then bring the bubble to the middle, and shift the vernier until its zero coincides with the zero of the limb. If the vernier is not movable, note the difference and apply it as a correction to each angle of elevation or depression. Notice that this adjustment is not necessary if only the vertical angle between two points is desired.

If the instrument has no level under the telescope, the method of making this adjustment is nearly the

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parallel to a line which is horizontal at D, and (2) the effect of refraction was omitted. Therefore the observations at D should be corrected for curvature and refraction (§§ 317–19). To make this correction, subtract $0.001(BD + 200)^2$ ft. from the reading at B, and $0.001[(2\,AC + BD) + 220]^2$ ft. from that at A; or, with sufficient accuracy, simply subtract $0.001(2\,AC + 220)^2$ ft. from *de* before using it in equation (1). Ordinarily this correction is less than the error of observation, and consequently may generally be omitted.

same as that given in § 126. Set the instrument midway between two points, level it carefully, *taking special care with the level parallel to the vertical circle*, clamp the telescope nearly horizontal, and determine the difference of level between the points. Move the instrument near one of the points, level it carefully, sight upon each point, and take the difference of reading. If this difference is equal to the true difference of level, as found when the instrument was midway between the points, the line of sight is horizontal. If the difference is not equal to the difference of level, alter the telescope in altitude (see the second paragraph of § 126) until the differences are the same. Finally, when the line of sight has been placed horizontal, move the vernier to coincide with the zero of the circle.

ART. 4. USING THE TRANSIT.

128. PRACTICAL HINTS. The beginner should not neglect the preliminary matters of planting the tripod, bringing the plumb-bob over the point, and leveling the instrument. There is great difference in the skill and rapidity with which different persons will set up the transit; but generally there is an unnecessary waste of time and hard labor. A very neat way of doing it is as follows: Tighten the screws in the upper end of the legs until friction will just hold them wherever placed, and open them until they make an angle of about 30° with the vertical, let down the plumb-bob, and set the instrument over the point; then a gentle pressure on the legs of the tripod will bring the plummet over the point, and a slight movement of the leveling screws will bring the plate level.

If the observer has clearly in mind what he is trying to do, and thoroughly comprehends the effect of possible errors in his instrument, he can save much time and

trouble, thus leaving himself free to bestow his attention where it will do the most good. For example, a great deal of time is often wasted in getting the instrument precisely over the point. The care should vary inversely as the distance of the object sighted at; an inch may cause an error of three seconds if the object is a mile away, but an error of three minutes if 100 feet away. The direction of the instrument from the point, with reference to the object sighted at, also affects the value of the resulting error.

It is a waste of time to level up accurately, each time, regardless of the kind of work to be done. If only the horizontal angles between points on the same level are to be found, the instrument can be leveled with sufficient accuracy by the eye alone; if only vertical angles between points in the same vertical are desired, only the level parallel to the telescope need be read. On the other hand, in measuring a horizontal angle between a high and a low point, particular attention should be given to the level perpendicular to the telescope. In all instrumental work there are certain operations which should be carefully attended to, while to give equal care to others would be only a waste of effort.

If the instrument is not firm, examine the tripod-head and the iron shoes on the legs, to see that they are not loose. No instrument can stand firmly with any looseness in these parts. The clamps and tangent screws also should be examined to see that they fit snugly. The instrument may slip on the lower plate, owing to the leveling screws' not being tight enough.

Some engineers seem to think that the harder the tripod legs are forced into the ground, the tighter the leveling screws are, and the tighter the instrument is clamped, the more accurate the work; but the contrary is more nearly true. An instrument keeps its adjustments better and works more kindly, when handled delicately.

129. Measuring Angles. Although it is scarcely necessary, a brief description of the method of measuring a horizontal angle will now be given. Set the instrument over the point, and level it. Then clamp the upper movement, and read one of the verniers, say *A*, using the 0°–360° graduation. Focus the eye-piece on the cross hairs, turn the telescope by hand until it nearly bisects one of the points, and clamp the lower motion. Next focus on the object and turn the lower tangent screw until the intersection of the cross hairs exactly covers the point. Then loosen the clamp of the upper motion, direct the telescope to the other point, and read as before. The difference of the two readings will be the angle between the two points.

It is immaterial whether the instrument is turned to the right or to the left. If there is only one row of numbers running from 0° to 360°, and if the vernier passes the zero point in turning to the second station, 360° must be added to the smaller reading before taking the difference. If there are two rows of numbers running from 0° to 360°, it is immaterial which way the telescope is turned, since that graduation may be read which increases in the direction of the motion. It is convenient, though not necessary nor quite as accurate, to set the vernier to read zero at the beginning, instead of reading it where it may chance to be.

130. Angles Measured More Accurately. It sometimes happens that an engineer desires an angle with the utmost accuracy. There are two methods of making the observations when extreme accuracy is desired; viz., *by series* and *by repetition.* One or the other of these methods is always used in measuring the principal angles of a geodetic triangulation. The principles involved are useful in less accurate work.

131. *By Series.* Sight upon the first station, and read both verniers to eliminate eccentricity. Sight upon

the next station to the right, and read as before. Continue around the horizon, reading upon each station, and close by reading upon the first station again. If the last reading is the same as the first, it proves that the instrument did not slip or get moved. Reverse in altitude and azimuth, turn on the lower axis a little to eliminate personal bias, and read upon the first station. Proceed around the horizon towards the left, reading upon each station and closing upon the first. The reversal in azimuth and altitude eliminates eccentricity of line of sight, error of telescope slide, and inclination of horizontal axis. Reversing the direction around the horizon eliminates any twist of the tripod ; and shifting the horizontal circle diminishes the possibilities of accidental errors of graduation. The above observations constitute one "set."

To secure greater accuracy by increasing the number of observations, and also to eliminate periodic errors of graduation, shift the horizontal circle an aliquot part of the distance between verniers, and take another set. The amount that the circle should be shifted between sets is equal to the distance between verniers divided by the number of sets to be taken. For example, if the angle is to be read three times, and if there are two opposite verniers, shift the circle 60°.

The arithmetical mean of the observed values is to be considered the true angle.

132. *By Repetition.* Sight upon the first station, read both verniers ; and with the upper motion turn to the next station, and read as before. The last reading is only for a check. With the lower motion turn back to the first station, the reading remaining unchanged ; then unclamp above, and turn forward again to the second station. The angle will now have been measured a second time, but on a part of the circle adjoining that on which it was first measured, the second

beginning where the first ended. This operation may be repeated any number of times, the circle being read after the last sight upon the second point. The difference of the first and last readings divided by the number of repetitions gives the angle more precisely than would a single observation. Notice that the vernier need be read only at the beginning and end, although the second reading, as above, is a valuable check in determining how many times 360° should be added to the last reading in case the vernier has passed 0°. Next reverse in altitude and azimuth, and measure the angle as before, beginning however at the second station.

This method eliminates all errors of adjustment, and reduces the error of observation by increasing the number of observations. Of course, the mean of the observed values is assumed to be the true angle.

133. *Comparison of Methods.* Both methods seem to be about perfect, as far as the elimination of errors of adjustment, of graduation, and of observation is concerned. The method by series is preferred by most observers for triangulation work. Its peculiar advantages can be fully realized only with the precise instruments used in that kind of work. With ordinary engineering instruments, there is a limit beyond which it is useless to multiply observations by this method. For example, if the instrument reads to minutes and the serial readings agree to minutes, the multiplication of observations adds nothing to the accuracy of the result.

The method by repetition was once a great favorite with the best engineers, especially the French, for triangulation work; but the improvements in the manufacture of angle instruments has given precedence to the method by series for the most accurate work. However, the method by repetition is peculiarly suited to the precise measurement of angles with a coarsely-divided circle, as, for example, a common engineer's

transit. The principle of this method is certainly very beautiful, but its accuracy is largely dependent upon the freedom with which the instrument turns on its centers, and upon the stability of the clamp and tangent screw. Under ordinary conditions, the limit of this method is reached after a few repetitions.

134. Transit Surveying. Under this head will be discussed methods of doing transit work which are more or less applicable in running a line survey—as for a railroad,—or in finding areas, or in topographical surveying. There is no generally accepted method of doing transit work. The three methods in more or less general use will now be considered. The first, for want of a better name, will be called the *angle method;* the second may be appropriately named the *quadrant method;* and the third has been called *traversing*. The last might well be named the full-circle system.

135. Angle Method. This method consists in measuring and recording the angle which each line makes with the preceding one. The angle measured may be the one included between the two lines, or the angle between the second line and the first produced. In the latter case, the telescope is sighted along the first course, the vernier read, and the telescope transited. The telescope then points in the direction of the first line produced. It is next turned to the second line, and the vernier read. The difference of the readings, *i.e.*, the angle swept over, is equal to the angle between the first course produced and the second. This angle is sometimes called the angle of deflection, and is recorded as a deflection to the right or to the left.

136. Quadrant Method. The distinguishing characteristic of this method is that the angles are read and recorded as bearings, just as in compass surveying. The meridian through the first station is obtained by reading the needle, or by sighting upon

some line whose bearing is known, or it may be assumed—since generally the object of such surveys is not so much to get the true bearings as to get the relative bearings accurately. The bearing of any succeeding line is found by measuring the angle which it makes with the preceding line produced, and adding it to or subtracting it from the bearing of the preceding line. This method is doubtless a survival from the time when the magnetic compass was the instrument ordinarily used in measuring horizontal angles. The 0°-to-90° numbering on the horizontal circle of transits is made especially for this system.

137. Traversing. Traverse surveying, or running a traverse, or simply traversing, is conducting a survey in such a way that the readings of the plate will show the angles which each line of the survey makes with any chosen reference line.

To run a traverse, set the instrument up over the first station—the end of the first course. For the present we will assume that the first course is the reference line, *i.e.*, the meridian of the survey. Set vernier *A* at 0°, clamp the upper motion, turn the telescope upside down, and with the lower motion direct the line of sight to the other end of the first course, then clamp the instrument, and transit the telescope. Now loosen the upper motion, sight upon the next station, and read vernier *A* again. Move to the next station, loosen the lower motion (to prevent the possibility of disturbing the reading in leveling up), level up, invert the telescope, and *glance at the reading to see that it has not changed.* Sight upon the last station (using the lower motion), and clamp. Invert the telescope, loosen the upper movement, sight upon the next station, and read. Proceed in like manner for any number of lines. Each reading is the angle between its corresponding course and the first one.

Fig. 30 shows the positions of the several lines of a traverse survey, and the following table shows the notes for the same.

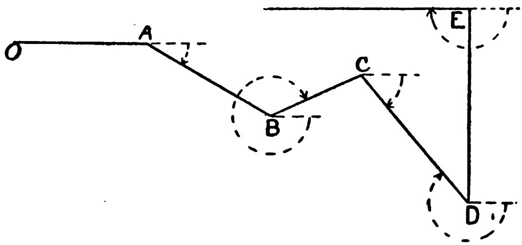

FIG. 30.

Stations.	Back-Sights.	Fore-Sights.	Remarks.
A	0° 00′	35° 52′	Azimuths read from vernier *A*, and counted from the south toward the west.
B	35° 52′	340° 31′	
C	340° 31′	41° 08′	
D	41° 08′	270° 00′	
E	270° 00′	180° 00′	

The instrument is first set at *A*, the line *O A* being regarded as the first course. The vernier is set at zero, and a back-sight taken to *O*, which is recorded opposite *A* in the back-sight column. The telescope is then transited, the upper motion loosened, the telescope directed to *B*, and the reading of this line, 35° 52′, recorded opposite *A* in the fore-sight column of the table. After the instrument is removed to *B* and the back-sight taken upon *A*, the vernier is to be read to make sure that it has not been changed, and this reading is recorded in the back-sight column. Writing this down will be evidence that the reading of the vernier was checked; and in actual work this will be found to be an important check against such errors as turning the wrong tangent screw or reading the wrong vernier. The angles marked in the diagram are the

corresponding angles of the fore-sight column in the table.

138. Instead of making the first course the reference line, as above, any other line, the meridian for example, may be taken as the reference line. In this case

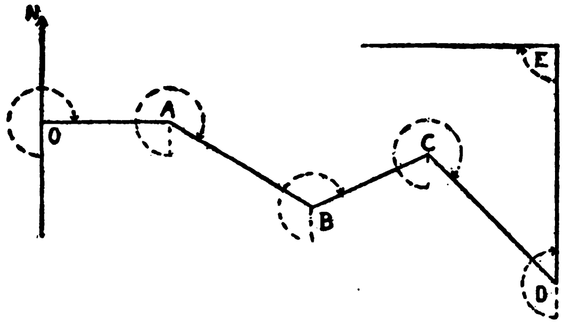

FIG. 31.

the instrument is first set up at the point of beginning, the vernier set as zero, and the first back-sight made in the direction of the meridian. The remainder of the field work and the record is as before.

It makes but little difference whether the first back-sight is made towards the north or the south end of the meridian, but a note in the remarks column should show which way it is made. However, it is more common, and therefore better, to count azimuths from the south toward the west.

139. Comparison of Methods. In the time required for the field work, there is very little difference between the three methods, the second and third requiring slightly less than the first.

The quadrant system is the oldest and the one most frequently used. Its chief advantage is in the facility of checking the vernier by the compass needle. One of the chief objections to this system is that four different directions are designated numerically by the same angle.

The principal disadvantage of the full-circle system is the greater labor required in checking the vernier by the compass needle; but even this could be removed by having the compass graduation of the transit run from 0° to 360°. However, this is more than overbalanced by the greater facility in computing and platting, and in the freedom from ambiguity and error. This system is very convenient in railroad surveying, and in surveys to find the area.* The full-circle system is indispensable in topographical surveying. In fact, the latter is the only kind of surveying in which it is employed to any considerable extent; but as its advantages become better understood, it will be more fully adopted.

140. SOURCES OF ERROR.† The errors of transit work may be classified as errors (1) of position, (2) of sighting, (3) of manipulation, (4) of adjustment, and (5) of reading.

141. Errors of Position. The instrument may not be set up over the point about which the angle is desired, nor over the point previously sighted at. An error of an inch can not make an error of more than 3 seconds if the object sighted at is a mile away; but it may make an error of 3 minutes if the object is only 100 feet away. The position of the instrument with reference to the true point and the object sighted at affects the values of the resulting error.

142. Errors of Sighting. The flag-pole may not be vertical, therefore sight as low upon it as possible. The intersection of the cross hairs may not exactly cover the point, owing to lack of care or to parallax in the instrument; but in either case the remedy is obvious. Always bring the intersection of the cross hairs to bear upon the point sighted at, for the vertical hair may not be vertical.

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* See Appendix II.

† For a discussion of Compensating *vs.* Cumulative Errors, see § 18.

143. Errors of Manipulation. The wrong tangent screw may be turned. This is a fruitful source of error, and one difficult to discover and impossible to correct. If the tangent screw has back-lash, the instrument must be handled so as to prevent it. Error is sometimes produced by the instrument's turning on the ball-and-socket joint, owing to the foot screws' not being screwed up tight enough. The upper part of the instrument should move so freely as not to twist the tripod.

144. Errors of Adjustment. The points to be considered are eccentricity, equality of standards, straightness of telescope slide, and adjustment of cross hairs. The eccentricity is always small, and easily eliminated by reading two verniers. The equality of the standards can affect only the horizontal angles between high and low points, and in ordinary practice the resulting error will be inappreciable. The straightness of the slide and the adjustment of the cross hairs are closely related; but neither will produce error either in measuring horizontal angles between points equal distances from, and equal distances above or below, the instrument, or in measuring vertical angles between points equally distant from the instrument. An error of adjustment of the line of collimation produces an error of double the amount in traversing, or in prolonging a line by back-sights and fore-sights. All errors of adjustment can be wholly eliminated by reversing, making a second observation, and taking the mean.

145. Errors of Reading. Errors may be produced by reading the wrong vernier, the wrong end of a double vernier, the wrong row of numbers, or by reading 28° instead of 32°, etc., or by forgetting to add the half-degree of the limb to the reading of the vernier and recording 20′ instead of 50′.

146. Limits of Precision. It is a little difficult to get sufficient data for a satisfactory discussion of this

subject without making observations specially for this purpose, which is not desirable. Work is done under very different conditions as to weather, speed, instruments, etc., and results without full information on all points are not very valuable.

The author's students, in the prosecution of the ordinary class work in topographical surveying, measured the angles of ten triangles. The length of the sides varied between 400 and 1,200 feet. The conditions as to time, targets, etc., were about those of actual practice. The instrument read to minutes, and certainly was not the best quality. All of the errors enumerated in §§ 140–45 were involved. This work gave 32 seconds for the probable error of a single direction; 50 seconds for the probable error of an angle; and 86 seconds for the probable error of the sum of the three angles of a triangle. The maximum error in the closing of a triangle was 3½ minutes.

The same class in measuring the four angles around a point and the three angles of a triangle, using chaining pins for targets, with sights about 100 feet long, obtained results about half as large as those above. The results of traversing, with flag-poles for targets and sights varying between 200 feet and 800 feet, gave results a little greater than half those of the preceding paragraph.

Ten measurements of an angle with an engineer's transit reading to minutes (the reading was estimated to 30 seconds), and "distinct signals at about 400 feet," gave a probable error of 19 seconds for a single value of the angle. Under the same conditions, the probable error of an angle as measured with a transit reading to thirds of minutes was 12 seconds.*

In locating the piers of the bridge across the Ohio

* Prof. Mansfield Merriman, in *Engineering News*, Vol. 10, p. 621.

River, at Cairo, Ill., by triangulation each angle was measured fifteen times with an engineer's transit reading to 10 seconds, the maximum sight being about 5,000 feet and the average about 2,500 feet. The average error of closure of the triangles was 1.5 seconds.*

In the topographical survey of St. Louis, Mo., the angles were measured with an engineer's transit reading to 10 seconds. Each series of courses began and ended at triangulation stations, which were about a mile apart. The azimuth was checked by comparing it with the azimuth of the lines of the triangulation. The average error of closure of azimuth was about 1′ 5″ for each series of courses, or about 11 seconds per line.†

147. A transit is sometimes used to determine areas. The legitimate errors in the balancing of the latitudes and departures in transit surveying can be discussed as for compass surveying (§ 52), the formulas deduced for that case being applicable to transit surveying. Such a discussion shows that ordinary work with the transit is proportionally as accurate as the best chaining. Therefore, in finding areas by the transit, as much care must be given to the chaining as to the measurement of the angles.

148. Care of the Transit. There are a great many small screws about a transit, and the general tendency is to overstrain them. This is especially true of the cross-hair screws. All straining of these screws beyond that necessary to insure a firm seat is more apt to cause the instrument to lose than retain the adjustment. Overstraining the leveling screws bends the plates and wears the screws unnecessarily. A very common fault is applying too much force in clamping the instrument. The operator frequently fails to appreciate the power of

* Journal of the Associated Engineering Societies, Vol. 9, p. 292.

† The Technograph, No. 5, p. 12.

a screw. Some instrument makers invite this abuse of their instruments by making the heads of the clamping screw too large. To avoid overstraining the clamp, it is best to ascertain by trial the minimum force required to produce sufficient hold for the action of the tangent screw, and then try to clamp only slightly in excess of this amount.

If the leveling or tangent screws get to working hard, take them out and brush with soap and water. The nuts can be cleaned by screwing a thin piece of soft wood through them. The tangent screw is intended only to complete the setting, and should never be used except to give a slight movement. The telescope slide and the centers should be examined occasionally; and if there is any fretting or cutting, take the piece out and burnish the rough place with some smooth hard tool, as the back of the blade of a pocket-knife.

To lubricate bearings exposed to the air, first thoroughly clean, apply a little watch-oil, vaseline, or rendered ox-marrow, and then wipe it off. The value of these three lubricants is in the order enumerated, vaseline and ox-marrow being too stiff in cold weather, and the ox-marrow also wearing out rapidly. Finely powdered plumbago is very good for exposed bearings. It is a wise precaution to carry in the field a gossamer water-proof bag to throw over the instrument in case of a shower, or in case the instrument must be left standing for any length of time exposed to dust.

To preserve the outer appearance of an instrument, never use anything in dusting it except a fine camel's-hair brush. To remove dust spots, first use the camel's-hair brush, and then rub with fine watch-oil, and wipe dry. If the oil is allowed to remain on, it will catch dust and dirt, and thus do more harm than good. On reaching the office, after using the instrument, dust it

off generally with a fine camel's-hair brush, examine the centers and all other principal movements to see if they run perfectly free and easy, and oil them if necessary.

To remove dirt and oxide that may have accumulated on the surface of a silver graduation, apply some fine watch-oil, and allow it to remain for a few hours; then take a soft piece of old linen and lightly rub until dry, but without touching the edge of the. graduations. If, after cleaning, the silver surface should show dark and bright spots (which would interfere somewhat with the accurate reading of the graduation), barely moisten the finger with vaseline and apply the same to the surface; then wipe the finger dry and lightly rub it once or twice around the circle, touching the graduation as little as possible. Such cleaning, however, must be resorted to only when absolutely necessary, and then only with the greatest care, as it is too apt to spoil the sharpness of the lines, which will decrease the accuracy of the reading.

It is not desirable to take the instrument apart unnecessarily, for even if the fittings are perfect, it requires considerable care to put them together properly. A little dust or dirt in a joint or bearing, or a screw left loose or tightened too much, may damage the instrument or cause errors in its use. As long as an instrument works well and the centers revolve freely, it is best not to disturb it.

For hints on the care of the telescope, see page 89.

CHAPTER VIII.

SOLAR TRANSIT.

149. THE solar transit is simply an engineer's transit to which is attached a solar apparatus (§ 55). There is great diversity in the form of the solar apparatus and in the method of attaching it to the transit.

ART. 1. CONSTRUCTION OF THE SOLAR TRANSIT.

150. SAEGMULLER'S SOLAR TRANSIT. The latest form of solar transit is shown in Fig. 32.* It consists simply of a telescope and level attached to an ordinary transit in such a manner as to be free to revolve in two directions at right angles to each other. When the transit telescope is horizontal, the auxiliary telescope and its level revolve in horizontal and vertical planes.

If the transit telescope be brought into the meridian and the vertical circle of the transit be set at the colatitude, the polar axis of the solar telescope will then be parallel to the axis of the earth; then if the solar telescope be turned on the polar axis, the solar sight-line will describe a line parallel to the equator. If the solar sight-line is perpendicular to the polar axis, the line of sight will describe the equator when the solar

* Invented by G. N. Saegmuller, in 1881. This form is made by G. N. Saegmuller, Washington, D. C., and by Keuffel & Esser Co., New York City; and somewhat the same form is made by Buff & Berger, Boston, Mass.

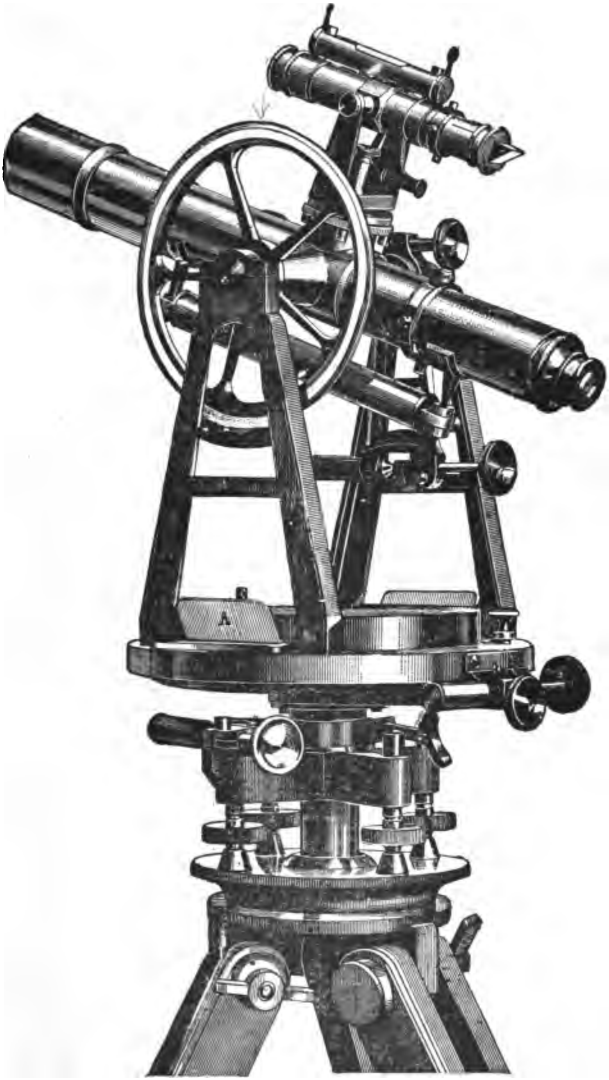

FIG. 32.—SAEGMULLER'S SOLAR ATTACHMENT.

telescope is revolved on the polar axis. If the solar sight-line makes an angle with the perpendicular to the polar axis equal to the declination of the sun, then when the solar telescope is revolved the line of sight would follow the sun's path in the heavens for the given day, *provided* the sun did not change its declination. Therefore, if the solar telescope is set at an angle with the perpendicular to the polar axis equal to the declination of the sun, and the solar telescope is directed to the sun, then the terrestrial sight-line will indicate a true meridian.

151. With the form of solar apparatus shown in Fig. 32 it is difficult to sight the solar telescope upon the sun accurately, owing to there being no clamp and slow-motion screw for the polar axis. The latest form of solar attachment has a clamp and tangent screw for the polar axis. See Fig. 33.

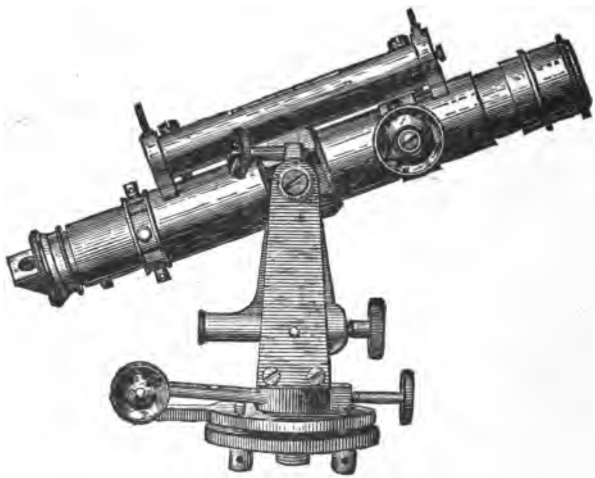

FIG. 33.—SAEGMULLER'S IMPROVED SOLAR ATTACHMENT.

Two pointers are attached to the solar telescope to aid in directing it to the sun. They are so adjusted

that when the shadow of the one is thrown on the other, the sun will appear in the field of view.

The solar telescope is provided with colored glass shades to protect the eye when observing. The objective and the cross hairs are focused in the usual way. There is a pair of horizontal cross hairs and also a pair of vertical ones, between which the image of the sun is brought.

152. Other Forms. Many solar transits consist of an ordinary engineer's transit to which is attached the solar apparatus of the solar compass (§ 56). The solar attachment is fastened to the center of the horizontal axis of the telescope, or to the end of the horizontal axis, or below the horizontal plate. All of these forms are inferior to the solar transit shown in Fig. 32 for one or more of the following reasons:

1. Most of them are more complicated. 2. All of them are more difficult to adjust. 3. All are deficient in precision. The solar sights of a solar compass consist merely of a small lens and a piece of silver with lines ruled on it placed at the focus of the lens, the coincidence of the sun's image with the lines being determined by the unaided eye or, at best, with a simple magnifying lens. This primitive telescope is probably about as accurate as the terrestrial sights of the common compass, but is much less accurate than the terrestrial telescope of the solar transit. It is obvious that the substitution of a telescope for the lens and the lines of the solar compass greatly increases the precision attainable. Obviously the power of the solar telescope should be in keeping with that of the terrestrial telescope. 4. The solar telescope can be used when the sun is partly obscured by clouds, at which time the ordinary solar apparatus fails altogether.

ART. 2. ADJUSTMENTS OF THE SOLAR TRANSIT.*

153. The adjustments of the solar transit consist of two distinct operations—the adjustment of the transit and that of the solar apparatus.

The transit should be in perfect adjustment, particularly the plate levels (§§ 38 and 39), the standards (§ 125), and the zero of the vertical circle (§ 127).

154. ADJUSTMENT OF THE POLAR AXIS. The polar axis should be vertical when the line of collimation and the horizontal axis of the transit are horizontal. To make this adjustment set the vernier of the vertical circle to read zero, and level the instrument by means of the plate levels. If there is a level under the telescope, the verticality of the vertical axis can be tested by revolving the transit on its vertical axis, and noticing whether the bubble of the level under the telescope remains stationary during the entire revolution. If it does not remain stationary, correct half the error by turning the foot screws. Since the level under the telescope is usually more sensitive than the plate levels, it is better to level the instrument by the latter than by the former. When these steps have been correctly made, the plane of the horizontal axis and the line of collimation are horizontal.

Then, to bring the polar axis vertical, revolve the solar telescope about the polar axis and notice if the bubble on the small telescope maintains a constant position. If it does not, correct half the movement by revolving the solar telescope on its horizontal axis, and the other half by means of the adjusting screws at the base of the solar apparatus (see Fig. 33, page 132).

* For general remarks upon adjustments, see § 37.

Notice that these adjusting screws are analogous to the foot screws of the main instrument.

155. Adjustment of the Cross Hairs. The line of collimation of the solar telescope and the axis of its level should be parallel. Bring the terrestrial telescope horizontal by setting the vernier of the vertical circle, or by reading the bubble under the telescope. Place the solar telescope as nearly as possible in the plane of the terrestrial telescope, and make the former horizontal by means of its bubble, and clamp it. Measure the distance between the axes of the two telescopes, and draw at this distance from each other two heavy black lines on a piece of paper. Set this piece of paper up, with the lines horizontal, at a convenient distance from the instrument and on about the same level as the telescope. Bring the terrestrial line of sight upon the lower mark, and see if the solar line of sight falls upon the upper mark. If it does not, move the cross hairs of the solar telescope until it does. Since moving the cross hairs is very liable to revolve the solar telescope on its horizontal axis, test the adjustment by revolving back to the horizontal position, and see if both bubbles come to the middle simultaneously.

Art. 3. Using the Solar Transit.

156. To Determine a Meridian. Notice that an observation with the solar transit involves four quantities, as follows: (1) the time of day, *i.e.*, the hour angle of the sun; (2) the declination of the sun; (3) the latitude of the place of observation; and (4) the direction of the meridian. In a general way, if any three of these are known, the fourth may be found by observation. The prime object of the solar transit is to find the true meridian when the other three elements are known.

157. The Time. The declination of the sun is tabulated for Greenwich or Washington noon, and hence a correction must be applied to find the declination at the time of the observation. An error of 30 minutes in the time can not make an error of more than 30″ in the declination, and the average error will be about 15″, which is less than can be laid off on the instrument. Therefore it is sufficient if the error in the time does not exceed 30 minutes; and ordinarily there will be no difficulty in finding it much closer.

158. The Declination. The declination of the sun, and also the hourly change in the declination, is given for each day of the year, in the "American Ephemeris and Nautical Almanac,"* for both Greenwich and Washington mean noon. Hence, to find the declination at any given time, it is necessary to know the time of day and the longitude from either Greenwich or Washington; and since the time generally employed in this country, *i.e.*, standard time, differs from Greenwich time by an integral number of hours, it is better to use the Greenwich declination. Therefore, for a point on, say, the meridian of New Orleans (90° west from Greenwich), the declination given in the ephemeris for Greenwich noon is the declination at 6 a.m. at the place of observation. For a point either side of the 90th meridian, the standard time does not indicate the true hour angle of the sun, *i.e.*, there is a difference between standard and local time; and hence for a place west of the 90th meridian the declination at noon at Greenwich is the declination at some time before 6 a.m., and for a place east of the 90th meridian the declination at Greenwich noon is the declination at some time after 6 a.m.

* Issued several years in advance by the U. S. Government, and for sale by book dealers. Price $1.25.

Since the difference between standard and local time is ordinarily less than 30 minutes, and since the hourly change in declination is less than 60 seconds of arc, it is sufficiently exact to assume that the declination of the sun given in the ephemeris for Greenwich noon for any day is the declination at 7, 6, 5, or 4 a.m. of the same date according as the point is situated in the Eastern, Central, Mountain, or Western time belt. Thus if the point of observation is in the Central time belt, the declination given in the ephemeris may be assumed to be the declination at 6 a.m. at the place of observation. Of course, this is strictly true only for points on the governing meridian of each time belt.

Knowing the declination for a given time of day at the point of observation, to find the declination for any other time of day it is only necessary to multiply the hourly change by the difference between the time for which the declination is given and the time for which it is required, and add it (algebraically) to the tabulated value. For example, if the declination at Greenwich noon on November 19, is $-19° 28' 24''$, and the hourly change is $-35''$, what is the declination at 11 a.m. at Chicago? The declination at Greenwich noon corresponds (nearly) to the declination at 6 a.m. at Chicago; and hence a change for 5 hours (from 6 a.m. to 11 a.m.) must be allowed for. Therefore the required declination is $-19° 28' 24''$ *plus* $(-35'' \times 5)$, which equals $-19° 31' 19''$. In a similar way the declination for any hour may be found.

159. *To Correct the Declination for Refraction.* Owing to refraction, all celestial objects appear higher than they really are. The declination to be set off on the solar apparatus is the apparent and not the real declination, and hence the declination as found above, before being set off on the instrument, must be corrected for refraction.

The refraction is zero for a point in the zenith, about 1′ for an altitude of 45°, and about 34″ at the horizon. It varies greatly with the temperature, pressure, and hygrometrical condition of the atmosphere,—particularly near the horizon. It is for this reason that all astronomical observations made near the horizon are very uncertain. Tables of mean refraction are frequently given in text-books on astronomy,* in logarithmic tables, etc.

Notice that refraction changes the apparent altitude, which is measured perpendicular to the horizon, while we desire to know the effect upon the apparent declination, which is measured perpendicular to the equator of the heavens. When the sun is on the meridian, the change in altitude has its full effect in changing the declination, but at other times the change in declination is less than the change in altitude. The correction to the declination due to refraction is

$$C'' = 57'' \cot\,(\delta + N),\dagger \quad . \quad . \quad . \quad . \quad (1)$$

where δ = declination,—plus when north and minus when south. N is an auxiliary angle such that

$$\tan N = \cot\phi \cos t, \quad . \quad . \quad . \quad . \quad (2)$$

where ϕ is the latitude, and t is the hour angle.

160. By means of equations (1) and (2) the refraction correction can be computed for any latitude, hour angle, and declination. Messrs. W. and L. E. Gurley, Troy, N. Y., publish a table giving this correction for each $2\frac{1}{2}°$ of latitude from 30° to $57\frac{1}{2}°$, for each 5° of declination, and for integral hour angles from 0 to 5.

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* For example, Loomis's Practical Astronomy, p. 364, and Chauvenet's Spherical and Practical Astronomy, vol. ii, pp. 604–7.

† See Chauvenet's Spherical and Practical Astronomy, vol. i, p. 171.

G. N. Saegmuller, Washington, D. C., publishes annually for gratuitous distribution a small pamphlet giving the declination of the sun and its hourly change for each day of the year. In the margin of the table is given the refraction correction for 40° of latitude from 0 hours to 5 hours of hour angle. In a supplemental table is given a series of co-efficients by which the refraction correction for 40° of latitude must be multiplied to obtain the refraction correction for any other latitude. As very complete explanations accompany these tables, and as it is necessary to have a table of declinations for the current year, it is not thought wise to consider this subject further here, but the reader who has any considerable solar work to do is advised to send for this little pamphlet, the title of which is "Solar Ephemeris and Refraction Tables."

161. The Latitude. The latitude is required that the polar axis may be set parallel to the axis of the earth. To find the latitude, a few minutes before the meridian passage of the sun level the transit carefully and point the telescopes toward the south. Then incline the terrestrial telescope—if the declination is north depress it, and if south elevate it,—until the reading of the vertical circle is equal to the declination of the sun uncorrected for refraction; and bring the solar telescope into the vertical plane of the terrestrial telescope, level it carefully, and clamp it. By moving the *terrestrial* telescope in altitude and azimuth, bring the image of the sun between the hairs of the solar telescope, and keep it there until the sun ceases to rise. Then the reading of the vertical circle, corrected for refraction, is the co-latitude.

162. The Observation. First focus the solar telescope for distinct vision of the cross hairs, and then carefully focus it upon the sun. It is necessary to make these adjustments before setting off the declination,

since the solar telescope is liable to be displaced if they are made afterwards.

Set off the declination by inclining the transit telescope until the vertical circle reads the declination—if the sun is south of the equator elevate the transit telescope, and if north depress it. Without disturbing the position of the transit telescope, bring the solar telescope into the vertical plane of the transit telescope, and also to a horizontal position by means of the level on the solar telescope. The two telescopes will then make an angle with each other equal to the declination, and the inclination of the solar telescope to its polar axis will be equal to the polar distance of the sun. Without disturbing the *relative* position of the two telescopes, set the vernier of the vertical circle to the co-latitude of the place of observation.

By revolving the transit on its vertical axis and the solar apparatus about its polar axis, taking great care not to revolve either telescope on its horizontal axis, bring the image of the sun into the solar telescope, and then the transit telescope must be on the meridian. If the vertical motion of the transit be clamped, the transit telescope may be turned down and the meridian marked; or the horizontal circle may be read, and the angle which any line makes with the true meridian can be found.

163. Sources of Error.* There are three sources of error in addition to those of adjustment and manipulation of the instrument, viz.: (1) error of declination, (2) error of latitude, and (3) error of refraction.

164. Declination and Latitude. Table II† (page 141) shows the errors in the direction of the meridian due to an error of 1 minute in the latitude or declination.

* For a discussion of Cumulative *vs.* Compensating Errors, see § 18.

† From "Theory and Practice of Surveying," by J. B. Johnson, by permission.

TABLE II.

ERRORS IN AZIMUTH DETERMINED BY SOLAR TRANSIT FOR 1 MINUTE ERROR IN DECLINATION OR LATITUDE.

Hour.	For 1 Min. Error in Declination.			For 1 Min. Error in Latitude.		
	Lat. 30°.	Lat. 40°.	Lat. 50°.	Lat. 30°.	Lat. 40°.	Lat. 50°.
	Min.	*Min.*	*Min.*	*Min.*	*Min.*	*Min.*
11.30 a.m. / 12.30 p.m.	8.85	10.00	12.90	8.77	9.92	11.80
11 a.m.... / 1 p.m.....	4.46	5.05	6.01	4.33	4.87	5.80
10 a.m.... / 2 p.m.....	2.31	2.61	3.11	2.00	2.26	2.70
9 a.m..... / 3 p.m. ...	1.63	1.85	2.20	1.15	1.30	1.56
8 a.m..... / 4 p.m.....	1.34	1.51	1.80	0.67	0.75	0.90
7 a.m..... / 5 p.m.....	1.20	1.35	1.61	0.31	0.35	0.37
6 a.m..... / 6 p.m.....	1.15	1.30	1.56	0.00	0.00	0.00

Several important conclusions may be drawn from this table and the equations from which it was deduced.

1. "The solar apparatus should never be used between 11 a.m. and 1 p.m., and preferably not between 10 a.m. and 2 p.m., if the best results are desired.

2. "At 6 a.m. and 6 p.m., when the line of collimation lies in a plane at right angles to the plane of the meridian, no small error in the latitude will affect the accuracy of the result.

3. "The best times of day for using the solar apparatus are from 7 to 10 a.m. and from 2 to 5 p.m. So far as the instrumental errors are concerned, the greater the hour angle the better the observation; but when the

sun is near the horizon, the uncertainties in the refraction may cause unknown errors of considerable size.

4. "For a given error in the setting for declination or latitude the resulting error in azimuth will have opposite signs in forenoon and afternoon. If, therefore, a 10-o'clock azimuth is in error 5′ in one direction from erroneous settings, a 2-o'clock observation with the same instrument should give an azimuth 5′ in error in the opposite direction.

5. "If the declination angle be erroneously set off, and the latitude angle be also affected by *an equal error in the opposite direction*, then the two resulting errors in azimuth will nearly balance each other.

6. "If the instrument is out of adjustment, the latitude found by a meridian observation will be in error; but *if this observed latitude be used* in setting off the co-latitude, the instrumental error is eliminated. Therefore always use for the co-latitude that given by the instrument itself in a meridian observation."

165. Refraction. The refraction correction to the declination (§ 159) is computed on the assumption that the refraction is a mean, whereas the actual refraction at any time and place may differ considerably from the mean or average. This difference is liable to be very much greater at low than at high altitudes. For this reason no observations should be made within 20° of the horizon, and preferably not within 30°. Fortunately most solar work is done in the summer, when the sun is high in the heavens, and this limitation is less serious then than in the winter.

Any error in the refraction correction to the declination has the same effect as an equal error in the declination. The refraction correction may be computed by means of equations (1) and (2), page 138; and the effect of any assumed per cent of error in this correction may be determined by an inspection of Table II, page 141.

For example, if the observation be made in latitude 40° on September 23 at 7 a.m., the refraction correction will be a little more than 3′; and if we assume that the refraction may be in error 20 per cent (probably a fair assumption for this altitude), the refraction correction may be in error nearly 40″, and from Table II we see that the corresponding error in azimuth would be a trifle over 50″.

If the observations are not made within 20°, or, better, 30°, of the horizon, and if the limitations in § 164 are observed, the error due to refraction can not be serious.

166. Limits of Precision. The author's students in ordinary class work adjust their own instruments, and determine a meridian with the Saegmuller solar attachment (the latitude and time being known accurately) with a maximum error of 4′ to 5′ between observations made in quick succession, with an average error of 2′ to 3′. Eight students made three observations each, and the greatest difference between the mean of each three and the true meridian (as determined by observations with an astronomical transit) was 4′.6. The average difference was 1′.6, the probable error (Appendix III) of the mean of three observations was 1′.8, and the probable error of a single observation was 1′.0.

167. The following results by Professor J. B. Johnson, of Washington University, St. Louis, Mo., may be taken as the best than can be done:*

"In order to determine just what accuracy was possible with a Saegmuller solar attachment, I spent two days in making observations on a line whose azimuth had been determined by observations on two nights on Polaris at elongation, the instrument being reversed to eliminate errors of adjustment. Forty-five observations

* Journal of the Association of Engineering Societies, Vol. 5, p. 35.

were made with the solar attachment on October 24, 1885, from 9 to 10 a.m., and from 1.30 to 4 p.m., and on November 7 forty-two observations between the same hours.

"On the first day's work the latitude used was that obtained by an observation on the sun at its meridian passage, being 38° 39′, and the mean azimuth was 20″ in error. On the second day, the instrument having been more carefully adjusted, the latitude used was 38° 37′, which was supposed to be about the true latitude of the point of observation. It was afterwards found this latitude was 38° 37′ 15″, as referred to Washington University Observatory, so that when the mean azimuth of the line was corrected for the 15″ error in latitude it agreed exactly with the stellar azimuth of the line, which might have been 10″ or 15″ in error. On the first day all the readings were taken without a reading glass, there being four circle readings to each result. On the second day a glass was used. On the first day the maximum error was 4′, the average error was 0′.8, and the probable error of a single observation was also 0′.8. On the second day the maximum error was 2′.7, the average error was 1′, and the probable error of a single observation was 0′.86. The time required for a single observation is from three to five minutes."

168. The Solar Transit in Mine Surveying. Since the standards of the solar telescope, as ordinarily made, are long enough to allow the small telescope to sight past the horizontal plates of the transit, the solar attachment can be used for oblique or vertical sighting, as is frequently required in mine surveying.

If the solar transit is used for this purpose, it should be adjusted as described in §§ 153–155, and the lines of sight of the two telescopes should lie in the same vertical plane. The last adjustment may be made by

leveling the instrument and sighting both telescopes upon a plumb-line. When this adjustment has been made the lines of collimation of the two telescopes are parallel, and any angle, say 90° from the horizontal, may be set off on the vertical circle, and the sight made through the small telescope. All instrumental errors are eliminated if, after making one observation, the transit is reversed on its vertical axis and another observation is made,—the mean of the two points being the correct one.

CHAPTER IX

PLANE TABLE.

ART. 1. CONSTRUCTION.

169. In its simplest form, the plane table consists of a drawing board mounted on a tripod, on which lines are drawn to represent the direction of any object as indicated by a ruler placed so as to point to the object. Any other parts are mere additions to render the operations more convenient and precise.*

170. COMPLETE PLANE TABLE. Fig. 34 (page 147) shows one of the most elaborate forms of plane table. An instrument of this form is virtually a transit in which the horizontal circle is replaced by the drawing board, the lines being drawn upon the paper instead of being read from the graduated limb. The lower plate of the transit is expanded, and becomes the drawing board; and the vernier or index is replaced by a straight-edge, which with the telescope and vertical arc is commonly called the alidade.

The alidade is usually about 20 inches long and 2½ inches wide. At the center is a standard which supports the telescope and which serves as a handle for the alidade. In the best plane tables the telescope is

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* "The invention of the plane table is ascribed to Praetorious in 1537, but the first published description appears to be that of Leonhard Zubler in 1625, who ascribes the 'beginning' of the instrument to one Eberhart, a stone-mason."

FIG. 34.—COMPLETE PLANE TABLE.

equal to that on an ordinary transit. In some forms the telescope does not transit on its horizontal axis, but is reversed in azimuth by lifting it out of its bearings. The telescope has no lateral movement with respect to the ruler, but both may be moved at pleasure on the table. The telescope is movable in a vertical plane, and is provided with a vertical circle. In the form shown in Fig. 34, the telescope is mounted centrally over the standard. In some forms the line of sight is placed over the beveled edge of the ruler, though this is not essential. It is only necessary that they should have a fixed horizontal angle with each other.

The mechanism connecting the board with the tripod is the most important part of the plane table. The board must be supported rigidly in a horizontal plane, and also be free to move in that plane. The difficulties of satisfying these conditions are increased by the weight and possible eccentric position of the alidade. The instrument should also have a substantial clamp and tangent screw for the motion in azimuth. In addition, portability must be considered. The form shown in Fig. 34 secures great stability with little weight, and allows the facilities necessary for manipulation. Plane tables of this class are usually supported upon three leveling screws.

A declinator, a small box carrying a needle which can swing 10° or 15° either side of the zero line, should accompany the instrument for use in determining the magnetic meridian. The zero line being parallel to one edge of the box, the magnetic meridian may at once be marked down on any portion of the map, and the bearing of any intersecting line may be determined by a protractor. Sometimes the needle is capable of swinging through 360°, in which case the magnetic bearing of any line may be read with the compass-box alone.

The board varies greatly in size, but usually is not

larger than 24 by 30 inches. It is very important that the board should be made of well-seasoned wood and be so constructed as not to warp. The securing of a plane upon which to work is one of the most difficult conditions to fulfill in the construction of a plane table. Brass and plate glass have been used for this purpose in Europe, but make the table excessively heavy. The paper is fastened to the board in any of several ways: (1) by thumb-tacks screwed into a metal socket set in countersunk holes so as to place the head of the tack or screw out of the way of the ruler; (2) by wrapping it around rollers at the ends of the board; (3) by pressing the end of the sheet against the end of the board with a strip of wood or metal connected to the board by screws; (4) by movable spring clamps gripping the edge of the board; (5) by pasting it down.

171. Light Plane Table. The form shown in Fig. 34 is quite heavy and difficult to handle on rough or obstructed ground. The mechanism connecting the board to the tripod is the chief source of weight. To reduce the weight of these parts, the form shown in Fig. 35 was invented.* In the figure, *a* represents a

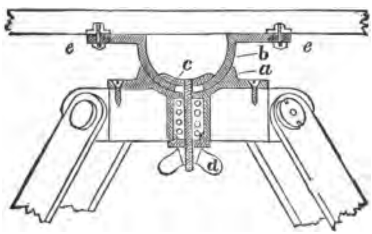

Fig. 35.—Gurley's Plane Table.

hemispherical concave metal cup fastened by screws to the wood top of the tripod, and *b* a convex hemispherical cup fitting into the cup *a* and being clamped to it at

* Patented and manufactured by W. & L. E. Gurley, Troy, N. Y.

will by the clamping piece *c* and nut *d*. A strong spiral spring in the hollow cylinder between *c* and *d* holds the two spherical surfaces of the two pieces together, and also allows the easy movement of the one within the other. The flange of the cup *b* supports the board and is connected with its under surface by three segments of brass, two of which are shown at *e*, *e*. These are brought down firmly upon the shoulder of the flange by capstan-head screws as shown, or released at will, thus allowing the board to be moved horizontally when desired.

Fig. 36 shows an improvement of the preceding form,

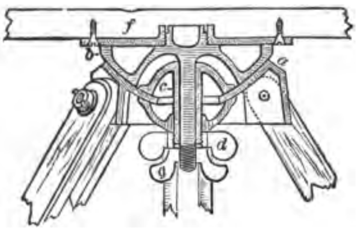

FIG. 36.—JOHNSON'S PLANE TABLE.

which has been adopted by the U. S. Geological Survey. The arrangement is essentially the same as in Fig. 35, with the addition of the winged nut *g*. When it is desired to level the table, the clamping nut *d* is released, the board is brought into position, and then securely clamped by the same nut. When it is desired to turn the table in azimuth, the nut *g* is loosened, which leaves the hemispherical surface *b* free to move around the concave part *a* of the tripod head. The only objection to this form of plane table is the impossibility of leveling the board sufficiently for the accurate determination of vertical distances with the vertical circle of the alidade. For work to which this limitation does not apply, this is a very excellent form of instrument.

172. Home-made Plane Table. A very fair plane table can be made very cheaply, if the engineer has a transit or leveling instrument in which the upper part is detachable from the leveling screws. This requires a good drawing board, say 18 by 20 inches, to the under side of which is screwed a casting of iron or brass similar to Fig. 37. The conical portion, *a*, should be made to fit the socket from which the axis of the level or

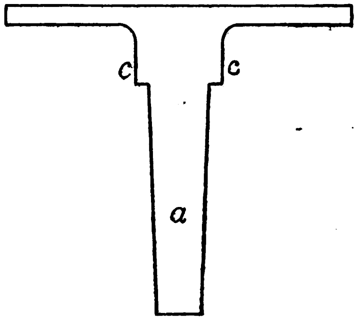

Fig. 37.

transit is taken ; and the cylindrical portion, *c c*, should fit the lower clamp. The alidade may be made of wood, similar to that shown in Fig. 38. One slit is narrow,

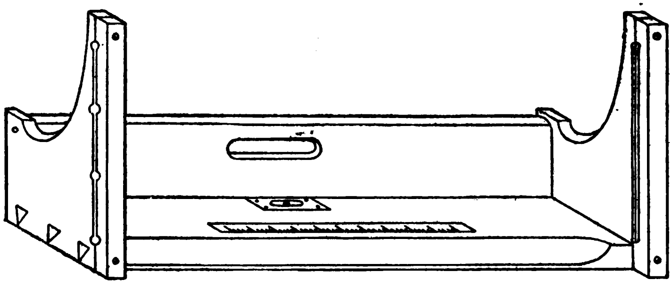

Fig. 38.—Alidade.

the other is wide and has a thread through the middle. The scale can be obtained by attaching a printed paper

scale. It is convenient to have a separate scale also. A level vial may be obtained of an instrument maker for a few cents, and may be fastened in the alidade with plaster of Paris. In fixing the vial in position, remember that, as usually made, level tubes are convex on one side and concave on the opposite. The former should be up, else the bubble will run to one end or the other, and never stop in the middle.

It is sometimes necessary to place a point on the board exactly over the corresponding point on the ground, which requires that the plumb-bob shall be suspended from the under side of the board immediately below the point given on the upper surface.* To aid in doing this, a plumbing bar, or frame, similar to Fig. 39,

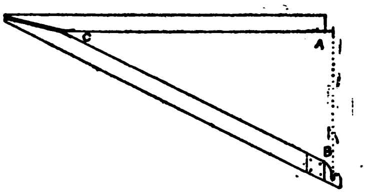

FIG. 39.—PLUMBING BAR.

is very convenient. It is made of two light bars of wood fastened together by a hinge at *C*. *A* is a piece of metal sharpened to a vertical edge. *B* is a piece of metal fastened into the end of the arm *B C*, and shaped to carry a plumb-line. The point *A* is placed upon the point on the paper, and the plumb-line is suspended from *B*. The plumbing bar is so made that if the under side of the upper arm is horizontal, the line joining *A* and *B* is vertical. The length of the arm *A C* should be a little greater than half the greatest dimension of the board, and *A B* should be a little greater than the distance from the top of the board to the bottom of the tripod head.

* For a method of obviating this difficulty, see § 188.

ART. 2. TESTS AND ADJUSTMENTS.*

173. THE SIGHTS. The sights of the elementary alidade (§ 172) should be perpendicular to its base. This can be tested sufficiently with a try-square. If this condition is not satisfied, the slits will not be vertical when the board is level, and sighting through the top of one and the bottom of the other will give a different line from that given by sighting through the tops or the bottoms of both.

174. EDGE OF RULER. The edge of the ruler should be a straight line. To test this, place the rule upon a smooth surface and draw a line along the edge; then reverse the rule end for end, place the edge upon the line, and again draw a line. If the two lines coincide the edge is straight.†

175. LEVELS ON ALIDADE. The bubble should be in the middle when the table is level. To make this adjustment, place the alidade in the middle of the table and bring the bubble to the center by means of the leveling screws of the table. Draw lines along the edge of the rule to show its exact position, and then reverse it 180°. If the bubble remains in the center, it is in adjustment. If it does not, correct one half by means of the leveling screws of the table, and the other half by the adjusting screws attached to the level.

It is next to impossible to make this adjustment accurately if the table is not a perfect plane. Great care

* For general remarks upon the adjustments, see § 37.

† "There is one deviation from a straight line, which, by a very rare possibility, the edge of the ruler might assume, and yet not be shown by the above test. It is when a part is convex, and a part similarly situated at the other end concave, in exactly the same degree and proportion. In this case, on reversal, a line drawn along the edge of the rule would be coincident with the other, though not a true right line." To determine whether this defect exists, move the alidade endwise and draw a third line.

is necessary not to disturb the table in making the adjustment.

176. Board. 1. The top surface should be a plane. Test it in all directions by a straight-edge (§ 174). If it is not perfectly flat, work it down with a smooth plane, a scraper, or sand-paper.

2. The face of the table should be perpendicular to the vertical axis of the instrument. To make this adjustment, set up the instrument, place the alidade on the table, and bring one of the bubbles, preferably the one on the telescope, to the middle; then reverse the table on its axis. If the bubble has not moved, the portion of the table covered by the alidade is perpendicular to the vertical axis; if it has moved, correct half the error by inserting washers between the table and the arms connecting it to the tripod head.

Turn the alidade 90° on the face of the board and repeat the test.

177. Telescope. 1. The line of sight of the telescope should be perpendicular to the horizontal axis. This adjustment is the same as that described in § 123 (page 109).

2. The horizontal axis of the telescope should be parallel to the top of the table. This is the same adjustment as that discussed in § 125 (page 111).

3. To make the line of collimation coincide with the fiducial edge of the alidade, level the table, set two needles in its face in range with some object, say 10 feet away; then place the fiducial edge against the needles, and direct the telescope toward the object. If the cross hairs bisect it, the adjustment is correct; but if they do not, it can be corrected by means of the screws attaching the standard to the rule.

The line of sight of the telescope is usually in the plane of the fiducial edge, although it is not necessary that it be either in the plane of the edge or parallel to

it; but it is necessary that the two should have a fixed horizontal angle with each other.

178. Zero of Vernier. The vernier of the vertical arc should read zero when the line of sight is horizontal. This adjustment is the same as that of § 127 (page 114). This adjustment is important when elevations are to be determined by vertical angles.

179. Level on Telescope. The bubble should be in the middle when the line of sight is horizontal. This is the same adjustment as that of § 126 (page 112). It is important only when elevations are to be determined by using the telescope as a level and measuring the difference of heights by a level rod. Since the level on the telescope is ordinarily more sensitive than the plate levels, difference of elevations can be determined more accurately by horizontal lines of sight than by vertical angles.

Art. 3. Using the Plane Table.

180. For many kinds of work the plane table is a very useful instrument. Even the simplest form (§ 172) may be used to advantage in obtaining the plat and area of the irregular tracts that occur in ordinary land and city surveying; and it is valuable in making plats of parks, cemeteries, mining property, etc. It has the great advantage of dispensing with all notes and records of the measurements (since they are platted as they are surveyed), and thus saves time and lessens the possibility of making mistakes.

181. Methods. Points may be located with respect to each other by any of four methods, viz., (1) radiation, (2) traversing, frequently called progression, (3) a combination of the first and second, here called radio-progression, and (4) intersection. The explanation of these methods will be worded for the determination of

a plat of a field, because that process includes all the operations involved in plane-table work with one exception—viz., placing the table in position at an undetermined point by observations upon three known points (see § 191).

With a little study the operator can discover many modifications and combinations of these methods. The facility with which plane-table work can be modified to suit circumstances is one of its excellencies.

182. Radiation. This is the most common method of using the plane table, particularly in topographical surveying. It is the simplest, though not the most accurate, method of finding the plat of a field.

Let *A*, *B*, *C*, *D*, *E*, *F*, Fig. 40, represent the corners

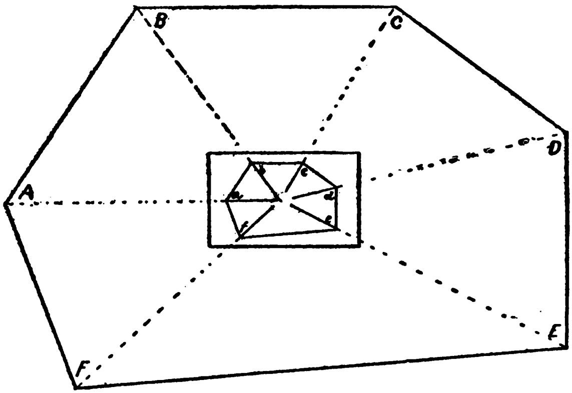

Fig. 40.

of a field the plat of which is to be found, and the rectangle near its center the plane table.*

Set the instrument at some point *O*, near the center of the field, from which all the corners are visible. Level

* The size of the table in this and the following figures is greatly exaggerated.

the table, and stick a pin or needle having a sealing-wax head in the prolongation of the plumb-line.* To determine whether the pin is set exactly right, revolve the board and notice whether the pin is stationary. Place the alidade against the pin, direct it to any corner of the field, as *A*, and draw a line. Measure *OA*, and set off this distance, to any convenient scale, from the needle along the line just drawn to *a*. In the same way plat *b*, *c*, *d*, etc., to represent the corresponding corners of the field. Join *a* and *b*, *b* and *c*, etc., and a complete plat of the field is obtained.

Notice that this method is deficient in checks upon the accuracy of the work. Any movement of the table during the progress of the work may be detected by sighting upon the first station.

183. The position of trees, houses, etc., may be determined in the same way. By using an alidade having a telescope with stadia hairs, the distance to the object may be determined from the stadia reading and the elevation from the vertical circle (see Chapter X.). Thus the map may be drawn in the field, which is a decided advantage in some respects, but seriously objectionable in others.

184. Traversing. This method is frequently called *progression*; but as it is strictly analogous to traversing with the transit (§ 137), the term traversing seems preferable. Let *A B C D* be the field to be surveyed. Select some point, *a*, Fig. 41, on the table to represent the first corner of the field, *A*. Estimate the dimensions of the field and so locate *a* that the plat will fall upon the board. Draw a line through *a* to represent *AB*; then measure *AB*, and lay off its length along this line to *b*.

* The center of the table should be permanently marked by setting flush with the face of the board a piece of brass, say one-fourth inch in diameter, having in it a hole just large enough to admit a common pin.

Set the instrument at *B*, so that the point *b* on the board shall be exactly over the corresponding point on the ground and the line *ab* on the plat shall have the

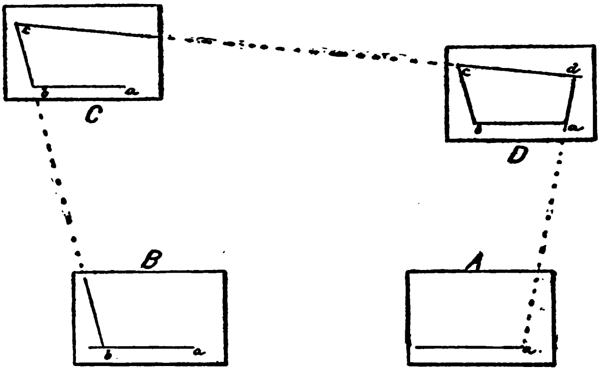

FIG. 41.

same direction as *AB* on the ground. To satisfy this condition with any considerable degree of accuracy* requires great patience and care. The difficulty is that having placed the point on the board over the point on the ground, it is necessary to destroy this condition in turning the board to make the direction of *ab* coincide with that of *AB*.†

Having placed *b* over the corresponding point on the ground and made the direction of *ab* coincide with that of *AB*, place the alidade against the needle at *b* and sight to *C*. Measure the distance *BC*, and lay it off

* The degree of accuracy required in setting the point on the board over the corresponding point the on ground depends upon the scale of the map and the distance to the object sighted at.

† A plane table with a shifting center would be very convenient for this purpose; but such tables are not made, owing to mechanical difficulties in their construction. A German plane table is provided with a double slide-rest motion for this purpose.

from *b* to *c*. Move the instrument to *C*, and proceeds as before. At the last station, *D*, determine the position of the first station, *A*, as though it were not already platted. The agreement of the two determinations of *a* is a check upon the accuracy of the work. The work may be checked as it progresses, by seeing whether any line, as *ca*, on the plat agrees with the corresponding line, *CA*, on the ground. The ability to check the work at every step is a valuable feature of this method.

185. Instead of trying to place the point on the paper exactly over the point on the ground, it is better to set the table level and approximately over the point, and then sight a line through the point on the paper to a temporary point having a corresponding position with reference to the point to be determined. For example, if in setting the table over *K* to determine the direction of *L*, the point *k* is 6 inches perpendicularly to the right from the line *KL*, set near *L* a point 6 inches perpendicularly to the right and sight at the point so marked. The line drawn on the paper is then parallel to the line on the ground. The true distance *K L* is to be measured and laid off on the paper.

186. This method is especially suited to the survey of a road, stream, or the like. Often the offsets required may be sketched in by the eye with sufficient accuracy.

When the paper is filled, put on a new sheet, and begin by fixing on it two points which were on the former sheet, and from them proceed as before. The sheets can then afterwards be united, so that all the points shall be in their true relative positions.

187. Radio-progression. This method combines the simplicity of the method by radiation with the checks of the method by traversing (progression). The chief advantage is the convenience with which the table is set up at each station, since the center of the table is always set over the station occupied,

Place a needle in the center of the board and set the table over any corner of the field, as *A*, Fig. 42. Take a sight to one of the adjacent corners, as *E*. Next sight upon the other adjacent corner, as *B*. To avoid confusion, mark the lines so as to indicate the side of the field to which they correspond. Move the table to the station last sighted at, set it up, and turn it until the alidade, when placed upon the last line drawn, bears upon the station previously occupied. Sight at the next station in order around the field, and proceed in a similar manner at all the corners. Fig. 42 represents

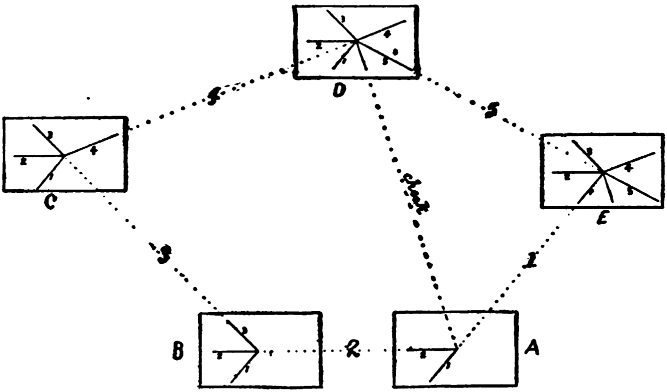

FIG. 42.

the table at each corner in succession, commencing at *A*. The sides of the field and the corresponding lines on the table are numbered similarly for convenience of reference. If the sighting is correctly done, the fore-sight from the last station will coincide with the back-sight from the first station. The work may be checked as it progresses by sighting at some other than the two adjacent points used above. *ad*, Fig. 42, is such a check line.

The lengths of the several sides are to be measured as usual.

It is obvious that the lines radiating from the center of the table are respectively parallel to the sides of the proposed plat. To draw the plat, it is only necessary to assume an initial point and draw lines through it parallel to any two adjacent sides of the field, and then lay off on these lines the lengths of the respective sides, to any convenient scale (see Fig. 43). Two other cor-

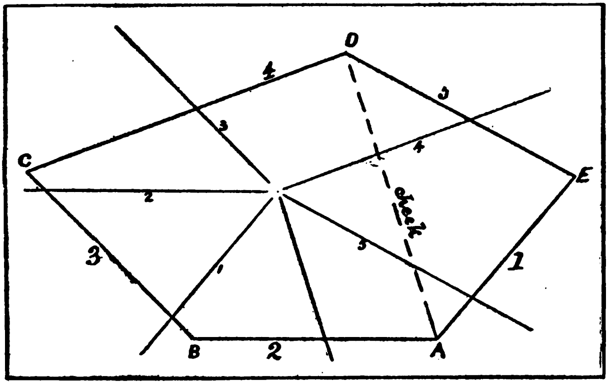

FIG. 43.

ners of the plat are thus determined. Through each of these draw a line parallel to the next side of the field; and do likewise for all the remaining corners.

The drawing can be checked as it progresses, by platting the check lines and noticing whether they pass through the corresponding point in the map. The closing of the plat checks the accuracy of all the work.

188. The principle of having the center of the table always over the point occupied may be applied in locating the position of trees, corners of buildings, points on streams, etc., by drawing through the point on the table corresponding to the point over which the instrument is set a line parallel to the edge of the alidade. This requires the use of two large and similar triangles in

drawing the line parallel to the edge of the alidade. The point occupied by the instrument being located on the paper, the alidade is placed against the needle standing in the center of the board, and the sight turned upon the point to be platted. Then without disturbing the alidade place one triangle against the edge of the alidade and the second triangle against the first, and slide one against the other until the edge of the second passes through the point on the paper corresponding to the point occupied. Next draw a line along the edge of the triangle, and on this line lay off the distance from the instrument to the point sighted at. Proceed in like manner for each point to be located.

Before moving the table to the next station, it is well to check the position of the table by re-determining the direction of the first line. For greater accuracy and speed in making this test, it is better to draw a line *through the center* of the board to mark the first position of the alidade. Having checked the position of the table, sight to the next station to be occupied, and mark the position of the alidade by a line through the center of the board. This line must also be transferred. Then move the instrument to the next station, set the tripod over the point, level the board, place the alidade on the last line drawn, through the center and turn the table until the lineof sight covers the point formerly occupied. Observations may now be made as at first.

If a considerable number of points are to be determined at each setting of the instrument, it is easier to spend some time in setting the point on the paper over the corresponding point on the ground, than to transfer all the lines; but if there are only a few points to be determined at each setting, it is easier to set the center of the board over the point and transfer the lines One advantage of keeping the center of the board over the point on the ground is that the alidade, which has

considerable weight—particularly one carrying a telescope—always stands centrally on the board and never near one edge. Notice that the two triangles take the place of the plumbing bar. As ordinarily made, plane tables have no means of hanging a plumb-line from the center of the instrument; but the deficiency is easily and cheaply supplied. If the two edges of the alidade are parallel, the triangle may be placed against either edge, as is most convenient.

189. Intersection. Measure any line, preferably central in the tract to be surveyed, on the ground, as *M N*, Fig. 44. Select some point on the paper to represent

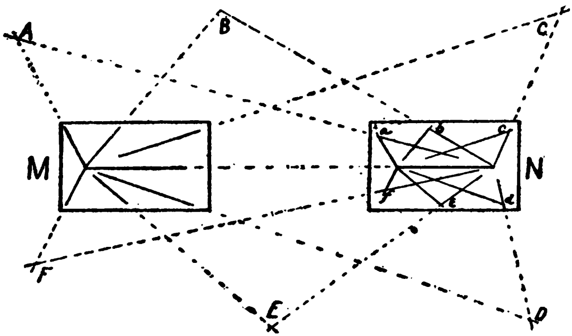

FIG. 44.

M, draw a line through it to represent the direction of the line *MN*, and lay off the distance *MN*. Then set the point on the paper representing *M* over the corresponding point on the ground, and orient the table so that the alidade when placed upon the line *MN* will sight to *N*, and clamp the instrument. Then sight to all the points whose location is desired, as *A*, *B*, *C*, *D*, etc., and draw lines to show the direction of these points. Each line and each point should be so marked that they can be certainly identified. Notice that if

there are any considerable number of lines it is not enough simply to put a letter or figure *on* the line, as in Fig. 44, since another line may be subsequently drawn through the number. Under these conditions the designation of the line should be placed in a small rectangle, one side of which coincides with the line to be identified. If necessary, the points sighted at may be marked by driving a numbered stake beside them.

Next, set the instrument over N, placing the point on the paper over the corresponding point on the ground, and orient the table so that the direction of the line on the paper corresponding to MN coincides with that line on the ground. Then sight at all the points and draw lines to correspond with their new directions. The intersection of the two lines of sight to each point will determine its position.

If there are other points not visible from M, as will be the case in a line survey, sight at them from N and measure a new base line, and proceed as before. Or the end of the new base line may be determined, as any other point, by the intersection of two sight lines from the ends of the original base line. Obviously, measuring the new base directly is the more accurate.

190. Before the introduction of the stadia, the method of intersection was the most usual and most rapid method of using the plane table. But this method affords no check upon the accuracy of the work, is often deficient in precision on account of oblique intersections, and requires so many lines upon the board as to cause confusion and error. It is well adapted to the mapping of harbors, shore lines, and generally to the plotting of inaccessible points. Of course, in this as in all triangulations, well-conditioned triangles give more satisfactory results; or in other words, avoid, if possible, angles less than 30° or greater than 150°.

191. The Three-point Problem. In this problem three stations *A*, *B*, *C*,* are plotted, as *a*, *b*, *c*, on the table, and the instrument being set up over a fourth point *D*, it is required to find the position of this point on the map. This problem is indeterminate when the point *D* lies in the circumference of a circle passing through *A*, *B*, and *C*, in which case the two-point problem (§ 194) may be applied. The three-point problem, which also occurs in the use of the sextant in locating soundings, has been much discussed, and many solutions have been proposed.† Only two will be given here.

192. Mechanical Solution. Fasten a sheet of tracing-paper on the board, and fix a point *d* to represent the station at which the instrument is set. With the alidade centring on *d*, direct the telescope successively to *A*, *B*, and *C*, and draw lines of indefinite length along the ruler's edge towards these stations. Then, if the tracing-paper be shifted until the three lines thus drawn pass through the points *a*, *b*, and *c*, the point *d* will indicate the position of *D*. The position of this point may now be transferred to the map by pricking through. The tracing-paper is then removed, and the table oriented.

193. Graphical Solution.‡ Let *a*, *b*, and *c*, Fig. 45, be the points on the sheet representing the signals *A*, *B*, and *C*, on the ground. Set the table at the point to be determined, *D*, and level it. Set the alidade upon the line *ca*, and by revolving the table, sight upon the signal *A*, and clamp. Then with the alidade centring

* The capital letters refer to the points on the ground, and the lower-case letters to the corresponding points on the board.

† The U. S. Coast and Geodetic Survey Report for 1880, pp. 180–84, gives a number of solutions elaborately illustrated.

‡ Known as Bessel's method by inscribed quadrilateral—see U. S. Coast and Geodetic Survey Report, 1880, p. 181.

on *c*, sight upon the middle signal *B*, and draw the line *ce* along the edge of the ruler. Set the alidade upon the line *ac*, direct the telescope to the signal *C* by revolving the table, and clamp. Then with the alidade centring on *a*, direct the telescope to the middle signal *B*, and draw the line *ae* along the edge of the ruler. The point *e* (the intersection of these two lines) will be in the line passing through the middle point and the point sought. Set the alidade upon the line *be*, direct *b* to the signal *B* by revolving the table, and the table will then be in position. Clamp it, center the alidade upon *a*, direct the telescope to the signal *A*, and draw along the ruler the line *ad*. This will intersect the line *be* at the point sought. To verify its position, center the alidade on *c*, and sight to *C*.

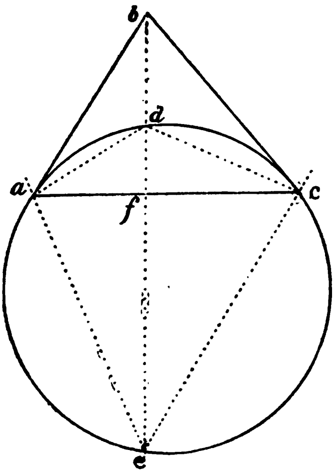

FIG. 45.—THREE-POINT PROBLEM.

The opposite angles of the quadrilateral *adce* being supplementary, the angles *ace* and *ade* are subtended by the same chord *ae*, and *cae* and *cde* are subtended by the same chord *ce*; and, consequently, the intersection of *ae* and *ce* at *e* must fall on the line *db*. Or, the segments of two intersecting chords in a circle being reciprocally proportional, the triangles *adf* and *cef* are similar, as also the triangles *cdf* and *aef*; and therefore *d*, *f*, and *e* must be in a right line passing through *b*.

194. THE TWO-POINT PROBLEM. There are several solutions* of this problem, only one of which will be

* U. S. Coast Survey Report, 1880, pp. 184–85.

given. Two points, *A* and *B*, not conveniently accessible, being given by their projections *a* and *b*, it is required to put the plane table in position at a third point, *C*. Select a fourth point, *D*, Fig. 46, such that the intersections of lines from *C* and *D* upon *A* and *B* make sufficiently large angles for good determinations. Put the table approximately in position at *D*, by estimation or by compass, and draw the lines *Aa* and *Bb*, intersecting in *d'*. Through *d'* draw a line directed to *C*, and on this line lay off, from *d'*, the estimated distance *CD*, and mark the point thus found *c'*. Set the instrument on *C* with *c'* over the point, and orient on *D* by the line *c'd'*. Draw lines from *c'* to *A* and to *B*. These will intersect the lines *d'A* and *d'B* at points *a'* and *b'*, which form with *c'* and *d'* a quadrilateral similar to the true one, but erroneous in size (since the distance *c'd'* was assumed), and in position (since the table was not properly oriented at either station).

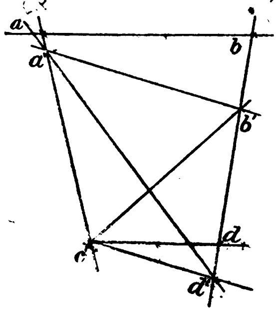

FIG. 46.—TWO-POINT PROBLEM.

The angles which the lines *ab* and *a'b'* make with each other is the error in position. By constructing now through *c'* a line *c'd''*, making the same angle with *c'd'* as that which *ab* makes with *a'b'* and directing the line *c'd'* to *D*, the table will be brought into position, and the true point *c* can be found by the intersection of *aA* and *bB*.

Instead of transferring the angle of error by construction, we may conveniently proceed as follows, observing that the angle which the line *a'b'* makes with *ab* is the error in the position of the table. As the table now stands, *a'b'* is parallel with *AB*; but we want to turn it

so that *ab* shall be parallel to that line. Therefore if we place the alidade on *a′b′* and set up a mark in that direction, and then place the alidade on *ab* and turn the table until it again points to the mark, *ab* will be parallel to *AB*, and the table is in position.

195. Sources of Error.* The sources of error in plane table work, in addition to those of chaining (§ 19) or stadia measurements (§§ 229–32) are as follows: (1) error of position of instrument; (2) error of sighting; (3) movement of the board between sights; (4) errors of adjustment of the instrument; (5) the inclination of the board; (6) error in marking the line upon the paper; (7) error in scaling off; and (8) the hygrometric effect of the atmosphere on the paper. The effect of any of these errors will depend upon the scale of the map. The first four are essentially the same as the corresponding errors in transit work, for discussion of which see § 140. If the board is not level, an error is produced in determining horizontal angles between points not in the same horizontal plane, and also in determining vertical angles between points not in the same vertical plane. The sixth and seventh depend upon the skill and care of the draughtsman. The changes in the dimensions of the drawing, due to the hygrometric state of the atmosphere, will vary with the weather and the kind of paper. The U. S. Coast Survey,† to determine this variation, cut several strips two meters long from three samples of drawing paper, and observed the variations daily for six months. For strips cut longitudinally the average variation was 9.0 millimeters, 13.0 millimeters, and 15.7 millimeters, and the maximum was 11.1, 15.5, and 20.9, respectively; and for

* For a discussion of Compensating *vs.* Cumulative Errors, see § 18.

† U. S. Coast Survey Report for 1862, p. 255.

strips cut transversely the average variation was 8.4, 8.1, and 12.1 millimeters, and the maximum was 10.3, 9.6, and 15.0, respectively.

196. Limits of Precision. Since there is so much variety in the kind of work, in the method of doing it, and in the conditions under which it is done, it is scarcely possible to state in general the degree of precision to be obtained with the plane table. Furthermore the plane table is designed more for rapidity than accuracy, although the best modern instruments are capable of a considerable degree of precision.

197. With Plain Alidade. In ordinary class work of the first term of surveying the author's students measured three angles of a triangle and four angles around a point, using a wooden alidade with slit and string for sights, with an average error, for the best thirty-two out of thirty-six results, of 1 minute and 44 seconds per angle (a probable error of 88 seconds). Under fair conditions the maximum error per angle should not exceed 3 minutes.

Under the above conditions the error of finding areas* was as follows: for radiation (§ 182), an average error of 1 in 586 and a maximum error of 1 in 274; for traversing (§ 184), an average error of 1 in 826 and a maximum error of 1 in 450; and for radio-progression (§ 187), an average of 1 in 1,111 and a maximum error of 1 in 640. For the sake of comparison it may be interesting to know that for the same students under the same conditions the average error with a magnetic compass (§ 51) was 1 in 1,220 and the maximum 1 in 580; and with a chain alone, the average error was 1 in 1,520 and the maximum 1 in 500.

198. With Telescope Alidade. Rarely, if ever, would the most elaborate form of plane table be employed in

* See second foot-note, p. 28.

finding areas; and, for reasons before stated, it is impossible to give a summary of the precision attainable with this form of plane table in topographical surveying. Therefore this subject will be concluded with a few references to descriptions of surveys made with the plane table, which contain data on precision, cost, and speed.

1. Professional Papers of the Engineer Department, U. S. A., No. 18,—Report of the Geological Exploration of the Fortieth Parallel, by Clarence King,—Washington, D. C., 1878, Vol. I, p. 762.

2. U. S. Geographical Surveys West of 100th Meridian, Wheeler, 1883, p. 47.

3. Report on the Third International Geographical Congress and Exhibition at Venice, Italy, 1881, Geo. M. Wheeler, Washington, 1885, pp. 79–81.

4. Science (New York City), July 29, 1887, p. 49.

5. Plane Table Methods used by the U. S. Geological Survey in Western Massachusetts, Louis F. Cutter, in Journal of the Association of Engineering Societies (Chicago), Vol. X, pp. 356–69.

199. Practical Hints. The board should be placed so low as to be readily reached, even at the most remote corners, and yet high enough to enable the observer to sight with comfort. This will bring it a little below the elbow. All beginners are apt to set the table too high. Care must be taken that no part of the body touch or rest against the edge of the board. In using the alidade, steady the standard with the left hand, while the right swings the rear end of the ruler in the proper direction.

Manilla paper is easier on the eyes in the sunshine than white, and also shows dirt less. Thumb-tacks and rollers for holding down the sheet are both objectionable, especially in high winds. The edges may be pasted underneath, or spring clamps may be used

to advantage. The sides of the sheet where they are turned under the table and come more or less in contact with the coat of the observer, should be protected by strips of paper about 4 inches wide, and 6 inches longer than the sides of the table, so as to fold under it and clamp on with the sheet itself. Tracing vellum is good for this purpose, as the points near the edge of the sheet can be seen through it.

A short linen coat with large pockets, in the style of a hunting-coat, probably affords the best means for carrying the requisite accessories for plane-table work; viz., scale, triangles, pencils, rubber, note-book, etc. By this means the weight of these articles is distributed in separate pockets, and they are always at hand when needed.

On beginning the work, set off at some point near the middle of the sheet, the magnetic meridian for the purpose of putting the table in approximate position at any subsequent station with the declinator.

Use as hard a pencil, and make as few lines, as possible. In locating points on contours, plot the distance at once along the edge of ruler by a detached scale, making only a dot at the point which should receive the number of the contour. Objects on a straight line may be quickly located by plotting the ends of the line and determining the intermediate points by the intersections of this line and lines of sight to the several points.

Always before leaving a station, and also at intervals when not otherwise employed, take sights to determine whether the board has been displaced.

It is well to have ready a light india-rubber cloth cover to slip over the board in case of a sudden shower, as well as to protect the paper from dust on the roads, mud in swampy ground, etc. A metal chart case should always accompany the table to protect the

sheet from sudden rain and other injury liable to occur in the transportation of the sheet to and from the field, and for its safe keeping when not in use. Its diameter should be not less than 3 inches, for no sheet can be rolled to a less diameter without serious rupture of the fiber of the paper.

CHAPTER X.

TELEMETERS.

200. TELEMETER is a term variously employed to designate some form of an instrument for determining distances by means of the visual angle subtended by a short base. There are a great number of instruments of this class, but the only ones of any considerable value in engineering practice are the stadia and the gradienter. These instruments will be discussed at considerable length in Arts. 1 and 2, respectively, and in Art. 3 various forms of telemeters will be briefly described.

ART. 1. THE STADIA.

201. The stadia is an instrument for determining the distance of a point from the observer by the visual angle subtended by an object of known size placed at the point. Ordinarily not only the distance but also the horizontal and vertical angles are observed, these three being sufficient to determine the direction, distance, and elevation of the point upon which the rod is placed. In 1820 Parro, an Italian engineer, first suggested the determination of distances in surveying by a visual angle and a rod.* He used the word *stadia* to designate the rod, but the term is now generally applied to the instrument as a whole. On the U. S. Coast and Geodetic Survey the term *telemeter* is used instead of stadia. In Great Britain the instrument by which the observation is made is called a *tacheometer*, and the rod is called a stadia.

* William Green, a London optician, in 1778 published a pamphlet giving a very full account of the stadia, but the credit of its practical introduction seems to be due to Parro.

The principles involved have long been well known, and have been applied in gunnery and military reconnoissance; but it is only lately that they have been used in engineering. The first notable use of the stadia was in 1836, in making a topographical survey of Switzerland. It seems not to have been introduced into America until nearly thirty years afterward, and has not been employed here to any considerable extent except in topographical surveys carried on by the U. S. Government. It has not come into as general use as its merits warrant, although it is more generally used in Europe than in the United States. The stadia is peculiarly adapted to topographical surveying, for it possesses the double advantage of giving both the horizontal and vertical co-ordinates, and this, too, by the most rapid method.

202. Principles. In all its forms the stadia is an application of the principle of similarity of triangles. The simplest form of the instrument is used in the familiar method of determining the distance of a man from an observer, by measuring on a rule held at arm's length the space covered by his height. There are several forms of this simple device, but none of them are of any practical value in surveying, owing to the impossibility of focusing the eye for two distances at the same time, and to the indistinctness of the farther object.

To employ the principle of the stadia with a telescope, it is necessary to introduce two parallel cross hairs and observe the amount of the rod intercepted between them. The hairs may be horizontal and the rod vertical, or *vice versa;* but the former is usually preferred, since the rod is then more steady, and there is less liability of its being obscured by brush, etc. The hairs may be fixed, the intercept on the rod being variable; or the hairs may be movable and be set to cover always the same two points on the rod, the vari-

able distance between the hairs being measured by a graduated screw. The fixed hairs are cheaper, more simple, more accurate, and in every way better than the movable ones; and are generally used.

In the stadia with a telescope the two similar triangles have a common apex at the optical centre (see first foot-note, page 106) of the objective, the base of one being the distance between the cross hairs, and that of the other the intercept on the rod. Since in focusing the telescope, the distance from the cross hairs to the objective varies with the distance of the object sighted at, the relation between the distance to the rod and the intercept is not as easily found as with the simple device previously mentioned. The formula for the stadia with telescope will be deduced presently.

203. Placing the Hairs. In placing the stadia hairs in the telescope three conditions must receive attention: they should be parallel, equally distant from the central one, and at a suitable distance from each other.

These conditions are the rigorous ones, but farther on it will be shown that no appreciable error will be produced if they are only approximately satisfied. It is necessary that the hairs should be parallel only in case the observation is not made exactly on the vertical hair. If the stadia hairs are not equally distant from the central one, it produces only a small error in the vertical angle. Finally, the distance between the hairs is immaterial, provided the rod is graduated to correspond. A formula will be deduced to meet the case in which the rod is already graduated and does not agree with the distance between the hairs (§ 215).

Any measuring telescope can be used as a stadia by adding a second horizontal hair; but it is much better to add two extra ones, one on each side of the ordinary one. With the stadia hairs thus placed, the field of view is symmetrical about the center, and the added

hairs interfere less with the ordinary uses of the telescope.

204. Instrument makers not infrequently fasten three horizontal cross hairs on the reticule (§ 80), the two outer ones being at such a distance apart that each foot of the intercept on the rod corresponds, at least approximately, to a hundred feet of the distance from the instrument to the rod. The space between these hairs is computed and fine grooves are engraved at the proper distance, into which the hairs are laid and fastened. If these hairs get broken the engineer can replace them (§ 81) with a little care.

Fig. 47 shows a common method of making the distances between the stadia hairs adjustable. The upper portion of Fig. 47 shows the reticule in place in the telescope tube, and the other shows details. The screws *d* and *f* can be operated from the outside of the telescope tube, and move (with reference to the ring *a*) the slides *b* and *c*, which carry the stadia hairs. The screws *d* and *f* are shown in Fig. 22, page 92, immediately in front of the ordinary cross-hair screws. *e* is a bent spring to take up lost motion in the screws *d* and *f*. The ring *a* and all parts connected therewith are adjusted in the telescope tube by the ordinary cross-hair screws, which are not shown. By means of the screws *d* and *f*, the distance of the stadia wires from the central wire, and from each other, can be varied at will. Lines are made upon the slides *b* and *c* to assist in placing the hairs parallel to each other. The following objections

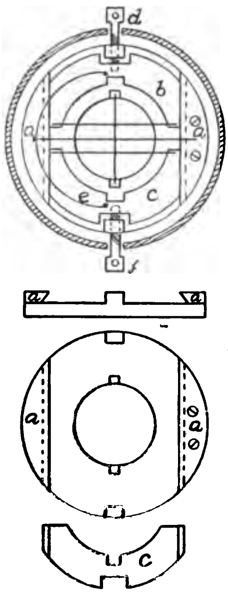

FIG. 47.

are sometimes offered to this construction: 1. The projecting heads of the screws *d* and *f* are liable to be struck and turned in handling the instrument or in carrying it through brush, and so produce a serious error with no adequate means of detecting it. 2. Since a spring must be inserted to take up lost motion, the screws have a tendency to work loose. 3. It is expensive.

205. Fig. 48 shows an inexpensive method of inserting adjustable stadia hairs, in an instrument not provided with them. The diagram represents the front view of the ordinary reticule. *a*, *b*, *c*, and *d* are small wire plugs which are free to turn, being held only by friction. The dark portion is in the plane of the face of the ring, and the light portion projects, say an eighth of an inch, above. The

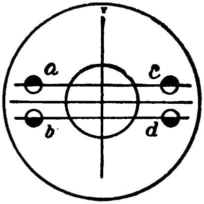

FIG. 48.

hairs are to be stretched in the line of the centers of *a* and *c*, and *d* and *b*, and fastened *at the outer edge of the ring*. Then by turning the plugs the hairs will be moved toward or from the central hair, according to which side of the plug is toward it. The wire plugs may easily be made to fit so tight as not to work loose and still turn freely enough for the above adjustment. With telescopes having inverting eye-pieces, which are much the best for stadia work, the wires can be turned, after removing the eye-piece, without taking the reticule out of the telescope tube. With an erecting eye-piece, the hairs can be adjusted without removing the ring from the telescope tube, by the use of a little wrench (made especially for this purpose by cutting a kerf with a hack-saw in the end of a small strip of brass), and operating it through a hole in the telescope tube (also made for the purpose), which can be closed by a metal or rubber band; or the ring may be removed from the tube to adjust the hairs. The telescope does not need to be

collimated to test the relative position of the stadia hairs.

This method provides a way of making the hairs parallel to each other, and allows a variation in the distance between the central and each outside hair equal to the diameter of the plug, which may be made of any size to suit the circumstances.

206. The maximum distance between the hairs is limited by the size of the field of view (§ 87). The field of view of the telescopes on ordinary engineering instruments differs with different makers, but is about as follows: for a magnification of twenty, 1° 30′; of twenty-five, 1° 15′; of thirty, 1°; and of thirty-five, 50′. With the smallest power mentioned above the distance between the stadia hairs can not be greater than one fortieth of the focal length of the objective, and for the largest power one seventieth. Since it is not possible to have the hairs at the extreme edge of the field of view, and since the outer portion of the field is not as good optically as the central portion, it is safe to say that with the higher powers the distance between the stadia hairs should not be more than a hundredth of the focal length of the objective. With the lower powers the distance between the hairs might be greater, but for long sights the rod would be inconveniently long.

On the other hand, if the hairs are very close together the intercept on the rod will be too small to be determined accurately. All things considered, it is probably best to make the distance between the hairs a hundredth of the focal length of the objective. A method of accurately making this adjustment will be described presently (§§ 214–15).

207. The Rod. In stadia work there are two kinds of rods—self-reading and target. A self-reading rod is one having a graduation such that the value of the intercept can be read through the telescope. A target rod is one

having a sliding target moved by the rod-man, in response to signals from the instrument-man, until it is in the plane of sight, when its position is read by the rod-man. The latter rod requires two targets, one of which is sometimes permanently fixed to the rod to save the trouble of setting two targets; but when the vertical co-ordinate is desired, this adds more complication than it saves. Target rods may be a little more accurate, but they are certainly very much less convenient. Self-reading stadia rods are generally preferred.*

Figs. 49–52 (page 180) show a few of the many graduations proposed for self-reading stadia rods. Fig. 49 is a graduation formerly much used on the U. S. Coast and Geodetic Survey, and is on the whole the best of the four shown. The distance from *a* to *b* corresponds to 10 feet on the ground. By dividing the oblique side of the triangle into fifths by estimation, the rod may be read to single feet. Notice that the division of the oblique side of the triangle is the principle of the well-known diagonal scale. Fig. 50 was used on the late U. S. Lake Survey, and is the standard on the surveys conducted by the Mississippi River Commission. It is better for short distances than Fig. 49, but not so good for long ones. The other figures are added to show the facility with which designs may be made. Fig. 52 is very good for short distances, and very poor for long ones. As a rule the graduation on self-reading stadia rods is not numbered, since it is generally considered easier to count the divisions included in the visual angle than to read both hairs and subtract.

Any self-reading leveling rod (§ 270) may be used as a stadia rod; but as a rule self-reading leveling rods have a great number of very small divisions, which become indistinct or wholly invisible at long distances;

* For a discussion of the relative merits of self-reading and target leveling rods, see § 270.

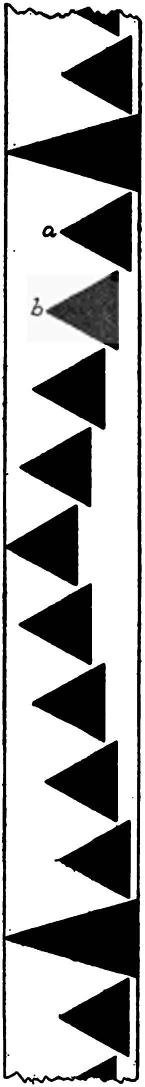

FIG. 49.

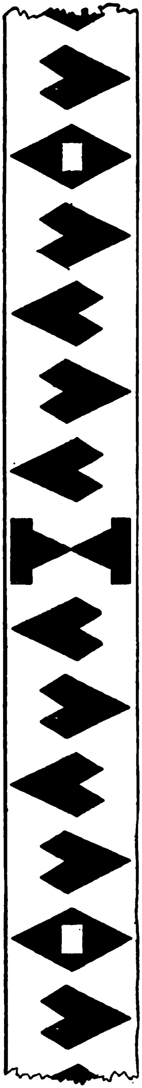
FIG. 50.

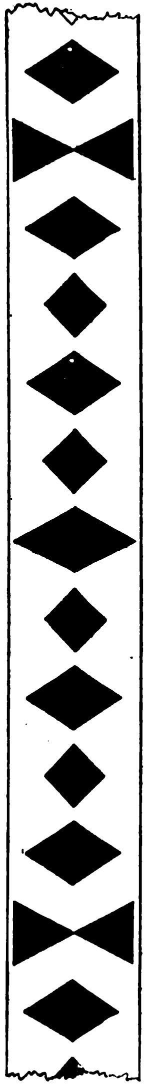
FIG. 51.


FIG. 52.

and hence most self-reading leveling rods are more suitable for short than for long sights. Leveling rods are generally read only at short distances, while stadia rods are frequently read at long distances.

208. Apparently stadia rods are not sold by instrument makers; but the engineer can easily make his own.

The rod must be light and convenient for transportation. Usually it is a board about 1 inch thick, 4 or 5 inches wide, and from 10 to 14 feet long. It may be stiffened by screwing a strip edgewise on the back. It may be hinged in the middle, and folded for ease of transportation and for the protection of the graduation; and when in use, it may be kept upright by a button or a bolt on the back. The rod should be provided with a disk level, or a short plumb, for keeping it vertical, and a handle by which to hold it. The graduation will keep cleaner and last much longer if the face of the board is recessed slightly to receive it. The face should be made perfectly white by a number of thin coats of paint, each being thoroughly dry before the succeeding one is applied.

In graduating a stadia rod, visibility is of the first importance. If the graduation consists of a number of small divisions, they will become invisible at long distances, and even at short distances will be very confusing. The graduation marks should therefore be made large and the smaller divisions be obtained by subdividing the larger ones by estimation. This method by estimation is almost, or quite, as accurate as when the smaller divisions are marked. It is often claimed that very small subdivisions can be read more exactly by estimation than by a direct graduation, owing to the liability of error in counting the graduation marks.

The pattern may be painted or stenciled directly upon the wood, or it may first be drawn or painted upon

paper and then fastened on the rod with varnish or any glue not soluble in water. Stencils may be made of writing paper which has been varnished or oiled. In either case the pattern should have a sharp outline, and should be marked with jet-black paint that dries with a dead, and not a shiny, surface.

209. To determine the vertical co-ordinates of the point on which the rod is set, it is necessary that the line of sight should be directed to a point at a known distance from the foot of the rod. The point at which the central visual ray is directed is most easily indicated by a target, which may be either fixed or movable, although the movable target is much the better. It may be made of a piece of rolled brass $2\frac{1}{2}$ to 4 inches wide and about an eighth of an inch thick, bent as shown in Fig. 53. The pieces *a* and *b* are made concave

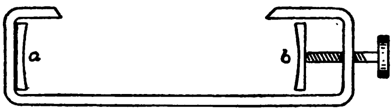

FIG. 53.

on their inner faces to fit the edges of the rod. *a* is soldered or riveted to the body of the target, and *b* is held in position by two plates (not shown in Fig. 53) attached to the top and bottom of it, which extend over the edges of the body of the target. The target is clamped at any point on the rod by the milled-head screw. A diamond is painted, preferably in red, on the face of the target (see § 268). The back is left open so the target may be moved up and down past the handle and plummet on the back of the rod.

210. FORMULA FOR HORIZONTAL LINE OF SIGHT AND VERTICAL ROD. In Fig. 54 let *a* and *b* represent the stadia hairs; *i* the distance between them; *s* the distance, *pq*, on the rod intercepted between the hairs; *f*

the principal focal distance of the objective; e a point at a distance f in front of the optical center of the objective, that is, e is the principal focus of the objective; c the distance from the plumb-line of the instrument to the optical center of the objective; y the distance from the outer focus, e, to the rod; and D the distance from the instrument to the rod. For convenience in printing represent $\frac{f}{i}$ by k.

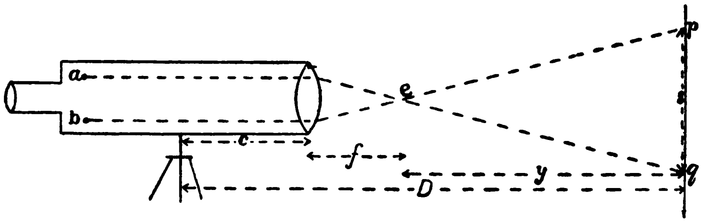

FIG. 54.

From the principles of optics, we know that all rays of light which pass through e are parallel to each other after emerging from the objective. Therefore there is some point q, which will emit a single ray of light that will pass through e, and, after traversing the objective will strike the cross hair a. If the telescope is focused for the point q, the objective will bring all rays emitted by q to a focus at a; and hence it is immaterial whether we consider the real course of the rays, or assume that all the light from q passes along the line qea.

Similarly we may assume that all the rays from p pass along the line peb.

From Fig. 54 we easily get $s : y :: i : f$, from which

$$y = \frac{f}{i} s = k\,s. \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

Notice that $k = \frac{f}{i}$, is a constant coefficient peculiar to each instrument, and also that the intercept s on the

rod varies as y—the distance of the rod from the outer focus of the objective. These relations may be seen directly from Fig. 54. Since the two rays from p and q are parallel after entering the telescope, it is immaterial where the cross hairs are; and, therefore, the distance of the rod from e is always proportional to the intercept s. In other words, the intercept on the rod is proportional to the distance of the rod from the point e, and any change in the position of the cross hairs in focusing upon the rod produces no change in this relation.

From Fig. 54 and equation (1) we get

$$D = ks + c + f. \quad . \quad . \quad . \quad . \quad . \quad (2)$$

211. As equation (2) is the foundation of all stadia formulas, it will be demonstrated by a slightly different method. In Fig. 55 o represents the optical centre of

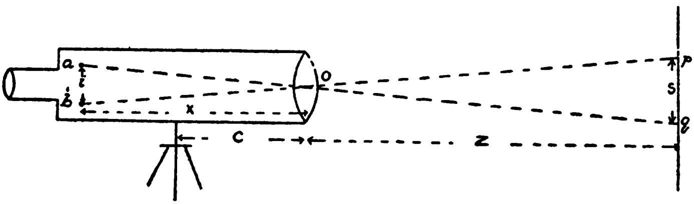

FIG. 55.

the objective; x the distance from the cross hairs to the optical center; and z the distance from the optical centre to the rod. The remainder of the nomenclature is as in Fig. 54. Since a lens may be regarded as a mathematical point which allows a very great amount of light to pass through it,* we may consider the rays of light as passing in a right line from q through o to a, and also from p through o to b. Then by similar triangles,

$$s : z :: i : x,$$

* See first foot-note on page 106.

from which

$$z = \frac{x}{i} s. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

From the theory of optics, we have the well-known relation for a convex lens,

$$\frac{1}{f} = \frac{1}{x} + \frac{1}{z}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

Combining equations (3) and (4) gives

$$z = \frac{f}{i} s + f. \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

Adding c to both members of (5) gives equation (2), page 184.

212. There are other and less simple demonstrations of the fundamental stadia formula—equation (2);—but as they all arrive at the same final form, they must involve the same approximations. Both of the preceding methods involve slight approximations: 1. Equation (1) assumes that the line qe, Fig. 54, and the horizontal line through a intersect in a vertical plane through the optical center, whereas they do not so intersect. 2. Equation (3) assumes that the lines qo and oa, Fig. 55, are one and the same right line, whereas they are not.

In addition to these, several relatively unimportant approximations are indirectly involved. Although the formula for the stadia is not mathematically correct, it is much more accurate than any observations that can be made with an ordinary telescope, and it will be shown later that the stadia is an instrument of considerable precision.

213. To find c and f. To find c, focus the instrument on a point 100 feet or more away, and measure with a

pocket-rule the horizontal distance from the vertical axis to the middle of the objective. Strictly, c is not constant in instruments in which the cross hairs are fixed and the objective movable; but if it is found within an inch it is more than sufficient.* The distance found as above is practically the minimum, but it is also the value corresponding nearly to the mean distance of the rod from the instrument.

To find f, focus the instrument on a point 100 feet or more away, and measure the distance from the cross hairs to the middle of the thickness of the objective.†

214. To find k. Set the rod vertical, at any convenient distance in front of the instrument, and having brought the line of sight horizontal, determine the intercept s by using a stadia rod (§ 207) or a self-reading level rod (§ 270). Then measure the distance D with a band-chain. From equation (2), page 184, we have

$$k = \frac{D - (c + f)}{s}, \quad . \quad . \quad . \quad . \quad . \quad (6)$$

from which it is an easy matter to compute k. For greater accuracy, make several observations at different distances, and take the mean of the corresponding values of k. The error of this mean will be consider-

* The variation in c is the same as the variation in x in equation (4), page 185. Assuming f to be 1 foot—a fair average,—we find that if $z = 10$ ft., $x = 1$ ft. 1.3 in.; if $z = 20$ ft., $x = 1$ ft. 0.6 in.; if $z = 100$ ft., $x = 1$ ft. 0.1 in.; and if $z = 1000$ ft., $x = 1$ ft. 0.01 in. Therefore, since the distance to the rod will ordinarily be more than 100 ft., the variation in c is less than one tenth of an inch, which is wholly inappreciable in stadia surveying.

† It is sometimes desirable to know f and the position of the optical center accurately. To find them proceed as follows: Remove the lens from the telescope tube, and set it up midway between two small movable white screens, one of which has a square ruled upon it with black ink. Shift the positions of the screens until the square and its image upon the other screen are of exactly the same size. The optical center will then be exactly midway between the two screens, and f is equal to one fourth of the distance between them.

ably less than the error of a single determination of the intercept s.

The above method may be employed to determine the k for each of the side intervals. If the stadia hairs are not adjustable, the side intervals will probably not be equal to each other; but a method of overcoming this will be explained later (§ 235, second paragraph). Of course, if the intervals are equal, the values of k will be equal to each other; and in any case the sum of the reciprocals of k for the side intervals must be equal to the reciprocal of k for the extreme interval, which affords an excellent check on the accuracy of the work.

215. For convenience of computation, k is ordinarily made 100 and s is measured in feet. This has the further advantage of permitting the use of an ordinary level rod as a stadia rod. If the hairs are adjustable, this condition may be readily attained. To adjust the hairs, carefully draw three diamonds with black ink on cardboard, and fasten two of these targets on a rod exactly 1 foot apart, and place the third one midway between the other two. Then from the plumb-line measure a distance $(100 + c + f)$ feet in front of the instrument, and set the rod vertical. With the tangent screw of the vertical movement, bring the central hair to bisect the middle target, and, next, with the stadia-hair screws bring the side hairs to bisect the side targets respectively. Test the adjustment by placing the outside targets 2 feet apart and setting the rod $(200 + c + f)$ feet from the instrument. In making these observations, the line of sight should be at least nearly horizontal (see § 219). When the hairs are adjusted as above, the stadia formula becomes

$$D \text{ ft.} = 100\, s \text{ ft.} + (c + f) \text{ ft.} \quad . \quad . \quad . \quad (7)$$

If the stadia hairs are not adjustable, k can be made equal to 100 (or any other number) by the following

method. Measure $(100 + c + f)$ feet in front of the instrument and set the rod vertical. Place the telescope horizontal or nearly so, fasten a target on the rod at about the height of the point covered by the upper hair, and bring this hair exactly to bisect the target. Then have an assistant place the other target so that it will be bisected by the lower stadia hair. If now the distance between the two targets is carefully measured and divided into one hundred equal parts, and the graduation is continued over the whole length of the rod, the number of units in the intercept will be equal to the number of feet the rod is from the front focus of the objective; that is to say, for this rod and this telescope, $k = 100$.

Since it would probably be impossible to mark each of the one hundred divisions upon the rod in such a way as to make them visible at any considerable distance, it is best to mark, say, each tenth one (see Fig. 49, page 180) and estimate the units. When k is determined as above, the stadia formula becomes

$$D \text{ ft.} = 10\, R + (c + f) \text{ ft.}, \quad . \quad . \quad . \quad (8)$$

in which R is the number of figures (spots) included in the intercept.

216. It is possible to determine both k and $(c + f)$ by the method explained in § 214. Make two observations for s at two known distances, D, and insert the results in equation (2), page 184, which gives two equations with two unknown quantities, from which k and $(c + f)$ can be determined by the ordinary rules of algebra. For greater accuracy make several observations, each two of which will give a value of each of the unknown quantities. Strictly speaking, the resulting equations should be solved by the method of least squares; but as the method of finding c, f, and k, as explained above, is very

much more simple and is abundantly exact, this method will not be discussed further, even though it is practiced by some prominent engineers.

Frequently in finding k, the intercept is assumed to vary as the distance from the center of the instrument, which is equivalent to omitting $(c+f)$ from equation (6), page 186. In this case, the distance as determined by the stadia is always in error, except when it is the same as the distance employed in finding k. For shorter distances the result is too small, and for greater distances it is too large. Sometimes observations are made at several distances and a mean value for k is computed, which simply averages errors and makes it impossible to find either the amount or direction of the error. This method is incorrect in principle, and inaccurate in practice, and has nothing to commend it. The method of § 215 is simple and gives strictly correct results; and if approximate results are desired the quantity $(c+f)$ in equation (2) may be disregarded in computing the distance, in other words, this quantity may be omitted or inserted at will without introducing error into any other part of the work.

217. To explain a method of eliminating $(c+f)$ from the formula, conceive that the rod has been graduated for the formula $D = ks + (c+f)$ and that it is standing $(100 + c + f)$ feet from the instrument. The intercept may be considered as divided into 100 divisions, each of which corresponds to a foot of distance on the ground. Then if the rod is shortened by cutting out of the intercept as many divisions, or units of the graduation, as there are feet in the distance $(c+f)$, the numbering of the graduations remaining unchanged, the apparent number of units between the stadia hairs will correspond to the number of feet from the rod *to the center of the instrument.* If care is taken that the point at which a portion of the graduation is cut out is always between

the stadia hairs, the reading will give the distance from the center of the instrument. This method is objectionable in that the point at which the graduation is omitted can not always be made to fall between the stadia hairs, owing to obstructions to sighting; and, further, in that the determination of the vertical coordinate is complicated thereby. The advantage of having the intercept proportional to the distance is so slight that it does not seem wise to obtain it by complicating both the graduation and the use of the rod. Furthermore, the stadia is not intended for an instrument of high precision, and the work for which it is generally used does not require extreme accuracy; therefore, as a rule, the term $(c+f)$ in the final formula may simply be omitted.

Parro, in 1823, showed that by placing an auxiliary lens between the objective and the cross hairs, the intercept can be made proportional to the distance from the *center* of the instrument. It is not known that such a telescope has ever been made.

218. Position of Rod for Inclined Line of Sight. The preceding formulas were deduced on the assumption that the central visual ray was horizontal and the rod vertical, *i.e.*, the central visual ray was assumed to be perpendicular to the rod. These formulas would be sufficient if the observations were made with a leveling instrument; but a level is too limited in its range to secure the full advantage of the principle of the stadia. In all that follows, it will be assumed that a transit is used.

The formula $D = ks + c + f$ may be used with an inclined line of sight, *provided* the rod is held perpendicular to the central visual ray. In this case D is no longer the horizontal distance from the instrument to the rod, but is the oblique distance from the horizontal axis of the telescope to the point on the rod covered

by the central visual ray. It then becomes a question whether, with an inclined line of sight, the rod should be held perpendicular to the central visual ray, or vertical.

The perpendicularity of the rod to the line of sight may be determined by a telescope or a pair of sights attached at right angles to the rod, which is directed toward the observing telescope by the rod-man. The verticality of the rod may be estimated by the rod-man, or it may be determined easily and accurately by attaching a plumb-line or a level vial.

Some prefer the rod perpendicular to the line of sight, but this position involves serious difficulties: (1) it is not easy to hold the rod steady in this position; (2) it is not always possible for the rod-man to see the telescope, especially at long distances or great vertical angles, or when undergrowth intervenes; and (3) the formulas for computing the horizontal and vertical co-ordinates of the point are more simple when the rod is vertical than when it is perpendicular to the line of sight. Only the case of the vertical rod will be considered here.

219. Formulas for Inclined Line of Sight and Vertical Rod. Let θ = the angle of the central visual ray with the horizontal = CIJ (Fig. 56). To measure θ, a third horizontal hair should be placed half-way between the other two, or, rather, stadia hairs are to be added on opposite sides and equally distant from the ordinary horizontal one; and θ is then determined as any other vertical angle by reading the vertical circle. θ will generally be small. Let 2α = the visual angle = BOD. 2α is always small, its maximum being about 35 minutes. AE is the actual intercept, and BD the value it would have if the rod were held perpendicular to the central visual ray. It is desired to find a relation between AE and BD.

The angle $CBA = 90° + \alpha$; and, since α is very small, $BC = AC \cos \theta$ nearly. Similarly $CDE = 90° - \alpha$, and $CD = CE \cos \theta$ nearly. Hence $BD = (AC + CE) \cos \theta = AE \cos \theta$ nearly. Notice that the two approximations

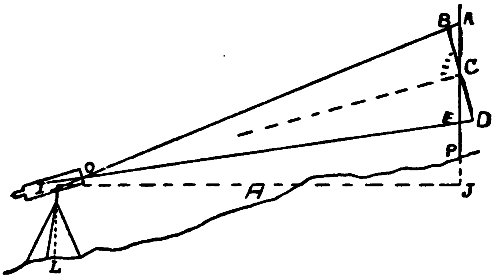

FIG. 56.

tend to neutralize each other, and that the final error involved is much less than the error of observing AE.*

Since BD is the intercept perpendicular to the line of sight, it is the value of s (§ 210) corresponding to the distance IC.

Hence equation (2), page 184, becomes

$$D = k\,s \cos \theta + (c + f), \quad . \quad . \quad . \quad . \quad (9)$$

in which D is the oblique distance IC (Fig. 56).

220. The Horizontal Distance. Let $H =$ the horizontal distance from the center of the instrument to the vertical through the foot of the rod $= IJ$ (Fig. 56). $H = IC \cos \theta = D \cos \theta$; and substituting the value of D from equation (9), we have

$$H = k\,s \cos^2 \theta + (c + f) \cos \theta, \quad . \quad . \quad . \quad (10)$$

or

$$H = k\,s - k\,s \sin^2 \theta + (c + f) \cos \theta. \quad . \quad (11)$$

* If the side hairs are equidistant from the central one, the true relation is $BD = AE \cos \theta (1 - \tan^2 \theta \tan^2 \alpha)$; and if the side hairs are not equidistant, the upper angle being α' and the lower α_1, the true relation is

$$AE = CB\frac{\cos \alpha'}{\cos (\theta + \alpha')} + CD\frac{\cos \alpha_1}{\cos (\theta - \alpha_1)}.$$

The second form is preferable, for it is always better to compute a correction to a quantity than the quantity itself. Notice that since $(c + f)$ is always small and $\cos\theta$ nearly unity, $(c + f)\cos\theta$ may always be taken equal to $(c + f)$, and often may be omitted entirely. Notice also that since $\sin^2\theta$ is small, the whole operation of reducing an observation by equation (11) may be performed mentally.

221. Vertical Distance. To determine the vertical co-ordinate of the point upon which the rod is set, the middle hair must be sighted upon a point of the rod at a known distance from its foot. The simplest way of accomplishing this is to provide the rod with a movable target (§ 209).

If the instrument is set over the reference point, the target is to be set at a distance from the foot of the rod equal to the height of the horizontal axis of the telescope above the point. The proper position of the target may readily be found by setting the rod up by the side of the instrument. Then the difference in level between the point under the instrument and the one on which the rod is placed is equal to the difference in elevation between the horizontal axis of the telescope and the target on the rod; and therefore to determine the height of any subsequent point, set the rod upon it, bisect the target, and read the angle from the vertical circle.

If the instrument is not placed over the reference point, bring the line of sight horizontal, set the rod on the reference point, and move the target until it is bisected by the middle cross hair. To determine the height of any subsequent point, set the rod upon it and read exactly as before.

222. To deduce a formula for computing the vertical co-ordinate, let $V =$ the height of the point on which the rod is placed, above the reference point; $V =$ the

distance the target is above the axis of the telescope = JC (Fig. 56). $V = IC \sin\theta = D \sin\theta$; and substituting the value of D from equation (9) gives

$$V = ks \cos\theta \sin\theta + (c + f) \sin\theta; \quad . \quad . \quad (12)$$

$$V = \tfrac{1}{2} ks \sin 2\theta + (c + f) \sin\theta. \quad . \quad . \quad (13)$$

Notice that generally $(c + f) \sin\theta$ may be omitted.

223. Reducing the Field Notes. This consists in finding the horizontal and vertical distances from the rod reading and the observed vertical angle. This can be done by using formulas (10) and (13). Although the equations are in a very convenient form for computation, it would be very tedious and slow to solve both equations for every observation.

Notice that $ks \cos^2\theta$ and $\tfrac{1}{2} ks \sin 2\theta$ are independent of the instrument and of the unit of linear measurement used, and therefore they can be tabulated for all cases. $(c + f) \cos\theta$ and $(c + f) \sin\theta$ depend upon both the instrument and the unit, and must be computed once for each particular instrument and rod. Obviously it is a great saving of labor and time if the results are computed once for all and tabulated.

We may compute $ks \cos^2\theta$ from equation (10), and $\tfrac{1}{2} ks \sin 2\theta$ from equation (13), for different values of ks and θ, and tabulate the results, in which case the horizontal and vertical distances can be taken directly from the table; or, we may tabulate only $\cos^2\theta$ and $\tfrac{1}{2} \sin 2\theta$ for different values of θ, in which case the horizontal and vertical distances are found by multiplying ks by the tabulated factor. The first method would be the better, if it did not require such voluminous tables.

In either case the results may be expressed in an arithmetical table or in a geometrical diagram. Arithmetical tables are capable of greater accuracy; but they

must be either very extended and therefore inconveniently large, or brief and give results by interpolation, which is slow and tedious. On the other hand, it is urged against geometrical diagrams that to be accurate they must be drawn to a large scale, and that therefore they are large and unwieldy. It is believed that with properly constructed diagrams the reductions can be made without sacrificing much, if any, accuracy, and with greater facility than by the use of tables.

224. Arithmetical Tables. Pages 196 to 198 contain a brief arithmetical reduction table. The column headed H gives the value of $\sin^2 \theta$ (see equation (11), page 192). The column headed V gives the value of $\frac{1}{2} \sin 2\theta$ (see equation (13), page 194). The terms $(c+f) \cos \theta$ and $(c+f) \sin \theta$ are seldom required; but they are given in a note at the bottom of the page, for use when great accuracy is desired.

Ockerson and Teeple, assistant U. S. engineers, have published tables* which give $\frac{1}{2} ks \sin 2\theta + 0.43$ *in metres* (for values of ks *in feet*) from 0 to 500, varying by 10, for each minute of θ from 0° to 10°. These tables give also $\sin^2 \theta + 0.43$ for the even minutes from 2° to 20°.

225. Geometrical Diagrams. There is a variety of methods of constructing diagrams for reducing stadia observations, but a modification of those proposed by Prof. S. W. Robinson† are believed to be the best.

226. *Horizontal Distance.* Fig. 57 (between pages 200 and 201) gives the term $ks \sin^2 \theta$ (see equation (11), page 192). To use the diagram, find the observed value of ks on the lower line, and from this point follow the inclined line to the radial line indicating the observed value of θ; then the elevation of this point as

* For sale by J. A. Ockerson, Chief Engineer Mississippi River Commission, St. Louis, Missouri.

† Journal of the Franklin Institute, Vol. 49, pp. 80, 81; and *Ibid.*, Vol. 51, pp. 15, 16.

TABLE III.

STADIA REDUCTION TABLES.

θ	0°		1°		2°		3°		4°		5°	
	H.	V.	H.	V.	H.	V.	H.	V.	H.	V.	H.	V.
0′	.0000	.0000	.0003	.0174	.0012	.0349	.0027	.0523	.0049	.0696	.0076	.0868
2	.0000	.0006	.0003	.0180	.0013	.0355	.0028	.0528	.0049	.0702	.0077	.0874
4	.0000	.0012	.0003	.0186	.0013	.0360	.0029	.0534	.0050	.0707	.0078	.0880
6	.0000	.0017	.0004	.0192	0013	.0366	.0029	.0540	.0051	.0713	.0079	.0885
8	.0000	.0023	.0004	.0197	.0014	.0372	.0030	.0546	.0052	.0719	.0080	.0891
10	.0000	.0029	.0004	.0203	.0014	.0378	.0030	.0552	.0053	.0725	.0081	.0897
12	.0000	.0035	.0004	.0209	.0015	.0384	.0031	.0557	.0054	.0730	.0082	.0903
14	.0000	.0041	.0004	.0215	.0015	.0390	.0032	.0563	.0054	.0736	.0083	.0908
16	.0000	.0046	.0005	.0221	.0016	.0395	.0032	.0569	.0055	.0742	.0084	.0914
18	.0000	.0052	.0005	.0227	.0016	.0401	.0033	.0575	.0056	.0748	.0085	.0920
20	.0000	.0058	.0005	.0232	.0017	.0407	.0034	.0580	.0057	.0753	.0086	.0925
22	.0000	.0064	.0005	.0238	.0017	.0413	.0035	.0586	.0058	.0759	.0087	.0931
24	.0000	.0070	.0006	.0244	.0017	.0418	.0035	.0592	.0059	.0765	.0088	.0937
26	.0000	.0076	.0006	.0250	.0018	.0424	.0036	.0598	.0060	.0771	.0089	.0943
28	.0001	.0081	.0007	.0256	.0018	.0430	.0037	.0604	.0061	.0776	.0090	.0948
30	.0001	.0087	.0007	.0261	.0019	.0436	.0037	.0609	.0062	.0782	.0091	.0954
32	.0001	.0093	.0007	.0267	.0019	.0442	.0038	.0615	.0062	.0788	.0092	.0960
34	.0001	.0099	.0008	.0273	.0020	.0448	.0039	.0621	.0063	.0794	.0093	.0965
36	.0001	.0105	.0008	.0279	.0021	.0453	.0039	.0627	.0064	.0799	.0094	.0971
38	.0001	.0110	.0008	.0285	.0021	.0459	.0040	.0633	.0065	.0805	.0095	.0977
40	.0001	.0116	.0009	.0291	.0022	.0465	.0041	.0638	.0066	.0811	.0097	.0983
42	.0002	.0122	.0009	.0297	.0022	.0471	.0042	.0644	.0067	.0817	.0098	.0988
44	.0002	.0128	.0009	.0302	.0023	.0476	.0042	.0650	.0068	.0822	.0100	.0994
46	.0002	.0134	.0010	.0308	.0023	.0482	.0043	.0656	.0069	.0828	.0101	.1000
48	.0002	.0139	.0010	.0314	.0024	.0488	.0044	.0661	.0070	.0834	.0102	.1005
50	.0002	.0145	.0010	.0320	.0024	.0494	.0045	.0667	.0071	.0840	.0103	.1011
52	.0003	.0151	.0011	.0326	.0025	.0499	.0045	.0673	.0072	.0845	.0104	.1017
54	.0003	.0157	.0011	.0331	.0026	.0505	.0046	.0678	.0073	.0851	.0105	.1022
56	.0003	.0163	.0011	.0337	.0026	.0511	.0047	.0684	.0074	.0857	.0107	.1028
58	.0003	.0168	.0011	.0343	.0027	.0517	.0048	.0690	.0075	.0863	.0108	.1034
60	.0003	.0174	.0012	.0349	.0027	.0522	.0049	.0696	.0076	.0868	.0109	.1040

For all values of θ found on this page $\begin{cases} (c+f)\cos\theta = (c+f) \\ (c+f)\sin\theta = 0. \end{cases}$

TABLE III.

Stadia Reduction Tables.

θ	6° H.	6° V.	7° H.	7° V.	8° H.	8° V.	9° H.	9° V.	10° H.	10° V.	11° H.	11° V.
0	.0109	.1040	.0149	.1210	.0194	.1378	.0245	.1545	.0302	.1710	.0364	.1873
2	.0110	.1045	.0150	.1215	.0195	.1384	.0247	.1551	.0304	.1716	.0366	.1878
4	.0112	.1051	.0151	.1221	.0197	.1389	.0248	.1556	.0306	.1721	.0368	.1884
6	.0113	.1057	.0153	.1226	.0199	.1395	.0250	.1562	.0308	.1726	.0371	.1889
8	.0114	.1062	.0154	.1232	.0200	.1401	.0252	.1567	.0310	.1732	.0373	.1895
10	.0115	.1068	.0156	.1238	.0202	.1406	.0254	.1573	.0312	.1737	.0375	.1900
12	.0117	.1074	.0157	.1243	.0203	.1412	.0256	.1578	.0314	.1743	.0377	.1905
14	.0118	.1079	.0158	.1249	.0205	.1417	.0257	.1584	.0316	.1748	.0379	.1911
16	.0119	.1085	.0160	.1255	.0207	.1423	.0259	.1589	.0318	.1754	.0382	.1916
18	.0120	.1091	.0161	.1260	.0208	.1428	.0261	.1595	.0320	.1759	.0384	.1921
20	.0122	.1096	.0163	.1266	.0210	.1434	.0263	.1600	.0322	.1765	.0386	.1927
22	.0123	.1102	.0164	.1272	.0212	.1440	.0265	.1606	.0324	.1770	.0388	.1932
24	.0124	.1108	.0166	.1277	.0213	.1445	.0267	.1611	.0326	.1776	.0391	.1938
26	.0126	.1113	.0167	.1283	.0215	.1451	.0269	.1617	.0328	.1781	.0393	.1943
28	.0127	.1119	.0169	.1288	.0217	.1456	.0271	.1622	.0330	.1786	.0395	.1948
30	.0128	.1125	.0170	.1294	.0218	.1462	.0272	.1628	.0332	.1792	.0397	.1954
32	.0129	.1130	.0172	.1300	.0220	.1467	.0274	.1633	.0334	.1797	.0400	.1959
34	.0131	.1136	.0173	.1305	.0222	.1473	.0276	.1639	.0336	.1803	.0402	.1964
36	.0132	.1142	.0175	.1311	.0224	.1479	.0278	.1644	.0338	.1808	.0404	.1970
38	.0133	.1147	.0176	.1317	.0225	.1484	.0280	.1650	.0340	.1814	.0407	.1975
40	.0135	.1153	.0178	.1322	.0227	.1490	.0282	.1655	.0342	.1819	.0409	.1980
42	.0136	.1159	.0180	.1328	.0229	.1495	.0284	.1661	.0345	.1824	.0411	.1986
44	.0137	.1164	.0181	.1333	.0231	.1501	.0286	.1666	.0347	.1830	.0414	.1991
46	.0139	.1170	.0183	.1339	.0232	.1506	.0288	.1672	.0349	.1835	.0416	.1996
48	.0140	.1176	.0184	.1345	.0234	.1512	.0290	.1677	.0351	.1841	.0418	.2002
50	.0142	.1181	.0186	.1350	.0236	.1517	.0292	.1683	.0353	.1846	.0421	.2007
52	.0143	.1187	0187	.1356	.0238	.1523	.0294	.1688	.0355	.1851	.0423	.2012
54	.0144	.1193	.0189	.1361	.0239	.1528	.0296	.1694	.0358	.1857	.0425	.2018
56	.0146	.1198	.0190	.1367	.0241	.1534	.0298	.1699	.0360	.1862	.0428	.2023
58	.0147	.1204	.0192	.1373	.0243	.1540	.0300	.1705	.0362	.1868	.0430	.2028
60	.0149	.1210	.0194	.1378	.0245	.1545	.0302	.1710	.0364	.1873	.0432	.2034

For values of θ on this page $\begin{cases} (c+f)\cos\theta = (c+f) \\ (c+f)\sin\theta \text{ varies between } 0.1(c+f) \text{ and } 0.2(c+f) \end{cases}$

TABLE III.

STADIA REDUCTION TABLES.

θ	12°		13°		14°		15°		16°		17°	
	H.	V.	H.	V.	H.	V.	H.	V.	H.	V.	H.	V.
0′	.0432	.2034	.0506	.2192	.0585	.2347	.0670	.2500	.0760	2650	.0855	.2796
2	.0435	.2039	.0509	.2197	.0588	.2352	.0673	.2505	.0763	.2655	.0858	.2801
4	.0437	.2044	.0511	.2202	.0591	.2358	.0676	.2510	.0766	.2659	.0861	.2806
6	.0439	.2050	.0513	.2208	.0594	.2363	.0679	.2515	.0769	.2664	.0865	.2810
8	.0442	.2055	.0516	.2213	.0596	.2368	.0682	.2520	.0772	.2669	.0868	.2815
10	.0444	.2060	.0519	.2218	.0599	.2373	0685	.2525	.0775	.2674	.0871	.2820
12	.0447	.2066	.0521	.2223	.0602	.2378	.0687	.2530	.0778	.2679	.0874	.2825
14	.0449	.2071	.0524	.2228	.0605	.2383	.0690	.2535	.0782	.2684	.0878	.2830
16	.0451	.2076	.0527	.2234	.0607	.2388	.0693	.2540	.0785	.2689	.0881	.2834
18	.0454	.2081	.0529	.2239	.0610	.2393	.0696	.2545	.0788	.2694	.0884	.2839
20	.0456	.2087	.0532	.2244	.0613	.2399	.0699	.2550	.0791	.2699	.0888	.2844
22	.0459	.2092	.0534	.2249	.0616	.2404	.0702	.2555	.0794	.2704	.0891	.2849
24	.0461	.2097	.0537	.2254	.0619	.2409	.0705	.2560	.0797	.2709	.0894	2854
26	.0464	.2103	.0540	.2260	.0621	.2414	.0708	.2565	.0800	.2713	.0898	.2858
28	.0466	.2108	.0542	.2265	.0624	.2419	.0711	.2570	.0804	.2718	0901	.2863
30	.0469	.2113	.0545	.2270	.0627	.2424	.0714	.2575	.0807	.2723	.0904	.2868
32	.0471	.2118	.0548	.2275	.0630	.2429	.0717	.2580	.0810	.2728	.0908	.2873
34	.0473	.2124	.0550	.2280	.0633	.2434	.0720	.2585	.0813	.2733	.0911	.2877
36	0476	.2129	.0553	.2285	.0635	.2439	.0723	.2590	.0816	.2738	.0914	.2882
38	.0478	.2134	.0556	.2291	.0638	.2444	.0726	.2595	.0819	.2743	.0918	.2887
40	.0481	.2139	.0558	.2296	.0641	.2449	.0729	.2600	.0822	.2748	.0921	.2892
42	.0483	.2145	.0561	.2301	.0644	.2455	.0732	.2605	.0826	.2752	.0924	.2896
44	.0486	.2150	.0564	.2306	.0647	.2460	.0735	.2610	.0829	.2757	.0928	.2901
46	.0488	.2155	.0566	.2311	.0650	.2465	.0738	.2615	.0832	.2762	.0931	.2906
48	.0491	.2160	.0569	.2316	.0653	.2470	.0741	.2620	.0835	.2767	.0935	.2911
50	.0493	.2166	.0572	.2322	.0656	.2475	.0744	.2625	.0839	.2772	.0938	.2915
52	.0496	.2171	.0575	.2327	.0658	.2480	.0747	.2630	.0842	.2777	.0941	.2920
54	.0498	.2176	.0577	.2332	.0661	.2485	.0750	.2635	.0845	.2781	.0945	.2925
56	.0501	.2181	.0580	.2337	.0664	.2490	.0754	.2640	.0848	.2786	.0948	.2930
58	.0504	.2187	.0583	.2342	.0667	.2495	.0757	.2645	.0852	.2791	.0952	.2934
60	.0506	.2192	.0585	.2347	.0670	.2500	.0760	.2650	.0855	.2796	.0955	.2939

For values of θ on this page $\begin{cases} (c+f)\cos\theta > 0.95(c+f) = (c+f) \\ (c+f)\sin\theta \text{ varies between } 0.2(c+f) \text{ and } 0.3(c+f). \end{cases}$

TABLE III.

Stadia Reduction Tables.

θ	18° H.	18° V.	19° H.	19° V.	20° H.	20° V.	21° H.	21° V.	22° H.	22° V.	23° H.	23° V.
0′	.0955	.2939	.1060	.3078	.1170	.3214	.1284	.3346	.1403	.3473	.1527	.3597
2	.0958	.2944	.1064	.3083	.1174	.3218	.1288	.3350	.1407	.3477	.1531	.3601
4	.0962	.2948	.1067	.3087	.1177	.3223	.1292	.3354	.1411	.3482	.1535	.3605
6	.0965	.2953	.1071	.3092	.1181	.3227	.1296	.3359	.1416	.3486	.1539	.3609
8	.0969	.2958	.1074	.3097	.1185	.3232	.1300	.3363	.1420	.490	.1544	.3613
10	.0972	.2962	.1078	.3101	.1189	.3236	.1304	.3367	.1424	.3494	.1548	.3617
12	.0976	.2967	.1082	.3106	.1192	.3241	.1308	.3372	.1428	.3498	.1552	.3621
14	.0979	.2972	.1085	.3110	.1196	.3245	.1312	.3376	.1432	.3502	.1556	.3625
16	.0982	.2976	.1089	.3115	.1200	.3249	.1316	.3380	.1436	.3507	.1560	.3629
18	.0986	.2981	.1092	.3119	.1204	.3254	.1320	.3384	.1440	.3511	.1565	.3633
20	.0989	.2986	.1096	.3124	.1207	.3258	.1323	.3389	.1444	.3515	.1569	.3637
22	.0993	.2990	.1099	.3128	.1211	.3263	.1327	.3393	.1448	.3519	.1573	.3641
24	.0996	.2995	.1103	.3133	.1215	.3267	.1331	.3397	.1452	.3523	.1577	.3645
26	.1000	.3000	.1107	.3138	.1219	.3272	.1335	.3401	.1456	.3527	.1582	.3649
28	.1003	.3004	.1111	.3142	.1223	.3276	.1339	.3406	.1460	.3531	.1586	.3653
30	.1007	.3009	.1114	.3147	.1227	.3280	.1343	.3410	.1465	.3536	.1590	.3657
32	.1010	.3014	.1118	.3151	.1230	.3285	.1347	.3414	.1469	.3540	.1594	.3661
34	.1014	.3019	.1122	.3156	.1234	.3289	.1351	.3418	.1473	.3544	.1599	.3665
36	.1017	.3023	.1125	.3160	.1238	.3293	.1355	.3423	.1477	.3548	.1603	.3669
38	.1021	.3028	.1129	.3165	.1242	.3298	.1359	.3427	.1481	.3552	.1607	.3673
40	.1024	.3032	.1133	.3169	.1246	.3302	.1363	.3431	.1485	.3556	.1611	.3677
42	.1028	.3037	.1136	.3174	.1249	.3307	.1367	.3435	.1489	.3560	.1616	.3680
44	.1032	.3041	.1140	.3178	.1253	.3311	.1371	.3440	.1493	.3564	.1620	.3684
46	.1035	.3046	.1144	.3183	.1257	.3315	.1375	.3444	.1498	.3568	.1624	.3688
48	.1039	.3051	.1147	.3187	.1261	.3320	.1379	.3448	.1502	.3572	.1629	.3692
50	.1042	.3055	.1150	.3192	.1265	.3324	.1383	.3452	.1506	.3576	.1633	.3696
52	.1046	.3060	.1155	.3196	.1269	.3328	.1387	.3457	.1510	.3580	.1637	.3700
54	.1049	.3065	.1159	.3201	.1273	.3333	.1391	.3461	.1514	.3585	.1641	.3704
56	.1053	.3069	.1163	.3205	.1277	.3337	.1395	.3465	.1518	.3589	.1646	.3708
58	.1056	.3074	.1166	.3209	.1280	.3341	.1400	.3469	.1523	.3593	.1650	.3712
60	.1060	.3078	.1170	.3214	.1284	.3346	.1403	.3473	.1527	.3597	.1654	.3716

For values of θ on this page $\begin{cases} (c+f)\cos\theta > 0.9(c+f) = (c+f) \\ (c+f)\sin\theta \text{ varies between } 0.3(c+f) \text{ and } 0.4(c+f). \end{cases}$

given by the scale at the left of the diagram is the value of $k s \sin^2 \theta$, which is to be subtracted from $k s$. For extreme accuracy add $(c + f)$. For example, if $k s = 740$ and $\theta = 10°$, what is H? On the bottom line find 740, and follow the inclined line to an intersection with the radial line marked 10°. By the scale on the left this point reads 22.4. Therefore $D = 740 - 22.4 + (c + f)$.

If θ is more than 13°, the method of using the diagram differs slightly from that just described. For example, if $k s = 820$ and $\theta = 17°$, what is H? On the extreme right-hand line of the diagram, find 820 and follow this horizontal line to its intersection with the radial line marked 17°, and then follow this oblique line to the scale at the top, we see the value sought is 70. Therefore $H = 820 - 70 + (c + f)$.

227. *Vertical Distance.* Fig. 58 (between pages 200 and 201) gives the value of $\frac{1}{2} k s \sin 2\theta$ (see equation (13), page 194).

This diagram is used in essentially the same manner as Fig. 57 (§ 226). If the observed value of θ is found in the left-hand triangle or section of the diagram, the value of $k s$ is to be found on the bottom line to the left of the center, and the final result is found on the left edge of the left-hand triangle. If the value of θ is found in the middle section, the value of $k s$ may be found as before, then following the inclined line up to the 6° line, and from this point following a horizontal line to the radial line corresponding to θ, and from thence following the oblique line to the scale on the upper edge of the drawing, find the value sought; or if preferred, the value of $k s$ and also of the final result may be found by the numbers written upon the face of the diagram. If the given value of θ is found in the right-hand triangle or section of the diagram, find the value of $k s$ on the bottom line to the right of the center, and

follow the more inclined lines to the intersection with the proper radial line; and then the reading of this point by the scale of the nearly vertical lines is the value of $\frac{1}{2}\, k\, s \sin 2\,\theta$. Fig. 58 was constructed very carefully, and can be used with confidence; and ordinarily will give results as accurate as the observations.

Concerning the term $(c+f) \sin\,\theta$ of equation (13), page 194, notice that it is always small, and varies but slightly for a considerable change in θ; and therefore the supplemental table on Fig. 58 gives this term with sufficient precision. The first two values of $(c+f)$ in this table are to be used when the metre is the unit, and the last two when the foot is the unit. For values of $(c+f)$ which differ from those given, the correction may easily be interpolated with ample accuracy. If the particular value of $(c+f)$ is known, the correction may be written outside of the diagram between the radii for the different degrees.

228. Another Geometrical Diagram. If cross-ruled paper is at hand, a diagram for reducing field notes may easily be constructed as follows: Establish a zero at one corner of the sheet and lay off 100, 200, etc., along one side—say, the bottom of the paper,—to represent the several values of $k\,s$, and lay off 10, 20, etc., parallel to the other side, to represent the values sought. Obviously the scale of the latter should be much larger —say, ten or twenty times—than that of the former.

If it is desired to construct a table for finding the vertical co-ordinate, compute, either by the tables on pages 196–98 or by ordinary trigonometric tables, the values of $1{,}000\ \frac{1}{2} \sin 2\ \theta$ for the various values of θ, and lay off the distances vertically above the 1,000 point on the scale representing the values of $k\,s$; and then connect the points so determined with the zero point. The completed diagram will appear something like Fig. 59, page 202. Since large values of $k\,s$ and θ seldom

occur together, the diagram may be truncated as shown. which greatly extends the range of a single diagram. With cross-ruled paper it is very easy to construct a diagram like that outlined in Fig. 59.

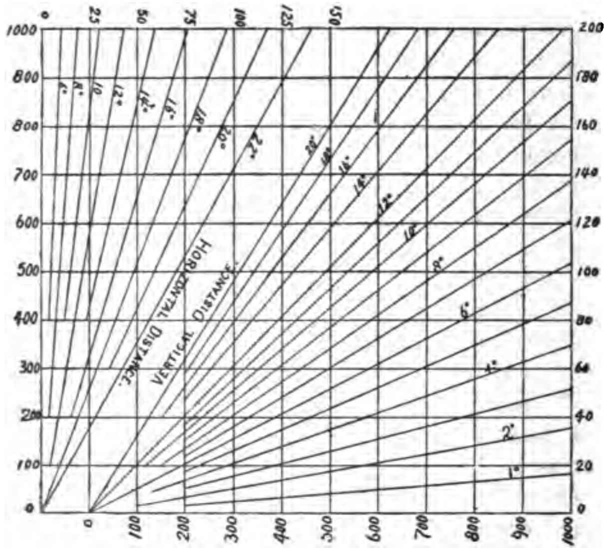

FIG. 59.—STADIA REDUCTION DIAGRAM.

229. SOURCES OF ERROR.* Stadia work is subject to many of the errors of ordinary transit work (see §§ 140–45). The additional errors peculiar to the stadia are (1) inclination of the rod, (2) error in the value of $k\,s$, and (3) error of observation.

230. Inclination of Rod. To investigate the effect of a slight inclination of the rod, let $k\,s =$ the intercept when the rod is vertical, and $k\,s' =$ the intercept when the rod makes an angle α with the vertical. Then $k\,s = k\,s' \cos \alpha$ very nearly (see § 219), and the error $= k\,s' - k\,s = k\,s'\,(1 - \cos \alpha) = k\,s' \sin^2 \frac{1}{2}\,\alpha = 0.000076\; k\,s'\,\alpha'^2$, α'

* For general discussion of Cumulative *vs.* Compensating Errors, see § 18.

being in degrees. It is convenient to remember that if the rod inclines 12° the resulting error in the distance is about one per cent. Notice, however, that the error increases as the *square* of the inclination, which shows the necessity of some means of plumbing the rod. Without some device for keeping the rod vertical (§ 208), this would undoubtedly be the principal source of error. On the U. S. Lake Survey, an inclined support was used to steady the rod and prevent its leaning toward or from the instrument. A light tripod is sometimes used for the same purpose.

231. Value of k s. If the value of *k s*, or the corresponding unit on the rod, is not correctly determined the results will be incorrect, although the relative position of points will be right. The correct value of *k s* is easily found (see §§ 214–16), and with the methods of attaching the hairs described in § 205 there is but little danger of its changing.

232. Errors of Observation. The principal item under this head is the inaccuracy in estimating the position of the hair on the rod. The amount of this error depends upon the size of the space on the rod corresponding to a unit on the ground, and upon the form of graduation. For those rods with which only one side of the hair is observed, this error is still further increased by the uncertainty due to the thickness of the hairs, and to any inequality in their thickness (§ 268). This element of uncertainty does not exist with the graduations shown in Figs. 49–52 (page 180).

Imperfect focusing, either of rod or hairs, is a source of error, because it is only when both are in focus at the same time that the assumed relations exist. If the rod is not in focus, the image covers too much space, which makes the distance too small. If the cross hairs are not in focus, the distance will be read too small or too great according as the hairs are on one side or the other

of the focus. If there is no parallax in the telescope (§ 94), there will be no error from this cause.

The indistinctness of the image due to an unsteadiness of the atmosphere produces an error by making the image too large and, therefore, the distance too small. The only remedy is to wait for better atmospheric conditions.

Another somewhat common error is miscounting the reading so as to make errors of 10, 100, etc., feet. These errors may be prevented by care, or checked by double readings ; or, instead of making an entirely new reading, the two halves of the visual angle may be read and their sum used to check the reading of the two outside hairs.

233. Limits of Precision. The stadia is designed to secure rapidity rather than accuracy ; but nevertheless with reasonable care a considerable degree of accuracy may be obtained. The claim is sometimes made that the stadia is more accurate than the chain; but from the nature of the principles involved, it can not be under equally favorable conditions, although under some circumstances the stadia is more accurate than the chain. The degree of precision is dependent upon the magnifying power of the telescope, the length of sight, and the ratio of the space on the rod to the corresponding space on the ground.

To ascertain the effect of the magnifying power, eight of the author's students determined a series of known distances with a telescope magnifying fifteen times and also with a telescope magnifying twenty-five times, all under essentially the same conditions. The average error in the first case was 1 in 282, and in the second 1 in 333.

To ascertain the relation between the length of sight and the error, sixteen of the author's students determined three series of distances each, the first being less

than 100 feet, the second between 100 and 200 feet, and the third between 200 and 300 feet. The conditions were essentially the same in all the observations. The average error in the first series was 1 in 182, in the second 1 in 263, and in the third 1 in 370. The relationship between the error and the distance depends upon the graduation, the magnifying power, and the atmospheric conditions; but for each particular case there is probably a distance at which the ratio of the error to the distance is a minimum. In a trial made by the author for the purpose of this record, with a graduation like Fig. 49, page 180 (1 foot on the rod corresponding to 100 feet on the ground) and a transit magnifying twenty-five times, this ratio was a minimum at about 700 feet.

234. The following data show the degree of accuracy attained in actual practice. They are not selected, but are all that could be discovered by personal inquiry and by search through engineering literature. Apparently the results were obtained with an ordinary engineer's transit. In comparing results we must distinguish between the error of simply finding the distance by the stadia, and the final error of a series of courses the lengths of which were determined by the stadia. The latter involves the error of measuring the angles as well as that of measuring the distances.

To show the degree of precision obtained in measuring horizontal distance, we have the following: Three measurements, each of six distances, from 50 to 500 ft., made to test the accuracy of the stadia measurements, the readings being made with targets, show an average error for a single sight of 1 in 1,100.* On the U. S. Lake Survey, three measurements of a base line gave errors of 1 in 1,000, 1 in 1,635, and 1 in 1,888.† On the

* Van Nostrand's Engineering Magazine, Vol. 30, pp. 319 and 476.

† Journal of the Franklin Institute, Vol. 49, p. 74.

Mississippi River Survey the maximum discrepancy permissible is 1 in 500, the maximum length of sight being 1,600 feet.

Only the following data can be found concerning the accuracy with which vertical distances can be determined. "Courses have been run with no more than ordinary care 1 to 6 miles, over heights of 150 to 200 feet, in which the final error in height ranged from 0 to 1.5 feet."* In the topographical survey of St. Louis, Mo., "the average error of elevations was less than 0.2 of a foot per mile."† The "Instructions for Topographical and Hydrographical Field Work on the Mississippi River," issued by the Mississippi River Commission, limit the maximum permissible error to 1 foot, the maximum length of sight being 1,600 feet.

Concerning the final error of a series of courses, the lengths of which were determined by the stadia, we have the following: "The stadia was used for getting the topography of some densely wooded timber land in the summer of 1863. The courses were so run as to connect points of triangulation from 1 to 4 miles apart. The distances from point to point along the courses ranged from 100 to 500 feet. The latitudes and departures were subsequently computed with the object of finding the error of stadia measurements. The results obtained were about 1 in 800, 1 in 1,000, and 1 in 1,100."‡ On the U. S. Lake Survey, in computing the co-ordinates of stadia work for 1875, the length of one hundred and forty-one lines varying between 3,200 feet and 22,000 feet (mean 8,080 feet) were compared with the lengths determined by triangulation or chaining, and the average error was found to be 1 in 649. The

* Journal of the Franklin Institute, Vol. 49, p. 74.

† The Technograph, No. 5 (1890–91), p. 13.

‡ Journal of the Franklin Institute, Vol. 49, p. 74.

maximum error permissible was 1 in 300.* In a survey of the Red River of the South, conducted by the U. S. Army Engineers, extending over about 70 miles, the stadia work and chaining were checked at eight points, and showed an average difference of 1 in 430.† In the topographical survey of St. Louis, Mo., "the error of closure, after making corrections for inclination and graduation of the rods, was about 1 in 800."‡ Without these corrections the error was about 1 in 500. Eighteen observations were made on two targets at distances from 50 to 500 feet, with a mean error for the mean of three observations (the individual observations are not recorded) of 1 in 1,500.¶

235. Practical Hints. To obviate the difficulty of estimating a fraction of a division of the rod for each stadia hair, set one of them at the beginning of a division. This will produce a slight, but generally inappreciable, error in the vertical co-ordinate. With long sights or small angles, it is still more convenient and sufficiently accurate, to set one of the hairs upon one of the more prominent divisions—as, for example, the even foot-mark. By remembering that either hair may be moved up or down, and by moving the telescope so as to produce the least displacement, this method of placing one of the hairs at an even division can frequently be used without any error, and will greatly facilitate the work.

If not enough of the rod is visible to read both side hairs, read first with one side hair and the middle one, and then with the middle one and the other side hair; and then add the two intercepts and compute the hori-

* Professional Papers, Corps of Engineers, U. S. A., No. 24—Primary Triangulation U. S. Lake Survey—p. 34.

† Report of Chief of Engineers, U. S. A., 1873, p. 638.

‡ The Technograph, No. 5 (1890-91), p. 12.

¶ Van Nostrand's Engineering Magazine, Vol. 30, p. 476.

zontal and vertical co-ordinates with the mean of the corresponding vertical angles.

In traversing with the stadia (§ 137 or § 184), check the work by reading the rod and the vertical angle on both fore-sights and back-sights. Frequently the work can also be checked by locating from each instrument station a point nearly in line between them; when the sum of the two distances to the object should be equal to the observed distance between the two instrument stations. Notice that this check is entirely independent of the check by back-sights and fore-sights.

If the target is not visible, owing to brush, etc., sight at any portion of the rod that is visible and note the point covered by the central visual ray. Make a record of this fact, and the next time the rod is visible throughout its entire length determine the correction for the former reading.

In observing for azimuth have the edge of the rod turned toward the instrument.

236. The many advantages of stadia measurements in surveying need not be dwelt upon, as they are self-evident to those acquainted with the principles. In broken country, the stadia can be readily used where the use of the chain is not practicable. The stadia is specially applicable to topographical surveying, to the topographical work of a railroad survey, and to determining the lengths of sights in leveling.

The stadia may also be used for underground surveying, where the chaining is peculiarly disagreeable and difficult. For this kind of work there is substituted for the rod a shallow box, lighted on the inside, with a glass front on which the graduation is painted.*

* Journal of the Franklin Institute, Vol. 55, pp. 384–87.

ART. 2. THE GRADIENTER.

237. The gradienter is a tangent screw having a micrometer head, attached to the horizontal axis of the telescope for the purpose of measuring a vertical angle in terms of its tangent. For a figure and description of the gradienter, see Fig. 28, page 101.

238. THE GRADIENTER AS A LEVELING INSTRUMENT. As its name indicates, the most important use of the gradienter is in locating grades in surveying railroads, irrigating ditches, etc. To locate a grade of, for example, 1.85 per cent, *i.e.*, 1.85 feet per 100 feet, bring the telescope level, and read the head of the gradienter screw; and then, if the screw is graduated so that one revolution corresponds to 1 foot at 100 feet, turn the screw 1.85 revolutions. The line of sight will then have a grade of 1.85 per cent up or down according to the way the screw was turned; and by setting a target on a rod at the height of the horizontal axis of the telescope, the ground corresponding to this grade may be easily found. By an obvious modification of the above process this device may be employed to determine the grade of any particular slope.

The gradienter is also very convenient in leveling up and down hills, over logs, etc., when extreme accuracy is not required.

Notice that to use the gradienter as a leveling instrument requires the direct measurement of the horizontal distances with a chain or tape.

239. THE GRADIENTER AS A TELEMETER. The gradienter may also be used to measure distances, in either of two ways: (1) By measuring the space on the rod passed over by the line of sight for a given number of revolutions of the screw; or (2) by observing the num-

ber of revolutions required to carry the line of sight over a constant space on the rod. The first is the more rapid, particularly for short sights; and the second is the more accurate, particularly for long sights, since the observations may be made on targets. Notice that there is not the same objection to the use of targets with a fixed intercept as with a variable intercept (§ 207).

240. Constant Number of Revolutions and Variable Intercept. *Line of Sight Perpendicular to Rod.* Let $D =$ the distance from one extremity of the intercept to the horizontal axis of the telescope; $H =$ the horizontal distance to be determined; $V =$ the vertical distance to be determined; $B =$ the length of the measured horizontal base; $S =$ the space on the vertical rod passed over by one revolution of the gradienter screw, at the distance B; $s =$ the space for one revolution, at the distance D. Then by the principle of similar triangles

$$B : D :: S : s; \text{ or}$$

$$D = \frac{B}{S} s, \qquad \dots \dots \quad (14)$$

Usually gradienters are so made that a single revolution of the screw carries the hair over 1 foot at a distance of 100 feet; that is, ordinarily $\frac{B}{S}$ in equation (14) is equal to 100. With some forms of instruments *two* revolutions are required to carry the line of sight over 1 foot at a distance of 100 feet. In either case the gradienter formula becomes

$$D \text{ ft.} = 100\, s \text{ ft.} \qquad \dots \dots \quad (15)$$

This is the fundamental equation for the gradienter, and corresponds closely with the fundamental equation for the stadia—see equation (2), page 184.

241. *Inclined Line of Sight and Vertical Rod.* It has already been shown (§ 218) that it is better to keep the rod vertical even though the line of sight is inclined. In Fig. 60, AE represents the intercept on the vertical

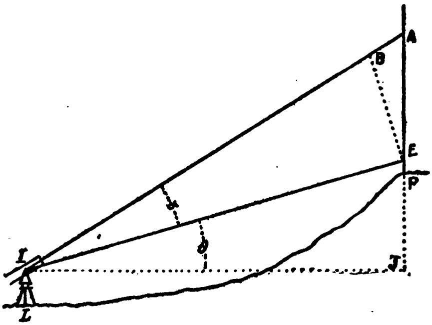

FIG. 60.

rod; BE the intercept perpendicular to the lower visual ray; $\theta =$ the angle of elevation of the *lower* visual ray; and $\alpha =$ the visual angle.

From the triangle AEB we get

$$\frac{BE}{AE} = \frac{\cos(\theta + \alpha)}{\cos \alpha} = \frac{\cos\theta\cos\alpha - \sin\theta\sin\alpha}{\cos\alpha}. \quad (16)$$

$$BE = AE(\cos\theta - \sin\theta\tan\alpha). \quad . \quad . \quad (17)$$

BE corresponds to s of equation (15), and hence

$$D \text{ ft.} = 100\, BE = 100\, AE(\cos\theta - \sin\theta\tan\alpha). \quad . \quad (18)$$

Since $AE = s$ and $\tan\alpha = \dfrac{S}{B} = \dfrac{1}{100}$ (§ 240), the preceding equation may be written thus:

$$D \text{ ft.} = s \text{ ft.} (100\cos\theta - \sin\theta). \quad . \quad . \quad (19)$$

Notice that equation (19) was deduced for the case in which θ represented an angle of elevation. If θ represents an angle of depression, equation (19) is still correct provided the angle is measured to the *upper* extremity of the intercept. But, for obvious reasons, it is better to measure θ always to the same extremity of the intercept—say, the lower,—in which case equation (19) may be used, if for angles of depression θ be considered as the observed angle *minus* the visual angle, *i.e.*, if θ is an angle of depression, subtract 34′ from it before inserting it in equation (19).

242. From equation (19) we easily get

$$H \text{ ft.} = s \text{ ft.} (100 \cos^2 \theta - \tfrac{1}{2} \sin 2\, \theta).^* \quad . \quad (20)$$

$$H \text{ ft.} = 100\, s \text{ ft.} - 100\, s \text{ ft.} \sin^2 \theta - \tfrac{1}{2}\, s \text{ ft.} \sin 2\, \theta.^* \quad (21)$$

Equations (20) and (21) correspond to equations (10) and (11), page 192, and may be reduced in the same way—see § 224 and § 226. Frequently the last term in each may be neglected.

243. To determine the vertical co-ordinate with the gradienter, place a target on the rod at a distance from its foot equal to the height of the horizontal axis of the instrument above the reference point, *and make the target the lower extremity of the intercept.*

From equation (19) we easily get

$$V \text{ ft.} = s \text{ ft.} (100 \cos \theta \sin \theta - \sin^2 \theta).^* \quad . \quad (22)$$

$$V \text{ ft.} = 100\, s \text{ ft.} \tfrac{1}{2} \sin 2\, \theta - s \text{ ft.} \sin^2 \theta.^* \quad . \quad (23)$$

Equation (23) corresponds to equation (13), page 194, and may be reduced in the same way—see § 224 and § 227. Frequently the last term can be neglected.

* If θ represents an angle of depression, subtract 34′ from it before using it in this equation.

244. Constant Intercept and Variable Number of Revolutions. Let $D =$ the perpendicular distance from the *center* of the intercept to the horizontal axis of the telescope; $H =$ the horizontal distance from the rod to the center of the instrument; $V =$ the vertical distance from the point under the instrument to the point on which the rod is placed; $S =$ the distance between targets; $N =$ the number of revolutions required to move the line of sight from one extremity to the other of the constant intercept, at the distance B; $n =$ the number of revolutions required to move the line of sight over the constant intercept, at the distance D.

Then by the principle of the similarity of triangles, $B : D :: n : N$, or

$$D = \frac{N}{n} B. \quad \ldots \ldots \quad (24)$$

Ordinarily gradienters are so made that if $B = 100$ feet and $S = 1$ foot, $N = 1$; and consequently for any value of S the number of units in S and N will be the same. Under these conditions, equation (24) becomes

$$D \text{ ft.} = \frac{100}{n} S \text{ ft.} \quad \ldots \ldots \quad (25)$$

This equation corresponds to equation (2), page 184, and equation (15), page 210, and is the fundamental equation for this method of using the gradienter. Notice that S, the distance between the targets, can be changed, as desired, to suit the nature of the work.

245. From Fig. 60, page 211, we may at once write

$$H \text{ ft.} = \frac{100 \cos^2 \theta}{n} S \text{ ft.}, \quad \ldots \quad (26)$$

in which θ is measured to the lower of the targets, the observed value being used for angles of elevation, and for angles of depression *N times* 34′ is subtracted from the observed value before inserting it in the formula (see § 241). The numerator of equation (26) may be found by Table III, pages 196–98, or by Fig. 57, between pages 200 and 201.

246. If the bottom target is placed at a distance from the foot of the rod equal to the height of the instrument above the reference point, then

$$V \text{ ft.} = \frac{100\frac{1}{2} \sin 2\theta}{n} S \text{ ft.}, \quad . \quad . \quad . \quad . \quad (27)$$

in which θ is measured as in § 245. The numerator of equation (27) can be found by Table III, pages 196–98, or by Fig. 58, between pages 200 and 201.

247. Stadia vs. Gradienter. It is sometimes claimed that the gradienter is superior to the stadia, but this is at least doubtful. When the gradienter is employed with a variable intercept (§§ 240–43), the labor and time required for the observations are essentially the same as with the stadia; but the formulas—equations (20) or (21), and (23)—are more complicated, particularly the correction of θ for angles of depression. When the gradienter is employed with a constant intercept (§§ 244–46), the time required to make an observation is greater than with the stadia, and the formulas—equations (26) and (27)—are considerably more complicated.

248. Limits of Precision. The gradienter as a distance-measurer is not as accurate as the stadia. This is chiefly due to the fact that with the former, two observations must be made upon each point and the instrument is liable to be disturbed between these observations. Further, changes of atmospheric refraction occur quickly, so that there is more risk of error

from two separate observations than if they were made simultaneously, as with the stadia.

In using the gradienter, to eliminate any back-lash or lost motion in the screw it is best to finish the setting with a motion always in the same direction.

249. Vertical Circle as a Gradienter. The vertical circle may be employed as a gradienter to lay off grades, or as a telemeter to determine the horizontal and vertical co-ordinates of a point.

250. To Measure Grades. To measure grades with the vertical circle, it is only necessary to change the degrees and minutes of arc into feet per hundred feet, by means of a table of natural tangents. For example, if we desired to run a grade line of one in a hundred, *i.e.*, a 1 per cent grade, we look in a table of natural tangents and find the angle whose tangent is 0.01, and set it off on the vertical circle. Similarly to run a 1.85 per cent grade, *i.e.*, a grade of 1.85 ft. per 100 ft., we would find the angle whose tangent is 0.0185.

If the angle of inclination of a particular slope has been measured, and we desire to express it in feet per hundred, we have only to find the natural tangent of the observed angle. For example, if the slope has an inclination of 43′, an inspection of a table of natural tangents shows that the grade is 1.25 per cent.

251. To Measure Distances. To use the vertical circle as a telemeter, notice that if the angle of elevation or depression of two points in the same vertical be observed, the distance between the points is the difference between the natural tangents of the observed angles.

To deduce a formula for this case, let $H =$ the horizontal distance from the instrument to the rod; $V =$ the vertical distance from the point on the ground under the instrument to the point upon which the rod is placed; $\alpha =$ the larger of the two observed angles; $\beta =$ the smaller of the two angles; and $s =$ the dis-

tance on the rod between the two points sighted at. Then, if α and β have the same sign, *i.e.*, if both are angles of elevation or angles of depression,

$$H = \frac{s}{\tan \alpha - \tan \beta}. \quad . \quad . \quad . \quad . \quad (28)$$

If α and β do not have the same sign,

$$H = \frac{s}{\tan \alpha + \tan \beta}. \quad . \quad . \quad . \quad . \quad (29)$$

If the target to which β is measured is placed at a distance from the foot of the rod equal to the height of the instrument above the reference point,

$$V = H \tan \beta; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (30)$$

$$V = \frac{s \tan \beta}{\tan \alpha \mp \tan \beta}. \quad . \quad . \quad . \quad . \quad (31)$$

ART. 3. VARIOUS TELEMETERS.

252. *Eckhold's Telemeter* consists of a telescope and micrometer-microscope firmly connected at right angles with each other, and both turning together in the same vertical plane. Below the rotation axis, in the plane of the telescope, is placed a finely graduated scale, which is read through the microscope. The intercept on the rod is of a constant length. To use this telemeter the telescope is directed to the top of the rod and the microscope read; the telescope is next directed to the bottom of the rod and the microscope is read again. The horizontal and vertical distances are computed by formulas essentially the same as equations (28) or (29), and (31) above.

Extravagant claims are made for this instrument, but they are not sustained by experience. It is inferior in accuracy to the stadia.

253. *Gautier's Telemeter* consists of a combination of two mirrors and a prism, such that when two successive observations are made from two points (a few feet apart) upon a third, the distance to the third point is the product of a factor read from the instrument, and the distance between the two points of observation.

"The accuracy of this instrument is extraordinary. With a base of 20 meters [66 feet], the error for distances below a kilometer [3,280 feet] is almost imperceptible. Distances from 3 to 6 kilometers [roughly 2 to 4 miles], and even more, have been measured by it, with bases of from 20 to 50 meters [66 to 164 feet], with a maximum error not exceeding one fourth of one per cent." Obviously such accuracy is impossible. 1. Only a low magnifying power is claimed for the telescope. 2. The observations are made by the coincidence of images, as in the sextant. 3. The instrument is held in the hand. 4. The error of measuring the base is multiplied in the result.

254. *Adie's, Smyth's, Clarke's, and Struve's Telemeters* each consist of two mirrors mounted on the ends of a rod, the direction of a third point being measured by the coincidence of images, as with the sextant. In Adie's telemeter the base is 36 inches, in Struve's 75 inches, and the others mentioned have bases intermediate between these two.

It is now generally conceded that the sextant is the best instrument for determining a distance when a measured base can be fixed.

255. *Other Telemeters.* There are many other forms of telemeters, several of which were invented for military use, but the above illustrate the principles of, at least, the most important ones. Only the stadia with fixed hairs and the gradienter have any value as engineering instruments.

CHAPTER XI.

SPIRIT LEVELS.

Art. 1. Construction.

257. Qualities Desired. The main qualities to be secured in a spirit-leveling instrument are stability of the instrument, defining and magnifying power of the telescope, and delicacy of the level bubble.

The stability of the instrument depends mainly upon the leveling screws and the manner of connecting the instrument with the tripod head (see Chap. II, Art. 2). The ordinary leveling instrument, as well as the transit, has four foot-screws; but as three give more stability and greater delicacy, it is more important to have three foot-screws on levels than on transits, for it is seldom necessary that the latter be exactly level (§ 128). Whether three or four foot-screws are used, the distance between opposite screws should be as great as possible. The center, or spindle, should be long and hard, and well fitted in the socket. Obviously the center of gravity of the instrument should be as near to the tripod head as possible, although many instruments are needlessly defective in this particular.

The optical qualities of the telescope have already been discussed (see Chap. VI).

However good the other parts of the instrument, the accuracy of the work depends upon the sensitiveness of the bubble. The bubble of a leveling instrument cor-

responds in importance with the graduation of a transit. It must be remembered that although a sensitive bubble may not remain exactly stationary, it will still give better results than a sluggish one which shows no movement when the inclination of the instrument is slightly changed. The liquid employed in level vials should have a minimum adhesion to the glass so as to settle quickly and accurately. Pure ether is best in this respect, but it has too large a co-efficient of expansion to permit its use in field instruments, which are exposed to great differences of temperature. Pure alcohol is frequently employed, but a mixture of alcohol and ether is generally considered best. The level vials of astronomical instruments and of some of the best engineering field instruments are provided with an air-chamber for regulating the length of the bubble, in which case pure ether can be used. A large air-bubble is more sensitive than a small one.

The scale by which the position of the bubble is read should be as close to the bubble as possible, to avoid parallax in reading; and therefore the graduation should be upon the glass tube. The glass tube should be protected by a metal case, but the former should be so fastened into the latter as to be free to expand and contract with changes of temperature. The common method of fastening the vial in the metal case with plaster of Paris is inadvisable for a sensitive bubble, since changes of temperature cause changes in the curvature of the tube and consequently in its sensitiveness.

258. Classification. Spirit-leveling instruments may be grouped in three classes. The first includes all instruments that can be adjusted by reversals. The wye level, Fig. 61, page 221, is the representative of this class. The second includes all leveling instruments that can not be adjusted by reversals. The dumpy level, Fig.

63, page 223, is the representative of this class. The third includes all instruments whose errors of adjustment may be eliminated by double observations. Fig. 64, page 225, is a representative. Instruments of this class are commonly called levels of precision,—sometimes geodesic levels;—and are used when extreme accuracy is sought.

259. Wye Level. The instrument shown in Fig. 61, is named the wye, or Y, level from the form of the supports of the telescope. A section of a wye level, by a different maker than that of Fig. 61, is shown in Fig. 62, page 222. The wye level is used far more than any other by American engineers. Its distinguishing characteristic is that the telescope may be revolved about its own axis, and turned end for end in its bearings. The only advantage of this construction is that it facilitates the adjustment of the instrument; while, on the other hand, owing to the nature of the construction the instrument does not hold its adjustment well. "The wye level is easily adjusted, and nearly always needs it."

Notice that the instrument shown in Fig. 61 has an inverting eye-piece, and that in Fig. 62 an erecting eye-piece. The latter is much more common, but the former is the better (see § 78).

To adjust the telescope, it must be free to revolve in the wyes about its own axis; but in using the instrument, it is necessary that the vertical hair should be vertical. Therefore, when the telescope is fastened in the wyes ready for use, there should be some means of knowing that the vertical hair is truly vertical. For this purpose some makers place a mark upon the wye and another upon the collar on the telescope; while others have a device (of which there are several forms) such that the clip passing over the telescope at the top of the wyes can not be fastened until the telescope is in a particular position. One of these devices

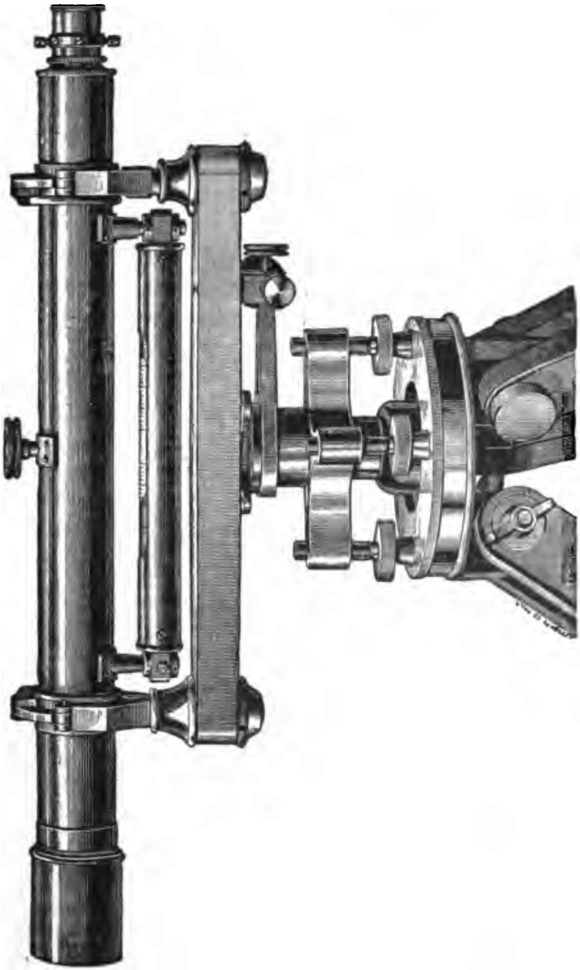

FIG. 61.—WYE LEVEL.

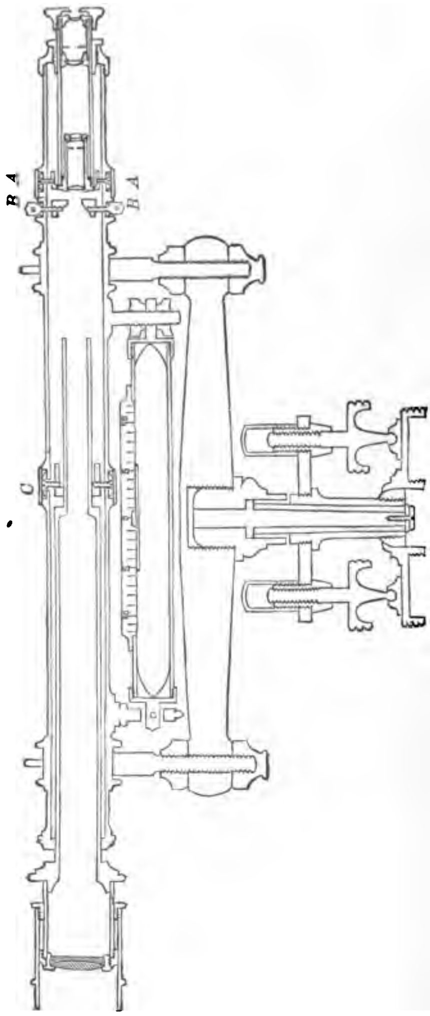

FIG. 62.—SECTIONAL VIEW OF WYE LEVEL.

is shown in Fig. 61, page 221, on the inner side of each clip.

260. Dumpy Level. This name is given to that form of leveling instrument in which the telescope is attached

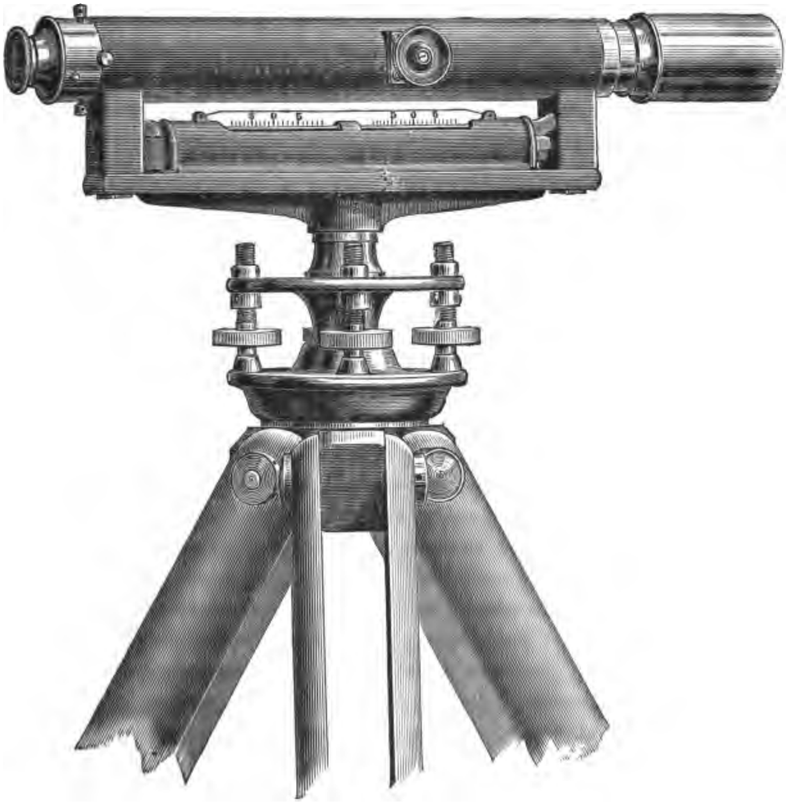

FIG. 63.—DUMPY LEVEL.

to the bar in such a way as not to admit of its rotation around its own axis, nor to allow of its reversion end for end. The telescope is usually inverting, and therefore shorter than the one commonly used on leveling instruments; hence the name dumpy level. A very good form

of this instrument is shown in Fig. 63, page 223. English engineers use the dumpy level almost exclusively. In the English books it is sometimes called the Troughton level, from the first manufacturer; and sometimes the Gavatt level, in honor of Gavatt's improvements.

In construction the dumpy is more simple and compact than the wye level, but is less convenient to adjust. It retains its adjustments better than the wye level, which is an important item in practice. If equally well made, it will do as accurate work as the more elaborate and more expensive wye level. The dumpy level usually has the inverting telescope, which gives it an advantage over the ordinary wye level (see § 78).

261. Levels of Precision. The distinguishing characteristic of this class of levels is that the telescope may be revolved about its own axis, and the level may be reversed end for end independently of the telescope. This construction enables the observer to eliminate all errors of adjustment, by making a double observation.

There is considerable variety in the form of this class of levels, but only two have been used to any appreciable extent in this country: the Swiss or Kern level, by the Lake Survey and Mississippi River Commission; and a modification of the Vienna or Stampfer level, by the Coast and Geodetic Survey. The construction of the two is similar, hence only the latter will be described here. The former is described in Professional Papers, Corps of Engineers, U. S. A., No. 24—Primary Triangulation U. S. Lake Survey,—p. 597, and also in Report of Chief of Engineers, U. S. A., for 1877, p. 1190.

262. The instrument shown in Fig. 64 is the level of precision employed on the U. S. Coast and Geodetic Survey.* The telescope may be reversed end

* Report for 1879, Appendix No. 15, pp. 202–11.

for end and revolved about its optical axis, the two positions in which the horizontal thread is horizontal being definitely fixed by projecting pins. The level may also be reversed end for end independently of

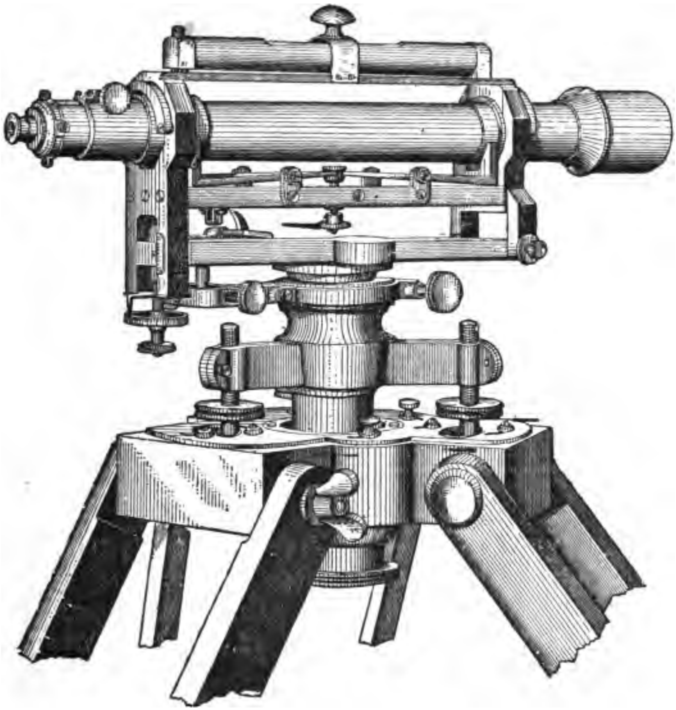

FIG. 64.—U. S. COAST SURVEY LEVEL OF PRECISION.

the telescope. One end of the telescope and level may be raised and lowered by the micrometer screw. Near the micrometer is a cam hook, by which the weight of the superstructure may be raised off the micrometer during transportation. Under the telescope are two false wyes on lever arms by which the telescope may be raised out of the wyes for transportation. The

whole instrument is secured to the tripod-head by a brass plate which fits over the feet of the leveling screws.

The aperture of the telescope is 43 mm. (1¾ inches nearly), focal length 410 mm. (16⅛ inches nearly), magnifying power 37. The value of one millimeter of the level scale is 1.5″ (radius = 450 feet). The diaphragm is glass, and has one vertical and two horizontal lines ruled upon it. The two horizontal lines are used as stadia hairs to determine the length of sight.

The instrument including the tripod weighs 45 pounds. A smaller size of this instrument, weight 23 pounds, is also used.

263. The following table contains some of the important information concerning the instruments used on four national surveys.*

TABLE IV.

DATA CONCERNING STANDARD LEVELS OF PRECISION.

	Great Britain.	India.	Switzerland.	France.
BUBBLE TUBE:				
Radius of curvature	360 ft.	540 ft.	600 ft.	360 ft.
Movement of line of sight corresponding to a movement of the bubble of 0.1 inch	5″	3½″	3″	3″ to 7″
TELESCOPE:				
Focal length	24 ins.	21 ins.	16 ins.	19 ins.
Diameter of objective	2⅛ ins.	2 ins.	1¼ ins.	1½ ins.
Magnifying power	50	42	42	36
RODS:				
Length	10 ft.	16 ft.	9.8 ft.	....
Graduated to	1–100 ft.	1–100 ft.	3–100 ft.	6–100 ft.
Read to	1–1,000 ft.	1–1,000 ft.	3–1,000 ft.	3–1,000 ft.
LENGTH OF SIGHT:				
Maximum distance from instrument to rod	330 ft.	660 ft.	330 ft.	430 ft.

* Proc. Inst. of C. E., Vol. 44, p. 181.

ART. 2. CONSTRUCTION OF LEVELING RODS.

264. The line of sight of the level is horizontal, and the distance of points below this line is measured by a graduated rod held vertically. Rods are generally graduated to read feet and decimals of a foot, and occasionally to feet, inches, and fractions of an inch; and rods graduated according to the metric system are sometimes used in this country.

There are two classes of leveling rods: (1) target rods, those having a target which is moved into the plane of sight, its position being read by the rod-man; and (2) self-reading or speaking rods, those having a graduation such that the position of the intersection of the line of sight and the rod can be read with the telescope. With the self-reading rod the rod-man has only to hold the rod vertical.

265. TARGET RODS. New York Rod. The target rod most frequently used in this country is the New York rod, shown in Fig. 65, page 230. This rod usually consists of two pieces of maple or satinwood sliding one upon the other, the same end always being held on the ground, and the graduations starting from this end. The graduations are to tenths and hundredths of a foot, the tenth figures being marked with a black figure and the feet with a larger red figure. The target carries a vernier reading to thousandths of a foot (see Fig. 10, page 67).

The front surface, *i.e.*, the one on which the target moves, reads to 6.5 feet. When a greater height is required, the horizontal line of the target is fixed at 6.5 feet, and the upper half of the rod, carrying the target, is slid upward, and the reading is obtained by a vernier on the side of the rod. The rod, when extended, may

be fastened in that position by means of a clamp at the lower end of the upper piece.

This rod is also made in three and sometimes four pieces. These forms are recent modifications to make the rod shorter when closed and longer when extended. The three-piece rod is 5 feet long when closed and 14 feet long when extended. The four-piece rod when closed is 5 feet long, and when extended is 16 feet long.

266. Boston Rod. The Boston Rod, Fig. 66, page 229, is formed of two pieces of mahogany or baywood, each about 6 feet long and sliding easily by each other in either direction. One piece is furnished at each end with a clamp and also a vernier; the other, or front piece, carries the target, and has on each edge a strip of satinwood inlaid, upon which divisions of feet, tenths, and hundredths are marked. The target is a rectangle of wood fastened on the front half.

When a reading of less than 6 feet is desired, the rod is placed target-end down, and the piece carrying the target is raised. When a reading of more than 6 feet is desired, the rod is placed target-end up, and the piece carrying the target is raised, the reading being taken from the other vernier. This rod is very convenient owing to its extreme lightness, but the parts are too frail to endure rough usage, and therefore engineers have generally given the preference to heavier and more substantial rods.

267. Philadelphia Rod. The Philadelphia Rod, Fig. 67, page 229, is a self-reading rod which is fitted with a target. The rod is made of two strips of cherry, each about $\frac{3}{4}$ inch thick by $1\frac{1}{2}$ inches wide and 7 feet long, connected by two metal sleeves, the lower one of which has a clamping screw for fastening the two parts together when the rod is raised for a greater reading than 7 feet. Both sides of the back strip and one side

FIG. 65.
NEW YORK ROD.

FIG. 66.
BOSTON ROD.

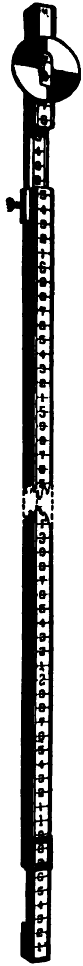

FIG. 67.
PHILADELPHIA ROD.

of the front one are recessed one sixteenth of an inch below the edges. These depressed surfaces are painted white, and divided into feet, tenths, and hundredths of a foot, according to the general principle illustrated in Fig. 72, page 233. The front piece is graduated from the bottom upward to 7 feet, the front surface of the rear half from 7 to 13 feet, also from the bottom upward; and the back surface of the rear half is figured from 7 to 13 feet, from the top downward.

This rod may be used either as a target rod or as a self-reading rod. The target, usually painted as Fig. 69, page 231, carries a scale (not a vernier) one tenth of a foot long divided to hundredths and half-hundredths of a foot, by which the rod is read to half-hundredths of a foot. When used as a target rod, for readings of less than 7 feet the target is moved up and down the front piece; and for readings greater than 7 feet the target is set at 7 feet and the back piece is run up, the reading then being obtained by a scale upon the back piece. When used as a self-reading rod, the observer notes, through the telescope, the point on the rod covered by the cross hair. For readings greater than 7 feet, the rear piece is fully extended, the whole front of the rod then becoming a self-reading rod 13 feet long.

268. The Target. This is a piece of brass or iron which can be moved up or down the rod, or clamped in any position. It carries a scale or vernier to subdivide the least space on the rod. The face of the target should be painted of such a pattern that it may be precisely bisected by the horizontal cross hair. Some of the many varieties are shown in Figs. 68, 69, 70, and 71, page 231.

Fig. 68 depends upon the nicety with which the eye can determine whether a line bisects an angle, which can be done very accurately by noticing the relative po-

sition of the two points formed on opposite sides of the hair. For data on the relative accuracy of the targets shown in Figs. 68 and 69, see Tables I and II, Appendix III.

Fig. 69 is the target of the New York rod, and essentially that of the Philadelphia rod. The design is

FIG. 68.

FIG. 69.

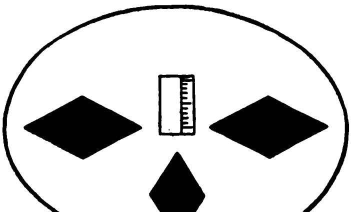

FIG. 70.

FIG. 71.

not good, because the cross hair may be above or below the middle of the target by its full thickness, as magnified by the eye-piece, without the error's being perceptible.

Fig. 70 is the same in principle as Fig. 68. For long sights Fig. 68 is the better, but for very short sights the diamond is larger than the field of view. Fig. 70

was designed to obviate this defect, and is an excellent target except for very long sights.

Fig. 71 depends upon the accuracy with which the eye can bisect a space. The only objection to this form of target is that for the greatest accuracy the width of the white band should be proportional to the length of the sight; and therefore if the width is right for short sights it will be too narrow for long ones. If the length of a sight were constant this would probably be the best form of target.

269. Level targets are usually painted red and white. Black and white are best for visibility; but red and white are most easily distinguished among trees, shadows, etc., and red gives the stronger contrast with the cross hairs. Probably red and white are the best for Fig. 69, and black and white for the other figures on page 231.

270. Self-reading Rods. A self-reading rod is one so graduated as to enable the observer to note at once the reading of the point which lies in the line of sight. The rod-man has only to hold the rod vertical; the observer notes and records the reading. The most common rod of this class is the Philadelphia rod, which may be used as a target rod also (§ 265). A few of the many patterns which have been proposed for self-reading leveling rods are shown in Figs. 72–75, page 233.

Fig. 72 illustrates the principle of the Philadelphia rod (§ 267). The figures indicating feet are red, the tenths black. The figures are six hundredths in height, placed with centers over the marks. Sometimes the bottom of the figure is placed on the line, in which case the figures are eight hundredths high. The angles of the figures indicate fractions of tenths. The graduation given is suitable for short sights; and for longer ones the figures are made larger, and only those for the even tenths are marked. The advantages claimed for

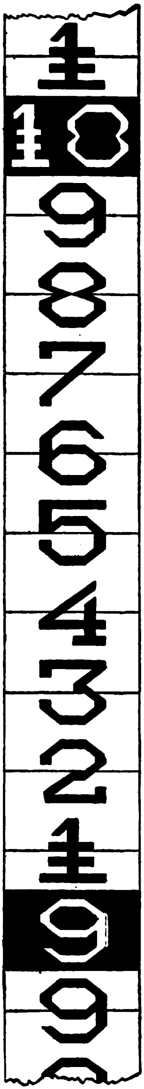

FIG. 72.

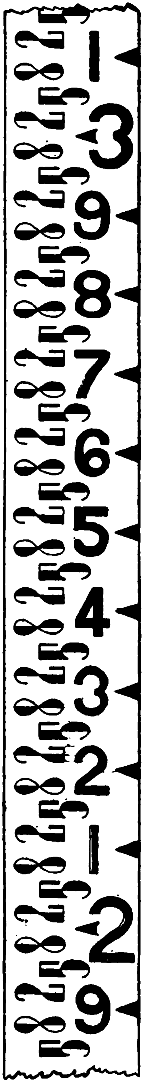

FIG. 73.

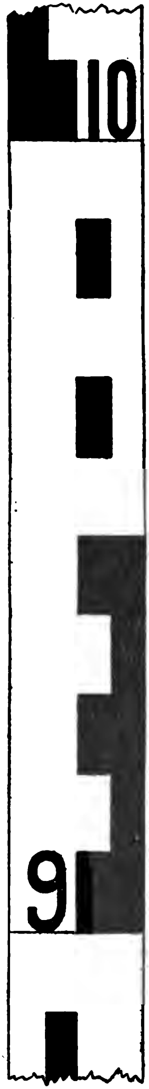

FIG. 74.

FIG. 75.

this general form of graduation are distinctness and visibility ; but in neither of these respects is it as good as Figs. 74 or 75, or as the stadia rods shown in Figs. 49–52, page 180.

Fig. 73 shows the graduation of the Francis rod,* which is a complete graduation to hundredths without visual division, and can be read without counting. The foot-marks are red, and larger than the tenths. It is best adapted to short sights.

Fig. 74, in its essential features, is the graduation of the favorite British level rod. The graduation shown is to tenths, the hundredths to be estimated. The graduation of the standard British rod consists of a number of black lines on a white ground, one hundredth of a foot wide and one hundredth of a foot apart. The lines indicating the tenths are longer than the others, and each alternate tenth is numbered. This graduation is very confusing to read. The one shown in Fig. 74 is designed on the principle that it is better to estimate the hundredths than to read them from a finely divided scale. This form of graduation reduces the difficulty of counting a number of small divisions, and also the possibility of gross mistakes. Fig. 74 has greater distinctness and visibility than the standard British rod, but it must be condemned for the same reason that the quadrant target of the New York rod was condemned in § 268.

The principle of the Texas rod,† Fig. 75, is of frequent application in making self-reading rods. It is superior to that of Fig. 74 in that it obviates the error due to the thickness of the cross hair. The points indicate tenths, but it can be read by estimation to hundredths. It is possible to make the reading much more

* *Engineering News*, Vol. 8, p. 415.

† *Engineering News*, Vol. 8, p. 289.

accurate by dividing the oblique side of the triangle, than can be done when the rod is graduated according to the principle of Fig. 74.

Any pattern suitable for a stadia rod (see Figs. 49–52, page 180) can be used in making a self-reading leveling rod. The pattern may be painted or stenciled directly upon the wood, or it may first be drawn or painted upon paper, and then fastened on the rod with varnish or any glue not soluble in water. Strips of cloth or paper containing scales for this purpose are sold by dealers in engineering stationery. For a few points applicable to the manufacture of home-made self-reading level rods, see §§ 208 and 209.

271. Target vs. Self-reading Rods. Probably the former are the more common now, but as the advantages of the latter are becoming better understood they are being more generally used. The chief advantage of self-reading rods is the saving of time. Setting the target to some exact point in accordance with directions given from a distance, is a tedious process at best. After a little familiarity with the pattern of the self-reading rod, the height of the line of sight upon the rod can be read very quickly.

On the other hand, target rods must of necessity be capable of greater precision than self-reading ones, but the difference in accuracy is not so great as might at first seem. The accuracy of leveling depends upon a number of things (§§ 310–19), of which the reading of the position of the line of sight upon the rod is one of the least important, and all of the others are independent of the kind of rod. Reading the position of the target to thousandths of a foot is unnecessary and useless, unless all other parts of the work are equally precise. The thing to be sought is proportionate accuracy in all parts of the work. If several independent readings of a rod be made upon the same point, the differ-

ence between the various readings will probably be considerably larger than the probable error of reading a self-reading rod (see Tables I and II, Appendix III).

A self-reading rod is generally used in precise leveling, two or three hairs, usually the latter, being read to reduce the error of reading. Three observations on a self-reading rod are probably more accurate than a single observation upon an ordinary target, and can be made in about the same time. The difference between the readings of the central and the two side hairs affords a check on the reading and also on the length of sight.

ART. 3. TESTING THE LEVEL.

272. THE BUBBLE TUBE. Its Form. A bubble tube is a glass tube bent or ground so that its inside upper surface is circular on a longitudinal section. Since the position of the bubble is determined by reading the position of its ends, a longitudinal section of the interior should be of uniform curvature, *i.e.*, circular, and the tube should not be in the least conical.

These conditions are satisfied if both ends of the bubble move over equal spaces for equal displacements of the bubble tube in altitude. This condition is not an absolute necessity, since the bubble should always stand in the middle when an observation is made. But in very delicate instruments it is nearly impossible to keep the bubble in the middle; and hence, if the above condition is satisfied, the bubble need not be brought exactly to the center each time, for its position may be noted and a correction applied.

To test the uniformity of the curvature of the tube, sight at the target of a leveling rod, and read the position of the bubble; and then move the target over equal

spaces, sighting upon it and noting the movement of both ends of the bubble for each position.

Or this test may be made without the aid of a telescope, by fastening the bubble tube to a board hinged so as to move in a vertical plane, and measuring the tangent of the angle of elevation. A micrometer screw* affords the easiest and best means of measuring the tangent. Such a contrivance is known as a level-trier.

A less delicate method of testing the above condition is to note whether the bubble expands and contracts equally both ways from the center during changes of temperature. In making these observations great care must be taken that the inclination of the level tube is not changed by external forces or by a change of temperature of the parts of the instrument.

273. Its Sensitiveness. The most important condition to be fulfilled by the level tube is that the bubble shall be sensitive. The sensitiveness depends upon the radius of curvature, or, in other words, upon the distance the bubble moves for any change of inclination.

To measure the sensitiveness, proceed as follows: Bring the bubble nearly to the center, and sight upon a rod held vertically. Raise or lower one end of the level, by operating the foot screws, until the bubble moves about as far to the other side of the center, and sight at the rod again. Let $h =$ the difference of rod readings; $D =$ the distance from the instrument to the rod; $m =$ the distance the bubble moved; $d =$ the length of one division of the scale; $n =$ the number of divisions the bubble moved; $I =$ the change of inclination of the line of sight; $R =$ radius of curvature of the level tube;

* A fair micrometer screw can be made of a tangent screw from a transit or leveling instrument, to which is attached a graduated cardboard disk.

and $V =$ the angular value of one division of the scale.

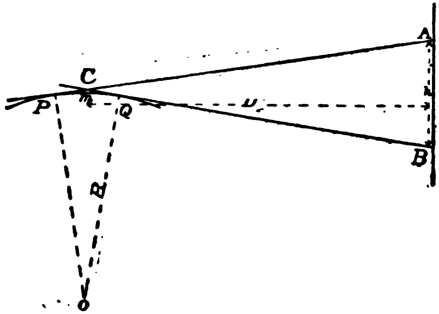

FIG. 76.

Then in Fig. 76, from the approximately similar triangles ABC and OPQ, we have:

$$R = \frac{m}{h} D = \frac{nd}{h} D \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

$$\tan I = \frac{h}{D} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Hence $$V'' = \frac{h}{\tan 1'' n D} = \frac{h}{0.00000485\, n D}.^{*} \quad (3)$$

The sensitiveness of a bubble is generally stated by giving the angle corresponding to a movement of one inch. Good engineer's levels have a motion of 200 to 120″ per inch, or a radius of curvature of 85 to 140 feet. Levels of precision have a motion of about 50 to 30″ per inch, or a radius of curvature of 340 to 600 feet.

274. SENSITIVENESS VS. MAGNIFYING POWER. The magnifying power of the telescope and the sensitiveness of the level should be so proportioned to each other that

* The engineer should carry the value of tan 1″ in his mind, as it is of frequent application. To assist the memory notice that the value is approximately 5 zeros and a 5.

the least perceptible motion of the bubble will cause sufficient motion of the cross hair on the rod to be easily noticed; and *vice versâ*, the least noticeable motion of the cross hair on the rod should cause a perceptible movement of the bubble. A higher power or a more sensitive level than that required by the above condition adds nothing to the accuracy of the instrument and is even worse than useless, for the former causes a loss of brilliancy of the object and the latter an annoyance in leveling the instrument. Of two levels in the writer's possession, one had a magnification of 17 and a radius of the bubble of 84 feet, and the other a magnification of 27 and a radius of 22 feet. The simple expedient of changing the bubbles improved both instruments very much. A third instrument is inferior to the other two in definition, although it has a bubble of 165 feet radius.

275. TELESCOPE SLIDE. The telescope slide should be straight and the optical center should move in the line of collimation. The *wye* level, *after having been collimated* (§ 278), may be tested by the method of paragraph 1, § 121. The slide is first tested for deviation from a vertical plane by setting a row of points with the telescope in a chosen position, and then revolving it 180° in the wyes and setting a second row. An easy method of accomplishing this is to set the instrument in a cut or low place, and sight at a plumb-line, marking the point on the head of a stake. Care must be taken not to disturb the instrument when moving the slide. If there is any play or looseness in the object-glass slide it will be almost impossible to make this test satisfactorily. Having completed this test, revolve the telescope 90° from its first position and repeat as above. If the two rows of points do not coincide, either the slide is not straight or it does not move in the right direction. If the direction of motion of the slide is adjustable, the

engineer can determine the source of error only by trial. In doing this remember that a movement of the back end of the objective slide disturbs the adjustment of the line of collimation. A better method of testing the slide of the wye level will be given when considering the method of collimating that instrument (see § 280.)

The slide of the *dumpy* level can not be tested by the method of paragraph 2, § 121, since the telescope can not be reversed about its own axis. It may be tested, *after having been collimated* (§ 285), by taking readings on a row of stakes as described in paragraph 2, § 121, and then moving the instrument to the other end of the row and sighting upon them again. If the differences of level relative to the stake first sighted at are the same both times, the slide is straight. This is not a very good test owing to the errors of observation, but it is the only one available.

276. Rings and Wyes of the Wye Level. The rings should be cylinders of the same size, and the wyes should present the same angle in the direction of the telescope tube. The last is not likely to be in error an appreciable amount. The wyes may be compared by taking them off and placing them side by side, or by carefully marking the angle of each upon a piece of paper.

The equality of the rings is a very important matter, but is often overlooked. Neglecting to examine this point makes the accuracy of the engineer's work depend upon that of an unknown workman.

The size of the rings can not be tested by any system of reversals, and can be examined only by means of some auxiliary instrument. They may be compared by calipering or by means of a delicate striding level. The student will do well to inquire into the accuracy of these methods. But, for the engineer, the best method is to take a test level, as will be described in § 284.

The test level should not be taken until all the adjustments have been made.

If the rings are not the same size, the instrument is, in effect, a dumpy level, and must be adjusted and used as such,—that is to say, the telescope must not be reversed in the wyes end for end, as in the wye level.

ART. 4. ADJUSTMENTS OF THE WYE LEVEL.*

277. LEVEL TUBE. The tangent at the middle of the level tube (the zero of the scale) should be parallel to the bottom of the wyes. The adjustment is made in two steps: the first is to bring the tangent and the axis of the telescope into the same plane, and the second is to bring the level parallel to the bottom of the wyes.

1. In making this adjustment, it is best to have the sun-shade on and have the object-glass run out for the mean length of sight, for then the weight will be symmetrical about the vertical axis and the level will not be affected by any unequal strain.

Clamp the vertical axis of the instrument, loosen the clips that hold the telescope in the wyes, rotate the telescope in the wyes until the level is about vertically under the telescope, and bring the bubble to the middle by moving the foot screws. Then rotate the telescope in the wyes, so that the level tube swings a few degrees to one side of a vertical plane. If the bubble changes its position longitudinally, it shows that the axis of the telescope and the level are not in the same plane. Correct *all* the error by moving the azimuth screws of the level.

After having made the bubble tube at least approximately parallel to the bottoms of the rings, if the bubble runs toward the same end when swung on both sides

* For general remarks on adjustments, see § 37.

of the vertical, the tube is conical (see § 272). Correct work can not be done with a tube of this form, owing to the effect of changes of temperature and to the uncertainty of the level's being vertically under the telescope.

2. Having made the first step, as above, bring the bubble to the middle, and carefully reverse the telescope in the wyes, end for end. If the bubble does not stand in the middle after reversal, correct *half* the difference by the altitude screws of the level, and the other half by the foot screws. Notice that if the angles of the wyes are the same and symmetrically placed, this brings the level parallel to the bottom of the wyes, whether the rings are of the same size or not.

If the rings are of the same size, this adjustment brings the tangent of the level parallel to the axis of the rings. The equality of the rings must be tested by a test level (§ 284).

278. Cross Hairs. The line of collimation should coincide with the line of the centers of the rings. To make this adjustment, *i.e.*, to collimate the instrument, focus upon some well-defined point (§ 94) 200 or 300 feet distant, and turn the telescope over in the wyes. If the intersection of the cross hairs has moved from the point sighted at, bring it half-way back by moving the screws in the diaphragm which carries the cross hairs (*B B*, Fig. 62, page 222), and correct the other half by the foot screws and the tangent screw to the vertical axis.

Since the cross hairs may not have been moved over exactly half the difference, and since the instrument may have been disturbed, the adjustment should be repeated. The instrument need not be level while making this adjustment.*

* The next two sections relate to the testing of the direction of motion of the object-glass slide. If the instrument is non-adjustable and has been

279. Since it is tedious to always bring the target exactly to the middle of the field of view, the horizontal hair should be as nearly horizontal as possible; and therefore, after having adjusted the line of collimation, bring the vertical axis vertical, close the clips at the top of the wyes (or bring the mark on the ring to coincide with the mark on the wye—§ 259), and bisect the target of a leveling rod with the horizontal hair. Move the telescope in azimuth, keeping it level, and then if the target is not bisected throughout the length of the hair, rotate the reticule until it is. The horizontal hair will now be horizontal, and the other one will be vertical or nearly so, since the two are supposed to have been placed perpendicular to each other. It will be at least nearly enough vertical to plumb the rod by.

Instead of adjusting the horizontal hair as above, it is customary to place the vertical hair vertical by sighting at a corner of a building. Since the rod may not be brought exactly to the center of the field, it is more important that the horizontal hair should be truly horizontal than that the vertical one should be vertical. For this reason, it is better to adjust the horizontal than the vertical hair.

280. When the instrument has been collimated for the above distance, focus upon a point very near the instrument, and test the adjustment of the line of collimation for this distance. If the instrument is not in adjustment for the second point, either the optical center is not in the axis of the rings, or the line of motion is not parallel to the axis of the rings. The engineer has no means of determining which of these conditions causes the error, except by trial. The former may be caused by the optical center's not lying in the

tested in this respect, the operations described in the next two sections may be omitted in making the adjustments. If the direction of motion is adjustable, the engineer should occasionally test it.

center of figure of the lens, or by the telescope slides not being straight. With an instrument which is provided with a means of adjusting the direction of motion of the object-glass slide (*C*, Fig. 62, page 222), the engineer may move the inner end of the slide until half the error is corrected; and then collimate again upon the first point, and test the adjustment upon the second. If the instrument is in adjustment for the two points at the same time, it is *probable* that the optical center lies in the axis of the rings and that the direction of motion coincides with that line.

281. It is generally held that if an instrument is in collimation for two different distances at the same time, the line of sight must be in the axis of the rings; but this is not necessarily true. In general, if either the intersection of the cross hairs or the optical center of the objective does not lie in the axis of the rings, the line of collimation will describe the surface of a cone when the telescope is revolved in the wyes; and for any given position of the optical center, it is possible to have such a position of the cross hairs and corresponding motion of the slide that the instrument may be in collimation for two points at the same time and still the line of collimation not be in the axis of the rings. That is to say, if the instrument is not in collimation, the line of sight will describe the surface of a variable cone, the points collimated upon being two positions of the vertex. Such a relation of parts could be obtained only by a series of successive approximations. The only check against the occurrence of this condition is a test level (§ 284).

Another particular case is that in which the line of collimation describes a cylinder. This occurs when the optical center and the intersection of the cross hairs are on the same side of, and equally distant from, the axis of the rings. This state of affairs will be revealed by

the instrument's being out of collimation the same amount for all distances. In this case the line of collimation will be parallel to the level (assuming the rings to have been found to be of the same size), and therefore the instrument is in adjustment. If the optical center does not lie in the line of the centers of the rings, the cross hairs may be moved, by a series of trials, until the line of sight shall describe the surface of a cylinder. Notice, however, that this can occur only in those instruments in which the line of motion is parallel to the axis of the rings.

282. Centering the Eye-piece. If after having brought the intersection of the cross hairs into the axis of the ring, the same field of view is not presented during an entire revolution of the telescope in the wyes, the fault is in the centering of the eye-piece; in other words, the optical axis of the eye-piece does not coincide with that of the objective. Some instruments are provided with an arrangement for correcting this by a motion of the inner end of the eye-piece,—see *A A*, Fig. 62, page 222. (The adjusting screws are covered by a ring which can easily be slipped off.) Strictly, this adjustment does not place the two axes parallel; it only makes the optical axis of the eye-piece intersect the optical axis of the objective in the plane of the image. However, the only effect is a slight blurring of opposite sides of the image.

Inverting telescopes have no such adjustment; and it would be an advantage if it were omitted in telescopes with an erecting eye-piece; but this can not be done, owing to mechanical reasons.

283. Wyes. The line of the bottoms of the wyes should be perpendicular to the vertical axis of the instrument. When the instrument is in adjustment, the bubble tube is parallel to the bottom of the wyes (§ 277), and therefore this adjustment is equivalent to

placing the bubble tube perpendicular to the vertical axis of the instrument. Notice that this adjustment is for convenience only, as it in no wise affects the accuracy of the work, but merely saves the labor of leveling up every time the telescope is moved in azimuth.

To make this adjustment bring the bubble to the middle by turning the foot screws, and turn the telescope 180° in azimuth. If the bubble does not stand in the middle after reversal, correct half the error with the nuts on the lower ends of the wyes (Fig. 62, page 222), and the other half with the foot screws. Since the telescope may not have been revolved exactly 180° in azimuth, test the adjustment for a position 90° from the one used as above, and then repeat in the first position.

284. Test Level. After having made the usual adjustments of the wye level, it is absolutely necessary that they should be verified by taking a test level.

To take a test level, drive two pegs into the ground, say, 400 feet apart, set the instrument exactly half-way between them, and carefully determine the difference of level. A line joining the two positions of the target is a level line, and the difference between the readings is the true difference of level however much the instrument may be out of adjustment (§ 126). Make several determinations of the difference of level for a check. Next, set the instrument near one of the pegs, say, 10 feet beyond it, and at least nearly in line with the other, and re-determine the difference of level, making several pairs of observations.

If the difference is the same both times, it is *certain* that all the conditions enumerated in the preceding discussion have been satisfied. That is, an agreement in the test level proves (1) that the slide is straight, (2) that the line of motion is parallel to the line of centers of the rings, (3) that the rings are of the same size, (4) that the level is parallel to the bottom of the wyes, and

(5) that the line of collimation is parallel to the line of the centers of the rings.

If the differences for the two positions of the instrument are not the same, the error is due to defect in one or more of the five particulars mentioned above. If the slide is not straight, the only remedy is in a new instrument. Some instruments are provided with a means of changing the direction of motion of the object-glass slide; but it is at least doubtful whether this is any advantage to an instrument. When no means of adjustment is provided, the only remedy is a new instrument. If the rings are not of the same size, the instrument can be adjusted and used as a dumpy level. If the fourth or fifth conditions are not satisfied, a re-adjustment is sufficient.

ART. 5. ADJUSTMENTS OF THE DUMPY LEVEL.*

285. COLLIMATION. It is required to make the line of collimation and the level parallel to each other. This may be accomplished in either of two ways: (1) by bringing the line of collimation parallel to the level, or (2) by bringing the level parallel to the line of collimation. In case both the level and the line of collimation are movable, the latter method is to be preferred; for then the intersection of the cross hairs can first be placed in the optical axis of the telescope, and not be disturbed by this adjustment. The best form of instrument is that in which the level is fixed, for then the maker can place the level parallel to the optical axis before he fastens the telescope to the standards; and when the line of collimation is adjusted, it will coincide with the optical axis.

1. To place the line of collimation parallel to the level, set the instrument upon a level piece of ground, and

* For general remarks on adjustments, see § 37.

bring the bubble to the middle of its race, read the rod upon a point, say, 200 feet distant, turn the instrument about, bring the bubble to the middle, and sight in the opposite direction upon a point at exactly the same distance as before. The difference of readings is the true difference of level. Make several determinations of the difference for a check. Then move the instrument near one of the points, say, 10 feet beyond it, bring the bubble to the middle, and sight upon each point. If the second difference is not equal to the first, correct the error by moving the cross hairs over a space on the farther rod equal, for the case supposed above, to $\frac{41}{40}$ths of the apparent difference of level (see the second paragraph of § 126). When the difference of readings is equal to the true difference of level, the line of collimation is horizontal and therefore parallel to the level.

2. To place the level parallel to the line of collimation, set the instrument midway between two points and determine the true difference of level. Next move the instrument near one of the points, and by means of the foot screws change the inclination of the telescope until the difference of the readings is the same as when the instrument was in the middle (see § 126). The line of sight is then horizontal (see Fig. 29, page 113). Without altering the inclination of the line of sight, raise or lower one end of the level until the bubble is in the middle. The level and line of sight are then parallel.

Notice that this adjustment does not necessarily make the line of collimation coincide with the optical axis of the telescope; but it can be shown* that *if the direction of motion of the slide is parallel to the optical axis*, the instrument will still give correct results when adjusted as above.

286. Wyes. For convenience, the level and line of collimation should be perpendicular to the vertical axis.

* Rankine's Civil Engineering, p. 84.

To make this adjustment, bring the bubble to the middle by means of the foot screws alone, and revolve the instrument 180° in azimuth. If the bubble is in the middle after reversal, it is adjusted; if not, correct half the error by raising or lowering one of the standards connecting the telescope with the tripod head,* and the other half by the foot screws. Repeat the operation until the bubble does not move in the tube when the instrument is turned in any position on its vertical axis.

In some instruments there are no means of adjusting the height of the standards or wyes, in which case it becomes necessary to make the level perpendicular to the vertical axis before adjusting the line of collimation, and to adjust the line of collimation by the first method.

ART. 6. ADJUSTMENTS OF LEVEL OF PRECISION.

287. As the adjustments of levels of precision do not commonly differ materially from those of the ordinary forms, the method need not be re-stated here. It is customary to adjust the instrument as carefully as possible, and then determine the error of adjustment. Each observation may then be corrected, thus affording a check between the members of a double observation. The tests and adjustments of the form shown in Fig. 64 (page 225) are the same as those of the wye level, with the following tests in addition.

288. The telescope when raised or lowered by the micrometer screw should move in a vertical plane. This adjustment is necessary, since it is not always practicable to have the line of sight exactly horizontal at the moment of sighting. To test this adjustment, care-

* The screws for raising or lowering the standards are on the under side of the cross-bar, and are to be turned with a screw-driver. Owing to the unexposed position of these screws, they are not liable to be turned accidentally, which is one of the advantages of this form of level.

fully level the instrument, and sight upon a plumb-line; and then move the telescope in altitude and see whether the intersection of the cross hairs follows the plumb-line. If it does not, the only remedy is to return the instrument to the maker.

289. The angular value of one revolution of the micrometer screw should be known approximately. This can be determined only approximately, owing to the eccentric position of the line of sight with reference to the pivots on which the telescope turns. The value of a revolution of the micrometer is useful only in correcting an observation made when the line of sight is not exactly horizontal, and hence an approximate value is sufficient for any case arising in practice. To determine an approximate value of one revolution, bring the line of sight nearly horizontal, read the micrometer, and sight upon a level target at a distance D. Then turn the micrometer screw one revolution, and read the rod again. Representing the difference of rod readings by d, the angular value of one revolution in seconds of arc is $\frac{d}{D \tan 1''}$. To reduce the error of observation, determine the value of several—say, three or four—revolutions and divide the result by the number of revolutions.

290. The collars, or rings, on the telescope, *i.e.*, the bearings in the wyes, should be of exactly the same diameter. If they are not, the inequality must be determined and a correction computed. The inequality of rings is the only instrumental error that can not be eliminated by any system of double observations. It may be eliminated from the final result by equal back- and fore-sights (§ 326); but to employ the check of double leveling (§ 306), a correction for inequality of collars must be applied to the rod reading.

To find the inequality of collars, set the instrument firmly, and level it carefully. Read both ends of the

bubble, and take the mean; reverse the bubble tube end for end, read again, and take the mean. Half the difference of these means is the inclination of the top of the rings, expressed in divisions of the bubble scale. Reverse the telescope end for end in the wyes, and repeat the entire process as above. To meet the possibility of the rings' not being perfectly round, revolve the telescope about the optical axis and determine the inclination as above with the telescope both direct and reversed. If

E_d = the inclination when telescope is direct and bubble tube erect,

E_r = the inclination when telescope is reversed and bubble tube erect,

I_d = the inclination when telescope is direct and bubble tube inverted,

I_r = the inclination when telescope is reversed and bubble tube inverted,

V = the value of one division of the bubble scale,

the correction to be applied to an observed reading at a distance D, is equal to

$$\tfrac{1}{2} \sin 1'' v D \left[\left(\frac{E_d - E_r}{4} \right) + \left(\frac{I_d - I_r}{4} \right) \right]$$

If the object-end ring is too large, the line of sight will be inclined downward when the tops of the rings are horizontal.

291. For the most accurate work, it is necessary to determine the absolute value of the unit of the rod, and also to test the uniformity of the graduation. The coefficient of expansion of the rod and the graduation of the attached thermometer should likewise be tested, as also the adjustment of the level by which the rod is kept vertical.

ART. 7. USING THE LEVEL.

292. A level line is a line parallel to the surface of still water. A horizontal line is a straight line tangent to a level line. Level and horizontal are frequently used as meaning the same thing, and many times there is no appreciable difference between the two (§ 319, equation 3).

The level is used (1) to find how much one point is above or below a level line passing through another point, (2) to obtain a profile of a line, and (3) to locate contour lines, grade lines, boundaries of embankments and excavations, etc. Whatever the ultimate purpose of the work, the immediate object for any setting of the instrument is to find how much one point is higher or lower than another, and this is ascertained by obtaining a horizontal line, and measuring how far each point is below this line. The line of sight of a properly adjusted leveling instrument is a horizontal line; and as the instrument is revolved in azimuth, this line marks a horizontal plane. The vertical distance of any point below this plane is measured by the leveling rod.

293. DIFFERENTIAL LEVELING. This consists in finding how much one point is above or below a level line passing through another point. If the two points are not too far apart, either horizontally or vertically, the difference of level may be found by setting the instrument between the two and taking a reading of the level rod on each point. The difference of the readings will be the difference of level required.

If the two points are so far apart, either horizontally or vertically, that the difference of level can not be found by a single setting of the instrument, establish one or more intermediate points, and determine the

difference of level between successive pairs of points. The algebraic sum of the difference of level between the successive pairs of points is the required difference of level; or, in other words, the sum of the rod-readings for the odd-numbered sights *minus* the sum of the readings for the even-numbered sights is equal to the algebraic difference of level.

If the elevation of the first point is given with reference to any datum, as, for example, sea-level, the elevation of the first point *plus* the reading of the rod on that point gives the elevation of the plane of sight; and the height of the plane of sight *minus* the reading of the rod on the second point gives the elevation of that point above datum. The rod reading on the known point is called the *back-sight*, and the reading on the point whose elevation is to be determined is called the *fore-sight*. Notice that the terms back-sight and fore-sight have no reference to directions. As shown above, the back-sight reading is essentially positive, and the fore-sight is essentially negative; and therefore the back-sight is sometimes also called a *plus-sight* and the fore-sight a *minus-sight*. The intermediate points established between two remote points for the purpose of determining the difference of level of the latter, are called *turning points*. *Bench marks*, or simply *benches*, are points of more or less permanent character whose elevations are determined and recorded for future reference. In the notes, benches are indicated by *B. M.* or *B*, with a subscript to show the number of the bench. Thus B_3 means the third bench from the beginning of the line.

294. Field Routine. The rod-man holds the rod on the starting point. He should stand directly behind the rod, and hold it vertically. The instrument-man, or leveler, sets up the instrument 100 to 300 feet (§ 325) from the end of the line, in the direction in which the

work is to proceed. It is not necessary that the instrument be set on the line, unless the distance run is also desired, in which case the instrument should be approximately on line. Having leveled the instrument, the leveler sights upon the rod, and, if it is a self-reading rod, notes the reading and writes it down as a "+ sight" (§ 293, third paragraph).

If a target rod is used, the leveler signals the rod-man the direction in which to move the target. There is great variety in the manner of making these signals. The important thing is that they should be easily seen and readily understood. A very good system of signals is that in which the arm raised above the shoulder indicates that the target should be moved up, the arm coming more nearly horizontal as the target approaches the desired point; when the target is right, the arm is swung horizontally, or in a circle, at which signal the rod-man clamps the target. Similarly, the arm held below the shoulder indicates that the target should be moved down. The rod-man should place the target as near at the proper place as he can by estimation, without waiting for signals from the leveler.

After the target has been lined in by the leveler, the rod-man should wave the rod slightly to and fro, being sure that it passes through the vertical position; and in the mean time the man at the instrument notices whether the target rises to the line of sight, *i.e.*, whether the highest position of the target coincides with the horizontal hair. If it does not, the leveler directs the rod-man to move the target. The instrument-man can tell whether the rod leans to the right or left, by comparing it with the vertical hair. If the rod is not vertical, he should hold one arm up vertically, stand where the rod-man can see him, and incline his body in the direction the rod should be moved. A good rod-man will seldom, or never, need this signal.

When the position of the target is satisfactory, the rod-man calls out, first, the number of the station or stake, which the leveler calls back as evidence that he understands; and then the rod-man calls out the reading, one figure at a time, which is repeated by the leveler as he records it in his note-book. These numbers may be read and called off while the rod-man is walking to the next station.

Having obtained the reading at the first station, the rod-man proceeds as far on the other side of the instrument as the instrument was from the first station (see § 326), and establishes a turning point by holding the rod upon some fixed point or upon a stake driven for the purpose. An iron pin 1 inch in diameter at the top and 6 or 8 inches long, tapering to a point, makes a good turning point; or a thin triangular plate of iron, having sides about 8 to 12 inches long with about 2 inches of the corners turned down, and having a small button attached to the middle of its upper surface, makes an excellent turning point for hard ground where there is no grass, weeds, etc. Whether the pin or plate is employed, a suitable handle should be attached to it.

A reading is taken upon the turning point, and the result is recorded as a "— sight." The instrument is then moved beyond the turning point, and set up again, a second reading is taken upon the turning point, and the result is recorded as a "+ sight." Thus the work proceeds, the rod and the instrument alternately being ahead.

295. The Record. The record for differential leveling is shown in Table V, page 256. The form is so simple that it does not need much explanation. The sum of the back-sights minus the sum of the fore-sights gives the algebraic difference of elevation. The length of sight, which may be obtained by the principle of the stadia (pp. 173–208), is not required except to check the equality

of the length of the back-sight and the fore-sight (§ 326), or to determine the distance run.

TABLE V.

FORM OF RECORD FOR DIFFERENTIAL LEVELING.

Sta.	Length of Sight.	+ Sights.	− Sights.	Elevation of Benches.	Remarks.
B_1		3.214		0.000	Corner of water-table at N.-W. corner of court-house.
1		3.617	2.431		
2		5.231	4.738		
B_2			3.214	+ 1.879	Marked point on W. abutment of White River bridge.
		+ 12.062	− 10.383		
		− 10.383			
		+ 1.879			
3		4.215	3.217		
4		2.831	5.272		
B_3			3.821	− 1.506	
					− 3.385
		8.925	− 12.310		+ 1.879
			+ 8.925		
					− 1.506
			− 3.385		

296. PROFILE LEVELING. The object of this form of leveling is to obtain a profile of the surface along an established line. Hence it is necessary to determine both the horizontal and the vertical distances from the initial point to the points at which rod readings are taken. When the line is established, stakes are driven at regular intervals, usually 100 feet apart. The stakes are numbered in order,—the first being 0, so that the number of any stake will indicate its distance from the beginning. The leveler is to obtain the elevation of the ground at each of these stakes and at as many intermediate points as may be necessary to enable him to draw a fairly accurate profile. The 100-foot stakes are called *stations*, and the points between the 100-foot stakes are

called *pluses*. Thus the tenth 100-foot stake is called station 10, or simply 10; and an intermediate point 20 feet beyond station 10 is referred to as station 10 + 20.

297. Field Routine. The field work is very much like that for differential leveling (see § 293–94). The rod is first held on the ground at the foot of the first stake, *i.e.*, at station 0, or on a bench near the beginning of the line. The number or designation of the point upon which the rod is held is entered in the first column of the record (see Table VI, page 261); and a brief description of the location of the point is recorded in the last column. The reading itself, which obviously is a + sight, is recorded in the second column.

A rod reading is then taken at each of the stations in order, the readings being recorded in the — S. column under the sub-head intermediates. The work thus proceeds until a point is reached which is about as far from the instrument as the point of beginning. If there is no firm point upon which to place the rod, the rodman drives a level-peg (§ 294, fifth paragraph), which he carries for the purpose, until the top of it is nearly level with the surface of the ground, and holds the rod upon it. This is a turning point, and the reading is recorded in the — *S.* column under the sub-head *F. S.* If the turning point is a regular station, it needs no other designation in the station column; if it is not a regular station, but is in the line, then it can be designated as a plus; and if it is not in the line, it is designated as peg or *T. P.*

The instrument is next moved forward, and a sight is taken upon the turning point that the height of the new line of sight may be determined with reference to the first station. The work then proceeds from the turning point as from the beginning. The first sight taken each time after setting up the instrument is a back-sight, or + sight; the last sight taken before re-

moving the instrument is a fore-sight, or — sight; and all others are intermediate sights, or simply intermediates.

The back-sights and fore-sights require the greatest care, since any error in them affects all subsequent work, while an error in an intermediate affects only the height of that single point. When the rod is set upon the ground, as is usually the case, and a target rod (§ 264) is used, it is sufficient to read on turning points to hundredths of a foot and on intermediates to the nearest tenth; although frequently, and generally improperly, the rod is read on turning points to thousandths and on intermediates at least to hundredths. With the Philadelphia rod (§ 267), it is rarely necessary to use the target except on turning points.

The rod-man is to place the rod at the foot of each stake, and at any point in the line where there is any considerable change of elevation. He is to determine the position of such points by stepping from the last station, and this distance is recorded in the first column of the record (see Table VI, page 261). For example, a reading having been taken on a point 50 feet from station 8, the record is made as in Table VI,—see the line under the record for station 8.

298. The Record. A great variety of forms of recording the notes have been proposed. The form shown in Table VI is frequently used, but a slight modification of it (see § 300, second paragraph) is by far the most common in this country. The common form and the forms shown in Tables VI and VII, pages 261 and 263, are examples of the *method by height of instrument.* The form shown in Table VIII, page 264, is an example of the *method by differences.* The last is not much used in this country, but apparently it is the favorite with British engineers.

299. *Modified Method by Height of Instrument.* The first, second, fourth, and fifth columns of Table VI, page 261, have already been explained (§ 297). Since the most convenient method of expressing the relative heights of a series of points is to refer them to some common datum, and since it is immaterial where this reference plane is chosen; therefore, if the elevation of the first stake—station 0 or B_1—has not been determined by previous operations, it is customary to assume it to be 10, 100, or 1,000 feet; that is to say, the datum is assumed at this distance below the first station. The elevation of the first station is assumed at some number supposed to be greater than any depression on the line to be leveled, so as to obviate + and – signs in the column indicating elevations. The assumed elevation of the first station is written in the first line of the elevations column,—see Table VI. The + *S.* on the first bench indicates that the line of sight was 1.763 feet above B_1; and this added to the elevation of B_1, gives the elevation of the plane of sight, or, briefly, the height of instrument, which is recorded in the third column of the table. The readings on stations 0, 1, 2, and 3 in-indicate the respective distances of these points below the line of sight; and these quantities subtracted from the height of instrument give their elevations. The instrument is then moved, and a sight is made back to station 3. The *B. S.* on station 3 added to the elevation of that station gives the new height of instrument, from which the succeeding minus sights are subtracted as before. The table and Fig. 77, page 260, will now mutually explain themselves.

300. After one page of the book has been filled, the computations should be checked. When correct, the difference of the sums of the back-sights and fore-sights on the page will be equal to the difference between the first and last elevation on the page. In this connection,

the word "**elevation**" is to be taken as applying to the elevation of a station or the height of the instrument. The two elevations to be used in checking can always be determined easily by a moment's consideration of the nature of the quantities.

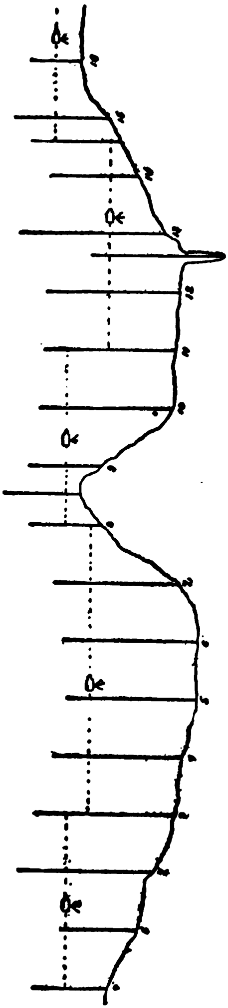

FIG. 77.—PROFILE LEVELING.

It is to facilitate this method of checking that the minus sights on turning points and on intermediates should be kept in separate columns; but as a rule, in ordinary practice, all the minus sights are recorded in a single column. With the exception of the separation of the minus sights, the form shown in Table VI is the one commonly employed in this country. Books containing rulings suitable for either of these forms are regularly sold by dealers in engineering stationery, the remarks column occupying the right-hand page, and the remainder of the table the left-hand page.

The computations of the elevations of the turning points and bench marks should be checked by the rod-man also, who should keep the notes of these points for this purpose. The elevations of the turning points and benches ought to be computed and checked when taken. The rod-man has ample time to do this while the instrument is being moved forward, and the leveler can, by watching his opportunity, compute his part

TABLE VI.

Form of Record for Profile Leveling.

Modified Method by Height of Instrument.

Sta.	+S. or B. S.	H. of I.	−S. F. S.	−S. Int's.	Elevations.	Remarks.
B_1	1.763	101.763			100.000	B_1 highest point of bowlder, 15 ft. to R. of Sta. 0+10 ft.
0				2.52	99.2	
1				5.46	96.3	
2				7.83	94.0	
3	1.931	93.967	9.727		92.036	
4				5.00	88.9	
5				7.25	86.7	
6				8.32	85.6	
7				7.54	86.5	
8	2.173	95.345	0.795		93.172	
+50				0.10	95.2	Highest point of Yankee Ridge.
9				3.72	91.6	
10				11.01	84.3	
B_2				7.321	88.024	Nail in root of lone cottonwood tree, 50 ft. right of line.
11	3.471	89.603	9.213		86.132	
12				3.40	86.2	
+65				10.2	79.4	Bottom of Bone Yard Branch, 10 ft. wide, water 3 ft. deep.
13				3.00	86.6	
14				1:72	87.9	
peg	5.384	94.156	0.831		88.772	
15				4.11	90.0	
16				1.2	93.0	
	14.722		20.566			100.000
			14.722			94.156
			5.844			5.844

without delaying the work. The elevations of the intermediates may be filled in afterward.

Finally, notice that the above tests check only the computations, and in no way prove the accuracy of the field work.

301. *Improved Method by Height of Instrument.* Some engineers object to the form of record shown in Table VI, page 261, (1) because the station and elevation columns are too widely separated for convenience in platting the profile and in referring to the notes, and (2) because the station column is too far from the remarks. For a form which meets these objections see Table VII, page 263, a form occasionally used in railroad surveying. The notes are the same as in Table VI,—the order of the columns only being changed,—and therefore need no explanation. One of the merits of this form is that wherever it is necessary to combine two numbers by addition or subtraction, they are found in adjacent columns.

302. *Form of Record by Differences.* The following objections are sometimes urged against the method of keeping level notes by the height of instrument (§ 299–301): (1) The elevations of the intermediates are not checked at all; (2) one of the checks on the benches and turning points requires the rod-man to keep a set of notes, which he can not well do with a self-reading rod; and (3) the notes should contain the data necessary to check them thoroughly at any time. The form shown in Table VIII, page 264, meets these objections, and may be used with either a target or a self-reading rod, and gives a check upon every point and a double check upon turning points.

The first four columns are copied from Table VI, page 261, and need no further explanation. The quantities in the column headed "Differ." are the differences between the readings on successive stations. If a read-

TABLE VII.

FORM OF RECORD FOR PROFILE LEVELING.

Improved Method by Height of Instrument.

B. S.	Elevations of T. P. and B.	F. S.	H. of I.	– S.	Elevation of Surface.	Sta.	Remarks.
1.763	100.000		101.763			B_1	Highest point of bowlder, 15 ft. to
				2.52	99.2	0	R. of Sta. 0 + 10.
				5.46	96.3	1	
				7.83	94 0	2	
1.931	92.036	9.727	93.967			3	
				5.00	88.9	4	
				7.25	86.7	5	
				8.32	85.6	6	
				7.54	86.5	7	
2.173	93.172	0.795	95.345			8	
				0.10	95.2	+ 50	Highest point of Yankee Ridge.
				3.72	91.6	9	
				11.01	84.3	10	
	88.024			7.321	88.0	B_2	Nail in root of lone cottonwood
3.471	86.132	9.213	89.603			11	tree, 50 ft. to R. of line.
				3.40	86.2	12	
				10.20	79.4	+ 65	Bottom of Bone Yard Branch, 10 ft.
				3.00	86.6	13	wide, water 3 ft. deep.
				1.72	87.9	14	
5.384	88.772	0.831	94.156			peg	
				4.11	90.0	15	94.156
				1.20	93.0	16	5.844
14.722		20.566					[illegible]00.000
		14.722					
		5.844					

ing is subtracted from a succeeding one, the difference is minus; and if from a preceding one, the difference is

TABLE VIII.

FORM OF RECORD FOR PROFILE LEVELING.

Method by Differences.

Sta.	+ S. or B. S.	− S. Int's.	− S. F. S.	Differ.	Eleva.	Check.	Remarks.
B_1	1.76				100.00	− 9.73	B_1 highest point of bowlder, 15 ft. to R. of Sta. 0 + 10 ft.
0		2.52		− 0.76	99.24	+ 1.76	
1		5.46		− 2.94	96.30	− 7.97	
2		7.83		− 2.37	93.93	100.00	
3	1.93		9.73	− 1.90	92.03	92.03	
4		5.00		− 3.07	88.96	+ 1.93	
5		7.25		− 2.25	86.71	− 0.79	
6		8.32		− 1.07	85.64	+ 1.14	
7		7.54.		+ 0.78	86.42	92.03	
8	2.17		0.79	+ 6.75	93.17	93.17	
+50		0.10		+ 2.07	95.24	− 9.21	Summit of Yankee Ridge.
9		3.72		− 3.62	91.62	+ 2.17	
10		11.01		− 7.29	84.33	7.04	
B_2		7.32		+ 3.69	88.02	93.17	Nail on root of lone cottonwood tree, 50 ft. to R. of line.
11	3.47		9.21	− 1.89	86.13	86.13	
12		3.40		+ 0.07	86.20	+ 3.47	
+65		10.20		− 6.80	79.40	− 0.83	Bottom of Bone Yard Branch, 10 ft. wide, water 3 ft. deep.
13		3.00		+ 7.20	86.60	+ 2.64	
14		1.72		+ 1.28	87.88	86.13	
peg	5.38		0.83	+ 0.89	88.77	88.77	
15		4.11	(4.11)	+ 1.27	90.04		
					9.96		
	14.71		− 24.67		100.00		
			+ 14.71				
			− 9.96				

plus. The elevations are determined by applying these differences, each with its proper sign, to the elevation of the preceding station.

The work is checked, as shown in the column headed "Check," by taking the difference between the *B. S.* and *F. S.* for each setting of the instrument, and applying it to the elevation of the preceding turning point, which should give the elevation of the succeeding turning point. Checking the elevation of the turning point checks the elevation of the preceding intermediate points. Each page may be checked by adding the *B. S.* and *F. S.*, and applying their difference to the last elevation, in essentially the same way as in the other forms. In Table VIII the elevation of station 15 was regarded as the "last elevation," and consequently the reading on that station was considered as the last *F. S.*, which it would be if the work stopped at station 15.

Notice that by this method each point is checked once and the turning points twice. Of course this method of keeping the notes requires more computing than the others, but it also more thoroughly checks the work. If rapid work is required, the quantities in the "difference" and "elevation" columns need not be worked out in the field, in which case the turning points will be checked only once.

303. Drawing the Profile. A profile is a vertical section of the line leveled, showing the relative heights and distances of the various points at which levels were taken. It is really a graphical representation of the station and elevation columns of the level notes, the first being the horizontal and the latter the vertical co-ordinates. The ground may be assumed to slope uniformly between the points at which levels were taken, for they were taken not only at the regular stations, but also at every considerable change of elevation; consequently, a line joining the points obtained by plotting the level notes, represents in detail the rise and fall of the line, as seen in a side view.

The profile may be made by drawing a horizontal

line to represent the datum, and laying off along it, to any convenient scale, the distances or stations from the first column of the field notes. The elevations corresponding to these points are next to be laid off at right angles to the datum line and above it. As the rise and fall of a line is always very small in proportion to its length, it is usual to make the vertical scale much greater than the horizontal. By thus employing two different scales, the irregularities of the surface are made more apparent to the eye, and also any subsequent use of the profile is rendered much easier and more accurate.

304. In practice, engraved profile paper is generally used, which is ruled in rectangles, to which any arbitrary values may be assigned. Three styles of profile paper are regularly sold by stationers. They are distinguished as A, B, and C, according to the scale of the ruling. The first has four horizontal and twenty vertical spaces to the inch ; the second, four horizontal and thirty vertical; and the third, five horizontal and twenty-five vertical. The paper is in rolls 10 yards long and 20 inches wide, and may be had printed on either opaque or translucent paper.

It is customary to make one space horizontally equal to one station, and one space vertically equal to one foot. In using engraved profile paper it is not necessary to plot the datum, but only to select a horizontal line near the middle of the paper and mark it, say, 100 feet, and number the other lines accordingly. The vertical lines being numbered to correspond with the stations, the points are easily and speedily plotted. A fine line drawn exactly through the plotted points completes the profile.

If, owing to the rise or fall of the ground, the surface line runs off the paper at the top or bottom, it is only necessary to change the numbering of the horizontal

lines and commence again at the other edge. The beginner is liable to plot the readings on benches, turning points, and plus stations as though they were regular stations. The readings on benches and turning points are not plotted, except when they are stations as well. The plus stations are plotted in their proper relative position, the distance on the profile being estimated.

305. PRECISE LEVELING. Precise, or geodesic, leveling is differential leveling (§ 293) performed with the utmost care. Leveling of the greatest accuracy possible is required in surveys to determine the figure of the earth, the relative elevations of the surfaces of the great lakes, the relative elevation of the water on opposite sides of an isthmus, the fall of the great rivers, etc.

306. Methods. There are two principal methods in leveling, according to the sequence of the instrument and rod, which may be called *single* and *double* leveling. Each may be performed with one rod or with two, which gives rise to four methods, according to the sequence of the instrument and the rod. There are also two methods of duplicating the work: viz., duplicate leveling once over the line, and duplicate leveling twice over the line. In all, then, there are six methods of leveling. The first five are represented in Fig. 78, and the sixth is simply the first method applied a second time.

In Fig. 78, I_1, I_2, I_3, etc., indicate successive positions of the instrument; A_1, A_2, etc., successive positions of one rod; B_1, B_2, etc., successive positions of the other rod; and *I*, *II*, *III*, etc., the several methods of leveling.

I, single leveling with one rod, is the method employed in ordinary leveling (§ 293 and § 296).

II, single leveling with two rods, is more accurate and also more rapid than leveling with a single rod. By the use of two rods the back-sight and the fore-sight

may be taken in quick succession, which saves time. Since less time intervenes between the sights, there is less liability of change in the plane of sight; and hence, for this reason also, this method is more accurate than

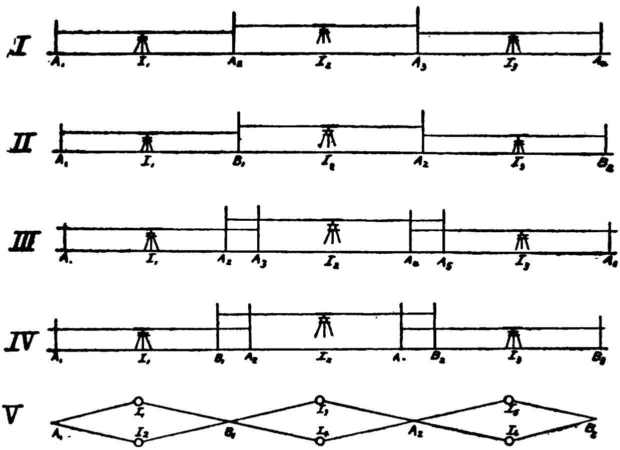

FIG. 78.—METHODS OF LEVELING.

the preceding one. This is the method used on the U. S. Lake Survey * and on the Mississippi River Survey.†

III, *double leveling with one rod*, affords a perfect check against errors of adjustment and observation, since the difference of the rod readings for the two fore-sights should be the same as the difference for the two back-sights following.

IV, *double leveling with two rods*, combines all the advantages of the second and third methods. The numerals adjacent to the rods show the order in which

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* Chief Engineer's Report, U. S. A., for 1880, pp. 2366 and 2429.

† Mississippi River Commission Report for 1881, p. 50.

they are sighted upon. This is the method employed on the U. S. Coast and Geodetic Survey.* On level ground or where the slope does not interfere, the distances A_1A_2, B_1A_2, etc., are about 220 meters (720 feet), and the distances B_1A_2, A_2B_2, etc., are about 20 meters (66 feet).

V, duplicate leveling—once over the line,—is simply duplicating the work without the labor of going twice over the line. Two rods are used, placed as in the figure. After having observed upon A_1 and B_1 the instrument is pulled up and re-set a little to one side and the two rods are sighted upon again. This method duplicates the work as far as instrumental errors are concerned, but is not as perfect a check as either the third or the fourth method.

A sixth method, *duplicate leveling—twice over the line,*—consists in an independent re-leveling of the line. The most reliable results are obtained by repeating the work in a direction opposite to that in which it was first done (§ 315).

307. Field Routine. After having planted the tripod firmly and leveled the instrument, read both ends of the bubble—estimating the fraction of a division. If there is a milled-head screw under one end of the telescope, the bubble can easily be brought to the middle each time; but if there is not, it is better to bring it nearly to the middle and apply a correction. It is not enough to read only one end, since the bubble is liable to change its length with a change of position or of temperature.

Next read the position of the three wires on the rod; and then read the bubble again for a check, and also to detect any change. Reverse the level end for end, and turn the telescope 180° about its optical axis, and repeat

* Report for 1879, p. 206.

the operations as above. The first reversal eliminates any inequality in the lengths of legs of the striding level; and the second eliminates any error of collimation. The mean of the several readings must be corrected for the difference in position of the bubble, and for inequality of the collars (§ 290).

Instead of reversing the level and telescope at the same time, the observations are sometimes made as follows: read upon the rod, reverse the level, and read again; reverse the telescope and read a third time; then reverse the level and make a fourth reading. The first method is the better.

308. On the Coast Survey the method of observing differs slightly from that described above.* Errors of level and of collimation are eliminated by reversing both the bubble and the telescope on each back-sight and fore-sight. Each observation is of a single wire on a target. The target is set but once for each station, the differential quantities being read by the micrometer under the eye end of the telescope. This seems not to be as good a method as the above; there are two objections to it, aside from the time and labor required to set the target. First, there is no sufficient check against errors in reading the positions of the target. Second, the micrometer is read for a central position of the bubble, the telescope is then moved to bisect the target, and the screw is read again; therefore there is no check on the stability of the instrument.

309. The Record. It is hardly wise to attempt, in this volume, an explanation of the method of recording the notes and making the computations. For a full explanation of the method employed on the U. S. Coast and Geodetic Survey, see the annual report of that Survey

* U. S. Coast and Geodetic Survey Report for 1879, pp. 206 and 207.

for 1879, pp. 210 and 211, or report for 1880, p. 139. For a full explanation of the form recommended by the U. S. Lake Survey, see annual report of the Chief of Engineers, U. S. A., for 1880, Part III, p. 2431.

310. Sources of Error. Probably in no other kind of surveying, with the possible exception of chaining, is it as important to distinguish between compensating and cumulative errors (§ 18) as in leveling. In general a clear comprehension of all the sources of error, their amounts, and the means of avoiding them, will be of great service in indicating the care necessary to secure a given degree of accuracy—and particularly is this true in leveling.

For convenience in discussing them we will classify errors of leveling as (1) instrumental errors, (2) rod errors, (3) errors of observation, (4) personal errors, (5) errors in recording and computing, and (6) errors of curvature and refraction.*

311. Instrumental Errors. The principal instrumental error is that due to the line of sight's not being parallel to the level. This may be caused either by imperfect adjustment, or by unequal size of rings, or by both. If the telescope slide is not straight, or does not fit closely,

* Owing to the unequal density of the earth's crust, the plumb-line does not always point to the center of the earth; and this gives rise to another source of error, *i.e.*, the local deflection of the plumb-line. This source of error has been suggested as the explanation of the discrepancy in the elevation above sea-level of a point at St. Louis, Mo., as determined by two lines of precise levels—one run from mean-tide of the Atlantic Ocean near New York City, and the other from mean-tide of the Gulf of Mexico near Mobile, Ala. This source of error is too complicated for discussion here. For a brief reference to this subject, see U. S. Coast and Geodetic Survey Report for 1882, p. 517.

If a line of levels were run fron mean-tide at the equator to mean-tide at the pole, what would the difference of elevation be? If the difference of elevation were determined with a mercurial barometer, what would it be? If with an aneroid barometer?

or does not move in the axis of the rings, it may produce an error. Of course the instrument must be focused so as to eliminate parallax.

All of the preceding errors are compensating, and the elevations of turning points may be made entirely independent of them, whatever their value, by always placing the instrument midway between the turning points.

312. Rod Errors. The principal rod error is caused by not holding the rod vertical. It is greater for a large rod reading than for a small one. It is compensating on turning points, and may be entirely eliminated by attaching a level or short plumb, or by waving the rod (§ 294, third paragraph). In waving the rod care must be taken that the face is not lifted by the rod's resting on its back edge when revolved backwards. To obviate this source of error and save the time consumed in waving the rod, a "corner target" is sometimes used. This consists of an ordinary target (Fig. 69 or Fig. 71, page 231), whose right and left halves are in planes at right angles to each other. The corner of the rod and of the target face the instrument, and if the rod is not vertical the central portion of the horizontal line of the target will be either above or below the horizontal cross hair. Another device for accomplishing the same purpose is a target similar to Fig. 69 or Fig. 71, in which the right and left thirds are, say, 2 inches behind the plane of the middle third. If the rod is not vertical, the horizontal line on the middle third of the target will be above or below that on the side portions. For several reasons neither of these devices is as rapid or as accurate as a short plumb-line or level—either a disk level, or two level vials at right angles to each other—attached to the rod.

With telescoping target rods, when extended, the slipping of the upper piece after the target has been pronounced correct and before the vernier has been

read, is a source of error. The target itself may slip, but this is not so probable, because of its inferior weight.

Another source of error is the settling of the turning point, due, in coarse or sandy soil, to its own weight, or to the impact of setting the rod upon it. The resulting error is cumulative. The remedy in the first case is to use a long peg, or to rest the rod upon a triangular plate having the corners turned down slightly (§ 294, fifth paragraph). Whatever the turning point, the rod should never be dropped upon it.

Finally, another small rod error is the error in the graduated length. This affects only the total difference of elevation between the two points. With the numerous home-made self-reading rods now in use, this is a much more important source of error than with the rods made by professional instrument makers.

"An important source of error in spirit leveling, and one very commonly overlooked, is the change in the length of the leveling rod from variations of temperature. It is quite possible that errors from this source may largely exceed the errors arising from the leveling itself."*

313. Errors of Observations. The principal error of observation is in reading the position of the bubble. Even if the bubble is kept in the middle, it is nevertheless read. Every leveler should know the error on the rod corresponding to a given difference of reading of the bubble, since he then knows how accurately he must read the bubble for the particular accuracy aimed at.

A small error arises from the fact that the adhesion of the liquid to the sides of the glass tube prevents the bubble from coming precisely to its true point of equilibrium (see the third paragraph of § 257). Even though it may finally arrive at the true point, it is liable to be

* Wright's Adjustments of Observations, p. 372.

read before it has stopped moving. The bubble should be re-read after the target is nearly adjusted, or, with a self-reading rod, after the reading has been taken and before the rod-man is signaled to move on.

Another small error is due to the effect of the sun in raising one end of the telescope by the unequal expansion of the different parts of the instrument. In ordinary leveling operations, the bubble is first brought to the middle and then the target is sighted in, leaving an interval for the sun to act. The error is greatest in working toward or from the sun. Ordinarily it is cumulative; for on the back-sight one wye is expanded, which elevates the line of sight, while on the fore-sight the other wye is expanded, which depresses the line of sight, the two errors affecting the difference of elevation in the same way. The error on the fore-sight is farther increased by the cooling of the wye which was expanded during the back-sight. The error due to the sun is always small, and can be nearly eliminated by noticing the position of the bubble *after* setting the target; and can be still farther reduced by shading the instrument, although "the Indian Geodetic Survey proved conclusively that the error was appreciable, even when the instrument was shaded."

Such a seemingly trivial thing as the unequal heating of the bubble tube produces an appreciable effect. The bubble always moves toward the warmer portion of the tube, owing to a change in adhesion of the fluid to the glass tube; and therefore, in accurate work, the leveling instrument should be protected from the direct rays of the sun, and the bubble tube should never be touched with the finger nor be breathed upon.

If the observer is compelled to change his position after reading the level and before sighting the telescope, his movement about the instrument may cause a change in the inclination of the telescope. "In some trials in

France, the inclination from this cause (one leg being in the line of sight) varied from two to one hundred seconds of arc according as one leg rested on the pavement or on vegetable soil." * This error is a minimum when two legs are placed in a line parallel to the route and the third at right angles to it. Sometimes, to eliminate this error, one man reads the bubble while a second sights the telescope; but this is objectionable, since it requires two skillful men and also divides the responsibility. It is better to provide the instrument with a mirror or a prism so that the bubble may be read from the position from which the telescope is sighted; for when the bubble is almost constantly in motion, the same man should see it and the cross hairs at the same time.

It has been found that appreciable errors are caused by the settling of the instrument on its vertical axis, and by the settling of the tripod legs into the ground. In spongy or clayey soil the tripod legs are sometimes gradually lifted up. These errors, though small in themselves, are more important than is generally supposed, inasmuch as they are cumulative (§ 18). They can be eliminated by re-running the line in the opposite direction.

If the reading is made on either side of the vertical hair, there is a possibility of error, owing to the horizontal hair's not being horizontal; and with wye levels not provided with a means of preventing the rotation of the telescope in the wyes, this possibility becomes a probability. Any device which insures the horizontality of the horizontal hair increases the rapidity and accuracy of the work. This error is compensating.

The inaccuracy of telling when the hair exactly covers the center of the target is a source of slight error,

* Wilfred Airy, in Proc. Inst. of C.E., Vol. 78, p. 458.

but not so slight as many think. In setting a quadrant level target, the instrument having a telescope magnifying thirty-five times and a bubble tube with a radius of 145 feet, the average probable error (§ 2 of Appendix III) for a class of fifteen was 1.4 thousandths of a foot at 100 feet and 2.25 thousandths of a foot at 300 feet (see Tables I and II, Appendix III). In running a line of levels this error would be compensating. Owing to this source of error, the difference in accuracy between a target rod and a self-reading rod is not so great as their difference in precision.

With a target rod, errors of one foot, one tenth, etc., are not uncommon. In double and duplicate leveling (§ 306) these errors are easily discovered and corrected. In ordinary leveling (§§ 293 and 296) they may be eliminated by having the rod-man and instrument-man read the rod independently and afterwards compare notes. In single differential leveling with one rod both men can read the rod without materially delaying the work, by the rod-man's reading the rod and recording it in a book carried for that purpose, and carrying the rod to the leveler if it be a back-sight or waiting for the coming of the leveler if it be a fore-sight, when the observer also reads the rod and records it in his book, *after* which the two records are compared. But in profile leveling this can not be done; and the only way to check the work is to duplicate it. With a self-reading rod the liability of this class of errors is somewhat reduced by reading three hairs.

314. Personal Errors. The errors previously described are liable to occur with any observer. They are due chiefly to the instruments and to the nature of the work, and would probably not materially differ for equally skilled observers. We come now to a class of errors which depend mainly upon inaccuracies peculiar to the individual. One observer may read a target higher or

lower than another of equal skill; or in reading the position of the bubble he may have peculiar views as to what constitutes the end of the bubble; or he may habitually read the bubble so as to get a distorted view of it through the glass tube. With skillful observers, all such errors are quite small and generally cancel themselves. In fact, the errors here classed as personal are possible rather than demonstrated as actually occurring; and yet there is nothing more certain than that in any series of accurate observations there is a difference between the results obtained by different individuals. This difference is known as the personal equation. In long lines of accurate leveling, it has been found that each man's way of performing the several operations has an appreciable effect upon the final result.

315. It is a curious fact, but one abundantly verified, that when lines are duplicated in opposite directions, the discrepancies tend to one sign and increase with the distance. This subject has been much discussed, and various explanations of the fact have been offered; as settling of instrument, settling of turning point, disleveling effect of the sun, unequal illumination of the target on back-sights and fore-sights, unequal illumination of the two ends of the bubble, effect of refraction in reading the bubble, the change of the position of the observer to read the bubble (see § 313, fifth paragraph), and personal bias in reading the target or bubble; but none of these reasons are entirely satisfactory. These discrepancies vary with different observers; are not even constant for the same observer; are nearly proportional to the distance; and seem to be independent of the nature of the ground, the direction in which the work is done, the season, or the manner of supporting the rod.*

* For a valuable discussion of this source of error, together with numerical results derived from actual work, see Report of the Mississippi River Commission for 1883, pp. 141–62; or Report of Chief of Engineers U. S. A., for 1884, pp. 2547–68.

The effect of this class of errors may be eliminated by each observer's duplicating his work in the opposite direction under as nearly the same conditions as possible. The accuracy is increased by leveling alternate sections in opposite directions, as is done in India; and it may be still further increased by reading the back-sight first each alternate time the instrument is set up.

316. Errors of Recording and Computing. Such blunders as recording the fore-sight in the back-sight column and, *vice versa*, the back-sight in the fore-sight column, although the result of gross carelessness, do nevertheless occur. To check against such errors, the rod-man and the leveler should read the rod independently, as explained in the last paragraph of § 313. In profile leveling, errors of this class are easily discovered, if the rod-man keeps the notes and works out the elevations of turning points and benches; and in any case, such errors are more likely to be discovered if the elevations are worked out immediately after the observations are taken.

Errors of computation, and the method of checking them, have already been discussed, incidentally, in §§ 299–302, which see.

317. Errors of Curvature and Refraction. The object of leveling is to find the distance that one point is above or below a level surface (§ 292) passing through some other point. The line of sight of a properly adjusted leveling instrument is a horizontal line, *i.e.*, a tangent to a level line. Therefore, if the two points sighted at are not equidistant from the instrument, the difference between the rod readings will not be the true difference of level, owing to the curvature of a level surface, *i.e.*, to "the curvature of the earth." The difference between a level and a horizontal line can be computed (§ 319); and hence if the length of sight is known, the effect of

curvature can be eliminated by applying a correction. The error is compensating, and may be entirely eliminated by setting the instrument midway between turning points.

318. Owing to the refraction of the atmosphere the beam of light from the target to the telescope is slightly concave downwards, and hence the line of sight is not a truly horizontal line. The difference between a horizontal line and the true line of sight can be computed for an average condition of the atmosphere (§ 319); and therefore if the length of sight is known, and if the air is in its normal condition, the effect of refraction can be eliminated by applying a correction.

But the atmosphere is not always in its normal condition; and hence if there is abnormal refraction or a change of refraction between sights, there may be residual errors of refraction, even though the correction for mean refraction be applied. As refraction varies greatly with the nature of the surface over which the light passes and also with the distance from the surface, the effect of the refraction may vary considerably. The most serious effect of variable refraction is the tremulousness or "boiling" of the atmosphere caused by the innumerable small currents of air of different temperatures or densities which exist when the atmosphere and the earth differ much in temperature. The beam of light in proceeding from the target to the telescope passes alternately through denser and rarer media, each of which produces a slight refraction of the ray, thus causing the target to "dance" and making it difficult to determine just when it is properly bisected by the cross hair. When this condition exists the only remedy is to shorten the length of sight, or wait for better atmospheric conditions. The atmosphere is usually in the best condition for seeing just before sunrise and a

little while before sunset, although the refraction is then greater. A cloudy day is better than a clear one.

Ordinarily this error is compensating; and will usually be eliminated by setting the instrument midway between the turning points.

319. *Correction for Curvature and Refraction.* To compute the correction for *curvature of the earth*, let AD, Fig. 79, represents the horizontal line; AB the level line; and AE the radius of the earth. Then DB is the correction for curvature. By geometry $AD^2 = DB(2AE + DB)$. Dropping DB from the parenthesis, as it is very small in comparison with $2AE$, and representing the length of sight by k and the radius of curvature of the earth by ρ, $BD = \frac{k^2}{2\rho}$. If BD is expressed in feet and k in miles, the correction for curvature becomes

$$BD \text{ ft.} = 0.667k^2 \text{ miles.} \quad . \quad . \quad . \quad . \quad (1)$$

To compute the correction for *refraction*, notice that D', Fig. 79, is the true position of the target and D its apparent position. Hence DD' is the correction sought. The ratio of the refraction angle, DAD', to the angle at the center of the earth, AED, is known as the coefficient of refraction, which we will represent by m.

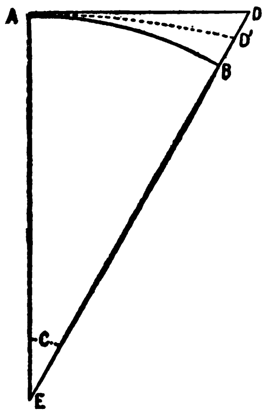

FIG. 79.

$$m\,AED = D'AD.$$

$$AED, \text{ in sec. of arc, } = \frac{k}{\rho \sin 1''}.$$

$$DD' = k \tan D'AD$$

$$= k\,D'AD'' \tan 1'' = \frac{m\,k^2}{\rho}.$$

The ordinary values of m vary between 0.06 and 0.08.

The value generally employed in computing the correction for refraction is 0.07. Then

$$D'D \text{ ft.} = 0.09k^2 \text{ miles.} \quad . \quad . \quad . \quad . \quad (2)$$

Combining equations (1) and (2), we have

$$BD' \text{ ft.} = 0.57k^2 \text{ miles.} \quad . \quad . \quad . \quad . \quad (3)$$

In general, the total correction for curvature and refraction, to be applied to the observed reading, is

$$BD' = BD - DD' = (1 - 2m)\frac{k^2}{2\rho} = 0.000,000,020,45k^2.$$

If $k = 220$ ft., $BD' = 0.001$ ft.; for $k = 300$ ft., $BD' = 0.002$ ft. Notice that the correction varies as the square of the length of sight. Numerous tables have been computed which give this correction directly for the different lengths of sight; or tables can be prepared which will give the difference of the correction with the difference of length of sight for an argument.

320. Limits of Precision. The probable error per unit of distance is generally adopted as a convenient measure of the precision reached. According to the theory of probabilities, the final error of a series of observations, affected only by accidental errors, will vary as the square root of the number of observations. In leveling, a method should be adopted which will eliminate all cumulative errors; and therefore, since only compensating errors remain, the final error of leveling a number of units of distance is assumed to vary as the square root of the distance.

This assumption would be true if only compensating errors remained uncorrected, and if the number of observations were strictly proportional to the distance leveled, *i.e.*, if the length of sight was constant and if

the inclination of the surface of the ground leveled over was the same (see Table IX, page 283). Since it is improbable that all cumulative errors will be entirely eliminated, that part of the final error which is due to cumulative errors will vary as the distance leveled. It has frequently been noticed that, considered individually, the errors of a number of short lines were well within the limits which were prescribed to vary as the square of the distance; yet when the sum for several lines were considered, the total discrepancy would exceed the limit. In other words, the error is not strictly proportional to the square root of the distance. One part of the error is proportional to the square root of the distance, and another portion varies nearly as the distance; hence, *the shorter the distance, the easier to attain a limit prescribed to vary as the square root of the distance.*

If the error was determined by duplicating the work in the same direction, and especially if at the same time, by methods III, IV, or V, Fig. 78, page 268, the difference will be the apparent error, and necessarily be too small. The result obtained by the adjustment of a net of lines by the method of least squares affords the best means of determining the degree of precision.

321. According to the Geodetic Association of Europe, levels of precision executed of late years in Europe show that the probable error of a line of levels of precision should never exceed 5 mm. $\sqrt{\text{distance in kilometers}}$ (0.0208 ft. $\sqrt{\text{miles}}$); that 3 mm. $\sqrt{\text{dist. in kilometers}}$ is tolerable, 2 mm. $\sqrt{\text{dist. in kilometers}}$ is a fair average, and 1 mm. $\sqrt{\text{dist. in kilometers}}$ is high precision.* The Coast Survey requires 5 mm. $\sqrt{2 \text{ dist. in kilometers}}$ (0.030 ft. $\sqrt{\text{miles}}$). The Mississippi River Commission's limit is 5 mm. $\sqrt{\text{dist. in kilometers}}$ (0.021 ft. $\sqrt{\text{miles}}$).

Of late years the Coast Survey's and Mississippi

* Report of U. S. Coast and Geodetic Survey for 1882, page 522.

River Survey's work are considerably within the limit of 2 mm. $\sqrt{\text{dist. in kilometers}}$. Table IX shows the degree of accuracy attained in precise leveling on the national surveys of Great Britain, India, and Switzerland.*

TABLE IX.

DATA SHOWING THE DEGREE OF ACCURACY ATTAINED IN PRECISE LEVELING, AND THE EFFECT OF DIFFERENT INCLINATIONS OF THE GROUND.

Reference No.	KIND OF GROUND.	AVERAGE DIFFERENCE PER MILE BETWEEN TWO OBSERVERS.		
		Great Britain.	India.	Switzerland.
		foot.	foot.	foot.
1	Nearly level, with very favorable weather	0.0230	0.0142	0.0125
2	Slightly undulating—gradients not exceeding 1 in 100	0.0238	0.0168	0.0148
3	Gradients entirely between 1 in 100 and 1 in 20	0.0379	0.0208	0.0183
4	Gradient entirely between 1 in 20 and 1 in 10	0.0566	0.0350	0.0308
5	Steep and rough ground—gradients frequently steeper than 1 in 10 . .	....	....	0.0416

To attain the preceding limits requires skillful observers, the best instruments, and plenty of time. Ordinarily there are not more than three or four hours of the day in which this class of work can be done; and as a general average not more than one or two miles can be run in a day (see § 323).

322. It is impossible to establish a limit for work less accurate than the best, since the conditions under which it may be done are too diverse. However, the difference in precision between ordinary leveling and precise leveling, is not as great proportionally as the difference in care and time given. A little increase in

* Wilfred Airy in Proc. Inst. of C. E., Vol. 44, page 181.

accuracy costs a very great increase of effort. Results of leveling of apparently greater accuracy than the foregoing are often given,* but an occasional accurate result —probably more largely due to good fortune than good management—gives no indication as to what results may be regularly expected.

Regularity of result and evenness of error is of more importance than occasionally a small disagreement. Naturally, it is usually the latter that is reported.

A line of "ordinary" levels was run on the bank of the Mississippi River, and checked upon the benches of the precise levels, with an average error of 0.034 feet $\sqrt{\text{dist. in miles}}$.† The conditions under which this work was done were about the same as those of a preliminary railroad survey, but it is probably more accurate than such surveys usually are.

In some of the branches of the A., T., & S. F. R. R., the instructions were to re-run the line whenever the difference between the levels on construction and location was 0.03 foot, between benches about 2,000 feet apart. This is equivalent to limiting the maximum admissible error to 0.048 feet $\sqrt{\text{dist. in miles}}$.

In the topographical survey of the city of St. Louis, Mo., "the limit of error allowed was 5 millimeters $\sqrt{\text{dist. in kilometers}}$ (0.0208 feet $\sqrt{\text{dist. in miles}}$). The average closure was 0.013 feet $\sqrt{\text{dist. in miles}}$. The probable error in the determination of a single mile of the work was 0.001 foot."‡

323. Speed. The amount of work that an observer

* For example, several text-books contain the statement: "A French leveler contracts to level the bench marks of a railroad survey to within 0.002 of a foot per mile." Compare this with § 321 and also with Tables I and II of Appendix III.

† Report of Mississippi River Commission for 1882, p. 2269; or Report of Chief of Engineers, U. S. A., for 1883, p. 2269.

‡ The Technograph, No. 5 (1890–91), p. 11.

should do in a day can not be stated definitely. It depends upon the accuracy required, the power and delicacy of the instrument, the method pursued, the ground, and very largely upon the atmospheric conditions. For the very best work, not more than three or four hours can be utilized, even in clear weather. The average daily run for several seasons on the Mississippi River, using a Kern level and method II (§ 306), was one and a half miles a day for the entire season, and two and a half miles for the days on which work was actually done.* On the U. S. Lake Survey, with the same instrument and method, the distance was about two miles a day for the days on which leveling was done.† On the Swiss levels of precision, 3 kilometers (1.8 miles) along railroads, and 2 kilometers (1.2 miles) along highways in the plains, was considered a fair day's work.‡

Professor J. B. Johnson,§ who has had large experience in levels of precision on the U. S. Lake Survey and on the Mississippi River, states that "with a wye level and a target rod, a single instrument should duplicate thirty miles per month, with no greater error than 0.05 feet $\sqrt{\text{dist. in miles}}$, or with a level of precision and speaking rod, do the same work with a limit of 0.02 feet $\sqrt{\text{dist. in miles}}$."

324. Practical Hints. For a number of points applicable in leveling, particularly to setting the tripod, see § 128.

If the instrument has once been leveled, and the bubble is found to have moved a little, bring it back with a slight pressure of the finger.

* Report of Chief of Engineers, U. S. A., for 1884, p. 2462.

† Professional Papers Corps of Engineers, U. S. A., No. 24—Primary Triangulation U. S. Lake Survey,—pp. 597-99.

‡ Proc. Inst. of C. E., Vol. 78, p. 456.

§ Jour. Association of Engineering Societies, Vol. 2, p. 160.

Instruments are usually provided with sun-shades to prevent trouble from the sun's shining into the telescope. If the metallic shade is not at hand, make one by rolling up a piece of paper and gumming, pinning or tying it together, or springing a rubber band around it. This is easier and better than holding the hat or note-book over the objective.

In ascending or descending a steep hill, it is desirable, for speed, that the line of sight should strike as near as possible to the bottom of the rod on the up-hill side, and to the top of the rod on the down-hill side. In selecting the position of the instrument to satisfy this condition, set the instrument up lightly, turn the telescope in the right direction, bring the bubble approximately to the middle by manipulating the tripod legs, and sight along the outside of the telescope. Even this rude observation will be valuable as showing whether the instrument should be moved up or down the hill, and it will save considerable time. With a little practice, the same observation may be made by drawing the tripod legs together and using them as a Jacob's staff, when the bubble can speedily be brought to its proper position by simply inclining the whole instrument.

If the up-hill rod is too near to be focused on, move either the rod or the instrument a little to one side. In short intermediate sights for which the telescope can not be focused, it is sufficient to sight by the bottom of the wyes or by the side of the telescope.

In closing at noon or night, be careful to set half-way between the last two turning points; and on resuming work, set near one of these points, and re-determine their difference of level. The same difference of level each time affords an excellent check upon the adjustments of the instrument (§ 284).

325. Length of Sight. The length of sight is limited by the power of the telescope, the atmospheric conditions, the accuracy desired, the time available, etc. Some errors increase directly, and others indirectly, as the number of sights taken in a given distance. It is generally assumed that *for the most accurate work* the rod should be at least 100 feet from the instrument and never more than 400 feet; and that *for ordinary work* the rod should be 300 or 400 feet away and never more than 500 or 600 feet.

On the U. S. Coast and Geodetic Survey the length of sight ranges from 50 to 150 meters (164 to 492 feet) according to the condition of ground and weather, the average being 110 meters (360 feet), the distance between the two rods on the same side of the instrument being 20 meters (see method IV, Fig. 78, p. 268). On the U. S. Lake Survey* the maximum length of sight was 100 meters (328 feet). On the Prussian Land Survey, the maximum sight has not exceeded 50 meters (164 feet), except for river crossings.† In the Swiss levels of precision, the length of sight on railroads with gradients under 1 in 100 was 100 meters (328 feet); on railroads with steeper gradients, from 50 to 100 meters (165 to 328 feet); on highways in the plains, from 30 to 60 meters (100 to 200 feet); and on mountain roads, from 10 to 25 meters (33 to 82 feet).‡

326. *Equal Back-sight and Fore-sight.* It is very desirable that at each setting of the instrument the lengths of the back-sight and the fore-sight should be equal; for, as has been seen, there are then a number of errors which cancel each other. *This is a very important point,*

* Professional Papers Corps of Engineers, U. S. A., No. 24—Primary Triangulation U. S. Lake Survey,—p. 598.

† Wright's Adjustment of Observations, p. 375.

‡ Proc. Inst. of C. E., Vol. 78, p. 456.

and should always be kept in mind. Leveling is the only kind of surveying wherein the instrumental errors may be thoroughly eliminated without duplicating the work. This is done by making the back-sights and fore-sights of equal length.

When stakes are set at regular intervals, there is no difficulty in determining the length of sights, and making them equal; and in other cases the distance can be determined by stepping. When extreme accuracy is desired, the rod-man approximates the distance by stepping, and the instrument-man measures it by the principle of the stadia (Chap. X). All levels should be provided with two extra horizontal cross hairs for this purpose.

In the precise leveling on the U. S. Lake Survey, and in surveys made under the direction of the Mississippi River Commission, as well as in the Swiss levels of precision, the difference between corresponding back-sight and fore-sight is not allowed to exceed 10 meters (33 feet).

In ascending or descending a hill, it is nearly impossible, and always very tedious, to make the back-sight and fore-sight equal. As the rod is about twice as high as the instrument, the down-hill sight will be about twice the length of the up-hill one. When the ground renders sights of unequal length unavoidable, keep notes of the distance; and, as soon as possible, take sights with corresponding inequalities in the contrary direction. When approaching a long incline make part of this compensation in advance.

327. Reciprocal Leveling. In crossing a river, it is absolutely necessary that the back-sight and fore-sight should differ considerably; and there are other somewhat similar cases in which they can not be made even approximately equal. In such instances the principles

of reciprocal leveling are applicable.* The method of procedure is as follows:

Establish a bench upon both sides of the river and determine the difference of level; move the instrument to the other side, and re-determine the difference of level. If the sights are taken in quick succession, the mean of the two results is the true difference of level. Simultaneous observations with two instruments would be still better.

The instructions on the U. S. Lake Survey were to (1) read upon a bench on the nearer shore with the telescope normal and also inverted, (2) read upon the bench on the farther shore five times with the telescope normal and five times with it inverted, and (3) read upon the nearer bench as before. The rod on the farther shore is provided with a target from 6 inches to 1 foot square, according to the distance. The line of sight should pass at least 10 or 12 feet above the water to eliminate abnormal refraction (§ 318). The observations must be corrected for curvature and refraction (§ 319). With a single Kern level (§ 261), this process has given for a river half a mile wide (the Ohio, at Cairo, Ill.) five results the mean of which had a probable error (App. III, § 2) of 0.5 mm. (0.002 ft. nearly).† Of course this must be regarded as an exceptionally accurate result, but it gives an idea of the practice in such matters. In another case, sixteen simultaneous observations on each side gave a probable error of 1 mm. (0.003 ft.) for a river crossing 600 meters (2,000 ft.) wide.

* The surface of water can not be assumed to be level except (1) where there is no current or wind, or (2) where the thread of the current is midway between the banks, and the line joining the two points of observation is perpendicular to the current, and there is no wind.

† Report of the Chief of Engineers, U. S. A., for 1880, Part III, p. 2432.

On the U. S. Coast and Geodetic Survey,* eleven sets of observations, consisting of twelve to sixteen pointings each, made at four different hours on three different days, across a river 2,200 feet wide, gave a probable error of 1.9 mm. for the mean of each set. In the above observations the line of sight was about 12 feet above the water; but an equal number of observations, made at the same time as the above, with the line of sight 5 feet above the surface, gave a result for the difference of level between the two bench marks on opposite sides of the river differing from the mean of the above observations by 23 mm. This shows the uncertainty of such observations, even though the results may not disagree among themselves.

328. If the line to be leveled passes over a stream with steep high banks, or over a narrow deep gorge or valley, establish a turning point on the farther side by reciprocal leveling. Then, to find the depth of the opening, level down the bank without much regard to equality in length of sights, or other refinements. This will usually be all that is necessary, and is much quicker done than leveling down one bank and up the other.

329. Bench Marks. These are permanent objects, natural or artificial, whose heights are determined and recorded for future reference. Any object not easily disturbed, and easily described and found, may be used as a bench mark, as, for example, the highest point of a bowlder, a nail in the root of a prominent tree, a stone door-sill, etc. A stake driven to the surface of the ground, with another projecting above the ground to mark its position, is frequently used in railroad and drainage surveying; but it is not very reliable at best, and is entirely unreliable after having stood over winter, owing to the "heaving of the frost." The precise loca-

* Annual report for 1879, pp. 212–13.

tion and description of every bench should be given very fully and definitely in the "remarks" column of the field notes. The description should be such that an entire stranger could find the bench by the aid of the notes alone. For convenience, the benches are numbered, the number, and the fact that it is a bench, being marked on or near it.

Bench marks should be established at frequent intervals along the line. They serve as points of beginning in case of accident,—as, for example, losing a turning point,—and are points on which to check in case the line is re-run. They are usually put in at each mile and half-mile from the beginning of the line; and they should also be put in at all places where they may be necessary or convenient—as, for instance, near where the line crosses a prominent road, on both sides of a river crossed, near the top or bottom of a high hill crossed, etc.

330. CARE. For remarks on the care of instruments, which are applicable to the level, see § 95 and § 148.

CHAPTER XII.

BAROMETERS.

332. THE difference of level of two places may be determined by ascertaining the difference in the height of the atmosphere above the places. This may be found in any of three ways; viz., (1) by determining how high a column of mercury, or other liquid, the column of air above it will balance, (2) by finding the pressure it will exert against an elastic box from which the air has been exhausted, or (3) by observing the temperature at which a liquid boils, *i.e.*, by observing the temperature at which the pressure of the atmosphere just balances the tension of the vapor. There are then three slightly different methods of barometric leveling according as the instrument used is a mercurial barometer, an aneroid barometer, or a thermo-barometer or boiling-point apparatus. As the thermo-barometer has been superseded by the aneroid, only the methods of leveling by the mercurial and the aneroid barometers will be considered here.

Barometric leveling is specially adapted to finding the difference of level between points at considerable horizontal or vertical distance apart. Under these conditions, it is the most expeditious, but the least accurate, of any of the methods of leveling. It is very valuable in making geographical surveys of large areas for determining the elevation of stations to be occupied by the topographer. It is also well suited to making a reconnoissance for a railroad or for a scheme of triangulation.

ART. 1. THE MERCURIAL BAROMETER.

333. CONSTRUCTION. There are two kinds of mercurial barometers, the cistern and the siphon. The former is the better and more reliable for hypsometrical purposes. The general form of the cistern barometer needs no description. Fig. 80, page 294, shows some of the details of Green's barometer, which is generally considered one of the best. The right-hand portion of Fig. 80 shows the cistern and the details at the lower end of the instrument, and the left-hand portion the vernier and scale at the upper end of the mercury column.

The cistern consists of a glass cylinder *F*, which allows the surface of the mercury to be seen, and a top plate *G*, through the neck of which the barometer tube *t* passes, and to which it is fastened by a piece of kid leather, making a strong but flexible joint. To this plate is attached also a small ivory point, *h*, the extremity of which marks the commencement or zero of the scale. The lower part, containing the mercury into which the end of the tube, *t*, is plunged, is formed of two parts, *i* and *j*, held together by four screws and two divided rings, *l* and *m*. To the lower piece, *j*, is fastened the flexible bag *N*, made of kid leather, furnished in the middle with a socket, *k*, which rests on the end of the adjusting screw *O*. These parts, with the glass cylinder, *F*, are clamped to the flange, *B*, by means of four large screws, *P*, and the ring, *R*. On the ring, *R* screws the cap, *S*, which covers the lower parts of the cistern and supports at the end the adjusting screw *O*. *G*, *i*, *j*, and *k*, are of boxwood ; the other parts of brass or German silver. The screw *O* serves to adjust the mercury to the ivory point, and also, by raising the bag so as to completely fill the cistern and tube with mercury, to put the instrument in condition for transportation.

The milled-head screw *D* moves, by means of a rack and pinion, the vernier *C*, so as to bring the horizontal

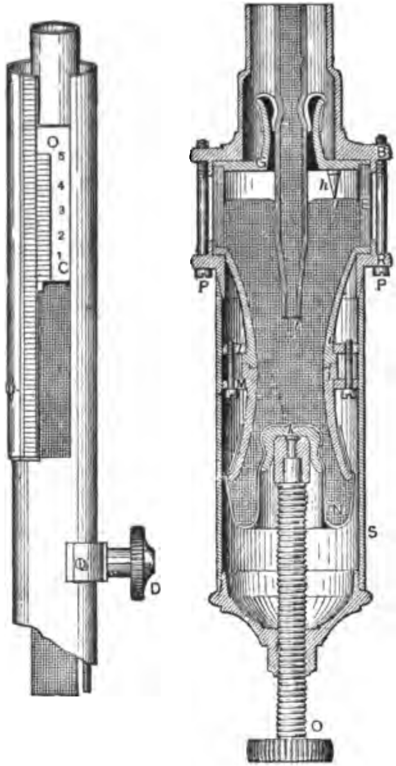

FIG. 80.—MERCURIAL BAROMETER.

line just below *C* level with the top of the mercury column.

334. Cleaning the Barometer. It frequently happens that the mercury in the cistern becomes so dirty that the ivory point, or its reflection in the mercury, can no longer be seen. This often occurs even though the barometer is in good condition in every other respect.

"The instrument can be taken apart and cleaned with safety and without changing in the slightest degree the zero of the instrument. Everything used in the operation must be clean and dry. Avoid blowing upon any of the parts, as the moisture from the breath is injurious.

"Turn up the adjusting screw at the bottom until the mercury entirely fills the tube, carefully invert, place the instrument firmly in an upright position, unscrew and take off the brass casing which encloses the wooden and leather parts of the cistern. Remove the screws, and lift off the upper wooden piece to which the bag is attached; the mercury will then be exposed. By inclining the instrument a little, a portion of the mercury in the cistern may be poured out into a clean vessel at hand to receive it, when the end of the tube will be exposed. This is to be closed by the gloved hand, when the instrument can be inverted, the cistern emptied, and the tube brought again to the upright position. Great care must be taken not to permit any mercury to pass out of the tube. The long screws which fasten the glass portion of the cistern to the other parts can then be taken off, the various parts wiped with a clean cloth or handkerchief, and restored to their former position.

"If the old mercury is merely dusty, or dimmed by the oxide, the cleaning may be effected by straining it through chamois leather, or through a funnel with a capillary hole at the end of a size to admit of the passage of but a small thread of the metal. Such a funnel is conveniently made of letter paper. The dust will adhere to the skin or paper, and the filtered mercury will present a clean and bright appearance. If chemically impure, it should be rejected, and fresh, clean mercury used. With such clean mercury the cistern should be filled as nearly full as possible, the wooden

portions put together and securely fastened by the screws and clamps, the brass casing screwed on, and the screw at its end screwed up. The instrument can then be inverted, hung up, and re-adjusted. The tube and its contents having been undisturbed, the instrument should read the same as before." *

With the instrument before the operator, these instructions are easily understood. In this case, as in using and caring for any instrument, a little care and a thoughtful inspection of the method of construction is worth more than any written description. If a little mercury has been lost during the operation, and there is none at hand to replace it, no serious harm has been done; but if much is lost, the open end of the tube may become exposed in inverting the instrument, in which case air may enter.

335. Filling the Barometer. It is no slight matter to properly fill a barometer. It can best be done by the manufacturer, who has all the facilities; but as it is sometimes necessary for the observer to re-fill his barometer, the following hints are given. Tubes require re-filling owing to the breakage of the glass or to the entrance of a bubble of air.

The mercury should be chemically pure and free from oxide, otherwise it will adhere to the glass and tarnish it. Moreover, if it is not pure, the height of the barometric column will not be correct. No mercury should be used except that which has been purified by distillation. For the best results, the mercury should be boiled in the tube to expel moisture and air; but this can not always be done, and fair results may be obtained without boiling.

In extended barometric operations in the field, a supply of extra tubes is carried, to be used in case a tube is

* On the Use of the Barometer on Surveys and Reconnoissances, Maj. R. S. Williamson, U. S. A., New York City, 1868, pp. 136 and 137.

broken. These tubes should be drawn out so as to be a little longer than they are required to be when fitted into the barometer. The open end should be cut off to such a length that it shall always be immersed and yet not interfere with the rise of the lower part of the cistern. When the instrument is finally put together, the cork in the upper end of the brass case should be adjusted so as to hold the closed end of the tube firmly.

336. By Boiling. "To fill a tube by boiling, an alcohol lamp is needed, although it can be done over a charcoal fire. The lamp being filled and put in order, begin to fill the tube by pouring in through the funnel as much warm mercury as will occupy about 5 inches; then, holding the tube with both hands above the mercury, heat it gently, and let the mercury boil from the surface downward to the end of the tube, and then back again, chasing all of the bubbles of air upward. A little practice will make this easy, the tube being held a little inclined from the horizontal, and constantly and rapidly revolved, always in the same direction, so that every portion of the metal may be heated gradually and uniformly. After this has been done, let the tube cool sufficiently to admit of its being held by the gloved hand, and then pour in enough warm mercury to occupy several inches more of the tube, which may now be held with both hands, one above and the other below the heated portion. After boiling this thoroughly free from air, repeat the same operation with more mercury added, until the tube is filled to the end. With care and practice the mercury may be boiled entirely free from air up to within an inch or less of the end of the tube. A tube filled in this way may have, in every respect, as perfect a vacuum as one prepared by a professional instrument maker." *

337. Without Boiling. The glass tube, which should

* Williamson's On the Barometer, p. 138.

be clean and dry, must have its open end ground straight and smooth, so that it can be closed air-tight with the finger, which should be covered with a piece of chamois or kid skin. Warm well both mercury and glass tube, and through a clean paper funnel with a very small hole (about $\frac{1}{60}$ of an inch) below, filter in the mercury to within one fourth of an inch of the top. Shut up the end and turn the tube horizontal, when the mercury will form a bubble which can be made to run from end to end by a change of inclination, and which will gather all the small air bubbles that adhered to the inside of the glass tube during filling. Let this bubble, which has grown somewhat larger, pass to the open end; then fill the tube completely with mercury, and shut it tightly. Next reverse the tube over a basin, when, by slightly relieving the pressure against the end, the weight of the column of mercury will force some out, forming a vacuum above, which ought not to exceed one half an inch. Closing up again tightly, let this vacuum bubble traverse the length of the tube on the several sides, that it may absorb those minute portions of air that were not drawn out at the first gathering, and which are now greatly expanded from removed atmospheric pressure. The complete absence of air is easily recognized by the sharp concussions with which the column beats against the sealed end, when the tube is held horizontally and slightly moved.

"A barometer filled without boiling will probably read lower by a few thousandths than if the tube had been boiled; but in a stationary barometer its error will probably not soon change, and carrying on horseback will be apt to improve it rather than otherwise, as it is then carried with the cistern uppermost and the bubbles will be jolted toward the open end. If possible it should be compared with a standard barometer." *

* Williamson's On the Barometer, p. 140.

338. Reading the Barometer. Read the attached thermometer first. It is more sensitive than the barometer, and is affected by the heat of the body, while the barometer is not so affected. The thermometer should be read as closely as possible, for a difference of 1° F. is equivalent to about 3 feet in altitude. Parallax should be carefully avoided in making this reading.

Then, by means of the adjusting screw at the lower end of the instrument (*o*, Fig. 80, page 294), bring the ivory point just to the mercury in the cistern. If there is a line of light visible between the point and mercury, the mercury in the cistern is too low; and if the point makes a depression, the mercury in the cistern is too high. If neither a line of light nor a depression can be seen, the adjustment has been correctly made. It is usually best to lower the screw till a distinct line of light can be seen, and then gradually raise it until the light disappears. When the mercury is bright, a shadow of the point can be seen, and if the shadow and the point itself form a continuous unbroken line, the screw at the bottom of the cistern has been properly adjusted. Before making the final adjustment, tap the barometer a little just above the cistern, to destroy the adhesion of the metal to the glass. Complete the contact of the mercury and the ivory point, at the same time being certain that the barometer hangs freely, *i.e.*, vertically.

Next, tap the barometer gently in the neighborhood of the top of the mercury column, to destroy the adhesion of the mercury. This is very important, since raising or lowering the mercury in the previous operation materially affects the form of the upper surface. Then take hold lightly of the brass casing of the barometer at a distance from the attached thermometer, that neither the case nor the thermometer may be unnecessarily heated, and by means of the milled-head screw

near the middle of the tube (*D*, Fig. 80), bring the front and back edge of the vernier into the same horizontal plane with the top of the mercury in the tube, and remove the hand to allow the instrument to hang vertically. Move the eye about, and if, in any position, a line of light can be seen between the mercury and the vernier, the latter must be moved down; if there is no line of light and a portion of the meniscus is obscured, the vernier must be moved up. As the top of the column is more or less convex, when the adjustment is correctly made a small place is obscured in the center while the light is seen on either side.

Finally, having adjusted the instrument as above, it may be read at leisure. On the best barometers the scale is divided to inches, tenths, and half-tenths, and the vernier reads to one twenty-fifth of a half-tenth ($\frac{1}{25} \times 0.05$), or two thousandths (0.002) of an inch. (See Fig. 15, page 69.)

339. Transporting the Barometer. "In transporting a barometer, even across a room, it should be screwed up, and carried with its cistern uppermost. For traveling it should be provided with a wooden and leather case. On steam boats or steam cars it should be hung up by a hook. In wheeled vehicles it should be carried by hand, supported by a strap over the shoulder or held upright between the legs; but it should not be allowed to rest on the floor of the carriage, for a sudden jolt might break the tube. If carried on horseback it should be strapped over the shoulder of the rider, where it is not likely to be injured unless the animal is subject to a sudden change of gait. When about to be used, it should be taken from its case while screwed up, gently inverted and hung up, when it can be unscrewed. While it has its cistern uppermost the tube is full, is one solid mass of metal and glass, and not easily injured; but when hung up, a sudden jolt might send

a bubble of air into the vacuum at the upper end of the tube, and the instrument would be useless until refilled." *

Art. 2. Aneroid Barometers.

340. The aneroid barometer consists of a cylindrical metallic box, from which the air is exhausted, having a thin corrugated metal top which readily yields to alterations in the pressure of the atmosphere. When the pressure increases, the top is pressed inwards; when it decreases, the elasticity of the lid tends to move it in the opposite direction. There are two general forms of aneroids, according to the method employed for reading the movements of the top of the vacuum chamber. In the common form these motions are transmitted by delicate multiplying levers to an index which moves over a scale. In the Goldschmid aneroid the movements of the top of the vacuum box cause an index to move through the field of a micrometer microscope by which the movement is measured.

341. Common or Vidi Aneroid. Fig. 81, page 302, shows the mechanism of the ordinary form of aneroid barometer. The outside case and the graduated dial are not shown. The movement of the top of the vacuum chamber, M, is communicated to the index through the levers l, m, and r, and the watch chain S which winds around the axis carrying the index. The broad curved spring R keeps the lever l in contact with the post on the top of the vacuum chamber.

There are several modifications of this form which differ in the mechanism employed to multiply the linear motion of the end of the vacuous box, and to convert it into angular motion. A spring is sometimes inserted

* Williamson's On the Barometer, p. 134.

between the two ends of the vacuum chamber to reinforce the elasticity of the corrugated ends.

Sometimes the vacuous box is not entirely exhausted, the claim being that the enclosed air renders the indi-

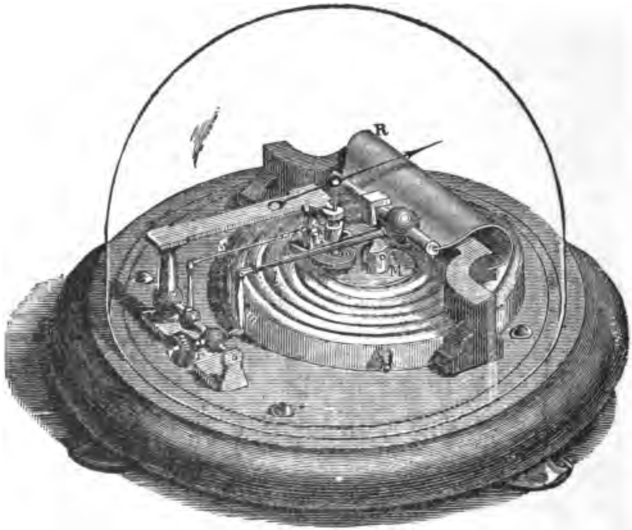

FIG. 81.—COMMON ANEROID BAROMETER.

cations of the instrument independent of changes of temperature. Such aneroids are said to be compensated; but the theory is incorrect, and some "compensated" aneroids are more affected by changes of temperature than uncompensated ones (see § 343, par. 2). An aneroid should have a thermometer attached to it.

The mechanism shown in Fig. 81 is enclosed in a brass or silver case, which varies from 2 to 6 inches in diameter, and from 0.5 to 3 inches in thickness. The smaller sizes are usually made of silver, in the form of a watch, and are known as pocket aneroids. The larger ones look like a short brass cylinder, and are

carried in a leather case supported by a strap over the shoulder.

342. The face of the instrument has a scale of inches, and sometimes also a scale of elevations. The former is graduated empirically by comparing its indications under different pressures with those of a mercurial barometer. The scale is marked to correspond to inches of the ordinary barometer column, the inches being divided into tenths, and the tenths usually into four parts. At the back of the instrument is a little screw which presses against one end of the exhausted box; and by turning this screw the index can be moved over the scale, and the instrument may thus be made to agree at any time with a standard mercurial barometer.

The altitude scale is obtained by converting the scale of inches into elevations by the use of some barometric formula (see Art. 4), and engraving the results upon the face, adjacent to the scale of inches. The altitude scale is at best of doubtful utility (see § 403).

343. Defects. The aneroid is a very convenient instrument, and where nice readings are not required it does very well; but for accurate hypsometrical results it is an inferior instrument. It has the following defects:

1. The elasticity of the corrugated top of the vacuum chamber is affected by repeated changes in pressure. This will produce error in the scale readings.

2. It is usually claimed that, in consequence of not completely exhausting the vacuum box, the indications of the aneroid become independent of the effect of changes of temperature of the instrument. The best that can be hoped is that for small changes the temperature correction is less than the error of observation. In instruments compensated for temperature, the effect of a change is sometimes the same as that in the

mercurial barometer, and sometimes the reverse. The effect of the temperature on any particular instrument can be determined only by trial.

3. The different spaces on the scale of inches are seldom correct relative to each other, owing probably to errors of observation and graduation, and possibly to differences of temperature and changes in elasticity. As a matter of fact, the scale is often only a scale of equal parts. The barometer scale is more accurate than the elevation scale, since the latter has all the inaccuracies due to the formula by which it is graduated (§ 403), in addition to those of the instrument itself. Before using the aneroid, it should be compared with a mercurial column under an air-pump, to determine the errors of its scale for different temperatures and pressures.

4. The weight of the machine affects its indication, *i.e.*, the reading of the aneroid will differ when held in different positions. In the best instruments this difference is sometimes as much as 0.008 of an inch, corresponding to a difference of elevation of about 8 feet. Usually the reading is higher with the dial is horizontal (face uppermost), than when it is vertical.

5. Like all combinations of levers, screws, and springs, the aneroid is liable to continual shifting of parts, when subject to the jars and jolts encountered in transportation and in use. The only remedy is frequent comparisons with a mercurial barometer.

6. The aneroid is deficient in precision, since the least reading is 0.025 of an inch, which corresponds nearly to 25 feet of elevation.

7. With most aneroids the spring ceases to act after the pressure has been lowered somewhat, *i.e.*, the instrument runs down. Before using it, experiments should be made to determine the range of pressure to which it may be exposed before the spring ceases to

act. In case an aneroid is to be used in an elevated region, if there is a mercurial barometer with the party, screw up the aneroid until the spring acts well, and set the instrument by the mercurial barometer so that there shall be a considerable difference, say 2 or 3 inches, between them. Of course this difference must be added to each reading of the aneroid.

344. "With all these defects a good aneroid is of much assistance on a survey or reconnoissance in mountainous districts, on side trips of one, or even several, day's duration, when the instrument has been previously compared with a standard mercurial barometer at various temperatures and in different elevations, and proper tables of corrections made. It should be compared before and after it is used in that way, to see if the zero has not changed in the meantime, and if the agreements are satisfactory the results can be relied upon. It is evidently important that there should be a good attached thermometer." *

345. Reading the Aneroid. In measuring heights with the aneroid barometer, it should be shielded from the direct rays of the sun, and care should be taken that it is not unnecessarily influenced by the heat of the body of the observer. If the instrument has a thermometer attached, that should be read first, as it is liable to be affected by the heat of the hand of the observer.

The barometer should be held in the same position for both observations—preferably with the face horizontal; and should be tapped gently with the finger just before taking the reading. Considerable care is required to determine exactly where the index points. Some instruments are provided with a small lens for this purpose (see last paragraph on page 94). Without

* Williamson's On the Barometer, p. 133.

the lens, the chief error is due to parallax. Of course, the index should move very close to the graduation and yet not touch the face of the instrument.

346. Goldschmid Aneroid. The common aneroid was invented about the beginning of this century, but was first made of a serviceable form by Vidi, of Paris, in

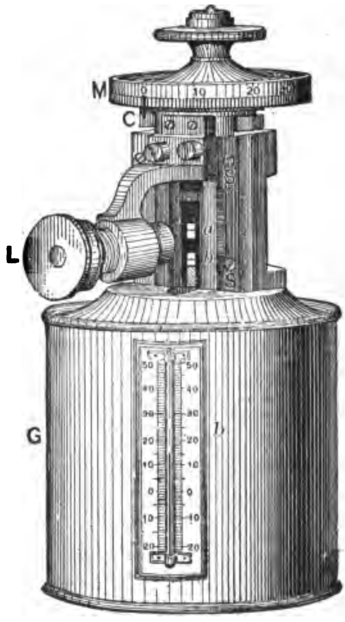

Fig. 82.—Goldschmid Aneroid.

1847. The defects of its complicated system of multiplying levers have long been recognized; and as early as 1857, Goldschmid designed a form of aneroid in which he dispensed with the transmitting and multiplying mechanism of the Vidi form.

Figs. 82 and 83, pages 306 and 307, are two views of one of the latest forms of the Goldschmid aneroid. Fig. 82

is an external view, showing the microscope *L*, and the micrometer *M*; and Fig. 83 is a section showing the compound vacuum chamber. Obviously, the greater the number of boxes, the larger the motion of the index *a*. The relative position of the movable index *a* and the fixed point of reference *b*, is observed by the telescope *L*, and the distance is measured by the micrometer *M*. The instrument is very delicate in its indications,

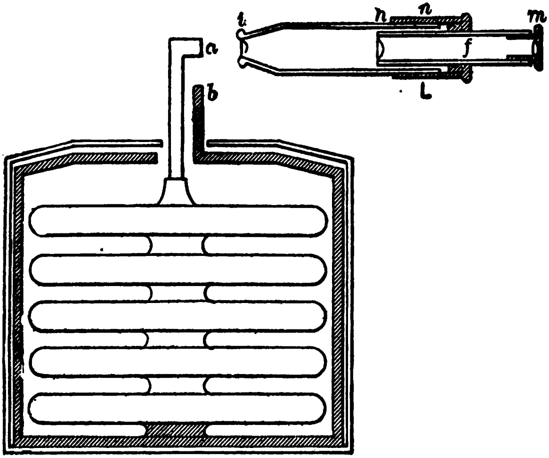

FIG. 83.—SECTION OF GOLDSCHMID ANEROID.

but is liable to serious disarrangement by ordinary handling. Different manufacturers make slightly different forms of the Goldschmid type, but all have essentially the same defect, *i.e.*, are not able to stand ordinary use.

It is doubtful if there is any advantage in an aneroid as complicated as that shown in Figs. 82 and 83. It seems probable that no form can be devised which shall be both delicate in its indications and able to stand rough handling. The chief advantage of the

common aneroid is its portability, combined with moderate accuracy. The mercurial and the aneroid barometers supplement each other; the first is delicate and the second is portable. These qualities can not be combined in a single instrument, nor can one be obtained more delicate or more reliable than the mercurial barometer.

ART. 3. THE OBSERVATIONS.

347. Outline of Method. To determine a difference of elevation with the barometer it is necessary to find at each of the two stations (1) the reading of the barometer, (2) the temperature of the barometer, and (3) the temperature of the air. Sometimes observations are made to determine (4) the amount of watery vapor in the atmosphere. Inserting these data in a barometric leveling formula (Art. 4) and reducing gives the difference of elevation.

Before discussing the method further, it is necessary to consider the errors to which barometric leveling is liable.

348. Sources of Error. For convenience of discussion, the sources of error will be considered under five heads; viz., (1) gradient, (2) temperature of air, (3) humidity, (4) instrumental errors, (5) errors of observation, and (6) effect of the wind.

349. Gradient Errors. Let A, B, and C designate three points at which the pressure is the same. The plane passing through A, B, and C is then a surface of equal pressure. If the air were in a state of equilibrium, this plane would be level; but under ordinary conditions it will be inclined in some direction. The inclination of this surface is called the *barometric gradient.*

Instead of considering only three points, we can in imagination project through the air a surface contain-

ing all points which have the same pressure. If the atmosphere were at rest, this surface would be a horizontal plane; but under the actual conditions, it is never a plane and is ever undulating. For small areas under ordinary conditions, this surface would probably not differ much from a plane.

Conceive another surface passed through all points at which the pressure differs from the preceding one by any constant quantity. With atmospheric equilibrium two such surfaces would be both level and parallel, but in the actual case they are neither level nor parallel. When widely separated surfaces are compared, the variations from parallelism are often so great that their inclinations above the same locality have opposite directions. The atmospheric gradient at the surface of the ground may therefore differ greatly in amount and direction from the simultaneous gradient at a considerable altitude above that point.

350. That the atmosphere is not in static equilibrium is shown by its being continually in motion and also by the variation in the height of the barometer. A variation of the barometric pressure indicates a change in the barometric gradient, and a gradient or a change in the gradient may produce an error in the result obtained by barometric leveling. For example, if A and B, Fig. 84, page 310, are two stations and the atmosphere is at rest, the surface of equal pressure, BC, is a horizontal plane, and AC is the difference of elevation which would be obtained by applying any one of the common barometric formulas. If the air is not in static equilibrium, the pressure at A will be greater or less than before, and the surface of equal pressure will lie above or below BC. If the pressure at A is greater than the average, the surface of equal pressure is above C, say at E, and AE is the corresponding difference of elevation; and similarly, if BD is the surface of equal

pressure, AD is the corresponding difference of elevation.

The problem is further complicated by the fact that the air above B also is in a state of oscillation. If the

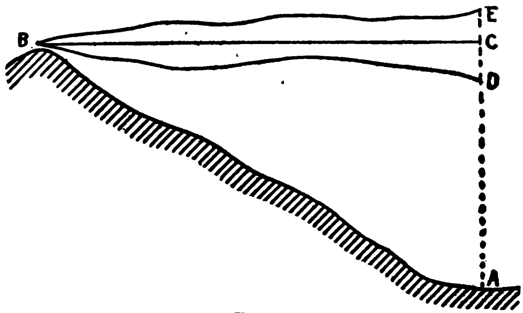

FIG. 84.

variations in pressure at the two stations were simultaneous and alike in amount, no error would be produced by the barometric gradient; but these conditions are seldom or never realized, and therefore there is always a possibility of error in barometric leveling, owing to the barometric gradient and also to a change in the gradient between the observations at the two stations.

351. There are four classes of barometric gradient; viz., (1) diurnal, (2) annual, (3) non-periodic, and (4) permanent.

352. *Diurnal Gradient.* It is a fact familiar to meteorologists that the pressure of the air everywhere undergoes a daily oscillation. The gradient introduced by this daily change is called the diurnal gradient. The pressure has two maxima and two minima which are easily distinguishable. Near the sea-level the barometer attains its first maximum about 9 or 10 A.M. In the afternoon there is a minimum about 3 to 5 P.M. It then rises until 10 or 11 P.M., when it falls again until about

4 A.M., and again rises to attain its forenoon maximum. The maxima occur when the temperature is about the mean of the day, and the minima when the temperature is at the highest and lowest respectively. The day fluctuations are the larger.

The daily oscillation is subject to variations in character and magnitude. It is greater in summer than in winter; and is greatest at the equator and diminishes toward the poles, but is not the same for all places of the same latitude. Within the United States it varies between 40 and 120 thousandths of an inch. Changes of altitude often cause a marked increase in the amount of the diurnal oscillation. The difference which pertains to latitude does not materially affect the ordinary hypsometric problem, but the difference depending on the altitude has a very important effect.

A change of 1 thousandth of an inch in the height of the barometer corresponds to a difference of elevation of from 0.8 to 1.0 foot. Therefore, if an observation were made at one station at about 10 A.M., and at a second station at about 3 P.M., the difference of elevation would probably be from 40 to 120 feet in error.

353. *Annual Gradient.* The annual progress of the sun from tropic to tropic throws a preponderance of heat first on one side of the equator and then on the other, which produces an annual cycle of changes in the pressure and gives rise to what has been called the annual gradient. The amount of variation in the barometric pressure is quite small near the equator, but increases rapidly toward the poles. The mean annual variation in the United States ranges from 120 to 200 thousandths of an inch, although the variation at any particular station, or for any one year, may be very much greater. For interesting diagrams showing the annual variation for a great number of stations, see Williamson's On the Barometer, pages 68–81.

354. *Non-periodic Gradient.* In addition to the diurnal and annual variations in the pressure, there are others due to the same general cause—the heat of the sun,—but so modified by the local conditions—topography, humidity, winds, storms, etc.,—as to make it impossible to discover the law of their action. These non-periodic variations are much greater in amount and more rapid in action than any of the others. For example, in a trial made for the purpose of this record, under apparently favorable circumstances, the barometrically-determined difference of elevation of two points having a difference of elevation of about 100 feet and being 25 miles apart on a plain, was in error about 400 per cent, owing mainly to non-periodic gradient. Usually these variations are greater immediately before and after a storm.

The errors arising from non-periodic gradients are approximately proportional to the force of the wind and to the horizontal distance between the two stations.

355. *Permanent Gradient.* Since the atmosphere, if undisturbed, would have no gradients, and since every disturbance produces them, it is easy to understand that any continuous disturbance will be accompanied by permanent gradients. The excess of solar heat received in the tropics, as compared with the polar regions, is of the nature of a continuous disturbance, and sets in motion the great currents of the atmosphere and the ocean. The joint action of these causes gives rise to a great system of permanent gradients. The annual gradients are only variations of the permanent gradient, caused by the annual progress of the sun from tropic to tropic.

The topographic conditions of the earth's surface also probably cause a somewhat permanent gradient.

356. *Conclusion as to Gradient Errors.* The result ob-

tained by barometric leveling may be in error to almost any degree owing to barometric gradient and to changes in the gradient. A description will presently be given (see §§ 371–81) of several methods of making the observations which eliminate at least part of the errors due to gradient.

357. Temperature of the Air. Variations in temperature is the chief cause of changes in barometric pressure, but the variation in the temperature of the air has another, and generally a more serious, effect upon the results obtained by barometric leveling. Let *A* and *B*, Fig. 84, page 310, be two stations the difference of elevation of which is to be obtained from observations of the barometer and thermometer made at each. Assumed that the pressure observed at *B* is the same as that at *C*—vertically over *A* and on a level with *B*. To use any of the barometric leveling formulas, the temperature of the column *AC* must be known; and in applying any of these formulas it is assumed that the mean temperature of this column is equal to the mean of the temperatures observed at *A* and *B*.

How admissible this assumption is will appear at once when the manner in which the air acquires and loses heat is recalled. The body of the atmosphere is heated directly by the sun, and gives off its heat by radiation into space. The surface of the earth is heated and cooled in the same manner, but many times more rapidly, so that by day it is always much warmer than the body of the air, and by night it is much cooler. A layer of air next to the earth receives its warmth from the earth, and hence differs widely in temperature from the remainder of the atmosphere. Not only is the greater part of the column inaccessible to us, but that portion to which our observations are restricted is the portion least representative of all.

"By measuring the difference of elevation of two

points with the spirit level, reversing the barometer formula, and computing the temperature of the air column, it has been found that in middle latitudes the average daily range of the temperature of the body of the air is about 4°, of the superficial layer from 10° to 20° near the seashore, and from 20° to 35° in the interior of continents." * There is a stratum of air near the surface of the earth which oscillates daily through this wide range, while the temperature of the upper and larger portion of the column *AC* is relatively constant; and therefore the mean of the observed temperatures absolutely fails to give the mean temperature of the column *AC* as required in the formula.

358. Nor does the trouble end here. Whenever the ground layer is cooler than the air above, it is of course heavier, and, like any other heavy fluid, it flows down hill and accumulates in valleys, forming lakes of cold air. The nightly layer of abnormally cool air is therefore thinner on eminences than in valleys, and the contrast increases as the night advances. When the conditions are reversed so that the lower layer is warmer than the air above it, it has a tendency to rise, but accomplishes the change in an irregular manner, breaking through the immediately superior layer here and there and rising in streams which spread out in sheets wherever the conditions of equilibrium are reached. Observers in balloons, as they ascend or descend, rarely find an orderly succession of temperatures. If, therefore, we could in any way determine the temperature of some point in the upper portion of the column *AC*, we should still be unable to deduce the mean temperature of the column with any considerable degree of accuracy.

359. *Conclusion as to Error in the Temperature of the*

* G. K. Gilbert, in the Report of the U. S. Geological Survey for 1880–81, p. 421.

Atmosphere. For an error of 5° F. in the mean temperature of the level stratum of air between the two stations, the resulting error is approximately 1 per cent of the difference of elevation (see § 390). This error may even under favorable conditions be two or three times this amount, and under unfavorable conditions five or six times as large.

Several methods of eliminating the error due to the uncertainty in the temperature of the air will be described presently (§§ 371–81); but even with the utmost care and the most elaborate system of observations, the determination of the temperature of the air is the chief source of error in barometric leveling.

360. Humidity. The barometer is influenced by the elastic force of the invisible aqueous vapor suspended in the atmosphere, in the same way that it is influenced by the dry air; and hence the attempt has been made to introduce a term into the barometric leveling formula, which shall take account of the amount of this watery vapor. The determination of the humidity of the atmosphere involves essentially the same difficulties as the determination of its temperature (see §§ 357–59). The observations are made in the stratum next to the earth, in which the amount of moisture is the greatest and the most variable. A change of position of a few feet, or a slight variation in the direction or force of the wind, will often cause a very important difference in the amount of watery vapor present.

The variations in the hygrometric state are still further increased by vaporization and condensation. Whenever a current of air moves upward its temperature is lowered by refraction, and a point may be reached where the accompanying vapor can no longer exist as such, and is condensed to cloud or even to rain or snow. On the other hand, whenever a current of air moves downward its capacity for moisture is in-

creased, and it acquires the power to take up water from whatever moist surface it comes in contact with. At the surface of the earth there is an almost continuous passage of moisture from ground to air, only a part of which is returned as dew. The daily circulation of the atmosphere incited by the heat of the sun carries the moistened air upward, and eventually it is condensed and returned to the earth in the form of rain or snow; but the condensation and succeeding precipitation are exceedingly irregular.

The irregularities of humidity are greater proportionally than the irregularities of temperature, but the error in the difference of altitude due to humidity is less than that due to temperature, because humidity is a much smaller factor of the hypsometric problem. In a very general sense, in temperate climates near the sea level, the amount of vapor in the atmosphere varies from 0.2 to 0.4 of an inch, or about one hundredth of the whole.

361. *Conclusion as to Humidity.* As the methods employed to eliminate errors of gradient and temperature (§§ 371–81) also eliminate errors due to humidity, the subject will not be discussed further here. See §§ 393–95.

362. Instrumental Errors. *Mercurial Barometer.* The errors that may exist in a mercurial cistern barometer are (1) error in the position of the index, (2) imperfect graduation of the scale, (3) error in the position of the zero and in the graduation of the attached thermometer, (4) impure mercury, and (5) air in the tube. The first is eliminated by comparing the instrument with a standard barometer, and either re-adjusting the zero of the scale or noting the correction to be applied to the reading. With a good barometer properly handled, the other errors will be inappreciable.

363. *Aneroid Barometer.* The index of the aneroid

is easily displaced, and hence it should be frequently compared with a mercurial barometer. The attached thermometer also should be tested. See §§ 343-45.

364. Errors of Observation. *Mercurial Barometer.* The principal errors of observation are in making contact between the ivory point and the mercury, and in adjusting the vernier to coincide with the top of the mercurial column (see § 338). Gilbert, from a comparison of three hundred and sixty pairs of observations made by the Signal Service and by the Geological Survey, found the average error of observation to be a trifle less than three thousandths of an inch.* This difference does not involve the personal equation between two observers, which, even for experts, may be nearly as much more.†

365. *Aneroid Barometer.* Owing to parallax, there is a liability of considerable error in reading the position of the index. The amount of this error will depend mainly upon the distance the index is from the scale; but as the aneroid is not an instrument of any considerable precision, this subject will not be discussed further. See §§ 343-45.

366. *Thermometers.* There is liability of error owing to parallax in reading the attached and detached thermometers. With the mercurial barometer an error of 0.1° F. in reading the attached thermometer produces an error of about 0.3 ft. in the elevation. Notice that this source of error is independent of, and in addition to, that discussed in §§ 357-59. The latter is usually very much the greater.

The chief source of error in determining the temperature of the air or of the instrument, is in the exposure of the thermometer. It should be very carefully shielded from both the direct and reflected rays of the

* U. S. Geological Report, 1880-81, p. 542.

† U. S. Coast Survey Report, 1870, p. 79.

sun, and the observer should be careful that the heat of his own body does not affect the thermometer (§ 338).

The brass scale is usually so thin that it undergoes changes of temperature more rapidly than the mercury; and therefore if the temperature of the surrounding air be gradually raised, the brass scale responds more promptly than the mercurial column and becomes relatively too long, while the reverse takes place if the temperature is lowered. The result is that a rising temperature gives too low a barometric pressure, and a falling temperature too high a one. If the change of temperature is rapid, the error from this cause may amount to ten or fifteen thousandths of an inch, which corresponds to a difference of elevation of 10 or 15 feet. The precaution generally recommended is to put the barometer in position and leave it with unchanged conditions for fifteen or twenty minutes before making the observation.

367. Wind. The wind may cause either a condensation or a rarefaction of the air in the room in which the barometer is hung, or even in the cistern of the barometer itself. This effect will vary with the velocity of the wind, the position of the openings with reference to the direction of the wind, the size of the room, etc. "On Mount Washington, a wind of 50 miles per hour caused the barometer to read 0.13 of an inch too low. The effect of the wind will vary as the square of its velocity. It may be nearly, if not wholly, eliminated by having two apertures, one each on the windward and leeward side of the enclosed space." *

The wind has a similar effect when the instrument is read in the immediate vicinity of any body which obstructs the wind. For example, if the barometer is observed on the windward side of a mountain, the

* G. K. Gilbert in Report U. S. Geological Survey for 1880–81, p. 562.

reading will be too high; and if on the leeward, too low. The only way to eliminate this error is to select stations not thus affected,—although it is not always possible to do so.

368. Limits of Precision. It is sometimes stated that "the barometer is the most accurate instrument for determining differences of level;" but it needs only a moment's reflection to see that this can not be true. The following results, given by Professor Guyot,* are frequently quoted as showing the great accuracy of barometric leveling:

Mount Blanc, by barometer, 15,781 feet,
by spirit level, 15,780 feet.
Mount Washington, by barometer, 6,291.7 feet,
by spirit level, 6,293 feet.
In North Carolina, by barometer, 5,248 feet,
by spirit level, 5,246 feet.
In North Carolina, by barometer, 6,701 feet,
by spirit level, 6,711 feet.

A few examples showing a small error prove nothing as to the accuracy of the method, for the agreement may be wholly accidental. A fair inference from the remarks of Professor Guyot accompanying the above data, is that the mean of several observations taken during one or two days will generally give as accurate results as those above; but it is probable that he did not intend to convey such an impression, for the very observations on which he relies for his spirit level altitude of Mount Washington resulted in a suggestion to modify the constants in the barometric formula.‡ In one instance his barometric results were in error 125 feet in a difference of altitude of 1,780 feet; and in another 75 feet in a difference of altitude of 6,280 feet.

* Smithsonian Miscellaneous Collection, Vol. I, Art. II, Group D, p. 34.
‡ U S. Coast Survey Report, 1854, p. 100*.

369. The difference of altitude computed from one, or even several, day's observations can not be relied upon as being more than a rough approximation. This has been shown by Williamson,* who has computed the difference of altitude between Geneva and St. Bernard by the formula used in the last three examples quoted above, for every day for two years, using the daily means of simultaneous bi-hourly observations. In several cases, the difference between the result by the barometer and the spirit level was more than 3 per cent. Under less favorable circumstances the errors were even more than twice as great.† The altitude computed by the monthly means of bi-hourly observations for different months of the same year, and also for the same month of different years, differ as much as 1 per cent.‡

The following differences between the results by the barometer and the spirit level do not indicate that high degree of accuracy in barometric hypsometry, *even when a long series of observations is used*, which was formerly supposed to be attainable by this means. The results by the barometer were obtained by computing the difference of altitude from monthly means of the mean of the daily observations, and taking the mean for the time stated.§

Vera Cruz and City of Mexico, 1 year's observations,
+ 5 metres in 2,282 metres.
Sacramento and Summit, 3 years' observations,
− 24 feet in 6,989 feet.
Portland and Mt. Washington, 6 years' observations,
+ 37 feet in 6,289 feet.
Geneva and St. Bernard, 12 years' observations,
− 2.6 metres in. 2,070 metres.

* Williamson's On the Barometer, pp. 194–205.
† U. S. Geological Survey Report for 1880–81, pp. 456–59.
‡ Williamson's On the Barometer, p. 236.
§ U. S. Coast and Geodetic Survey for 1881, p. 254.

For an interesting comparison of the absolute, and also the relative, errors of the various methods, see Chap. III of the Report of U. S. Geological Survey for 1880–81. For additional data concerning the accuracy of barometric leveling, see Report of the U. S. Coast and Geodetic Survey for 1870, p. 88; *ibid.*, 1871, pp. 154–75; *ibid.*, 1876, pp. 356–76; Williamson's On the Barometer, p. 205.

370. Although the barometer can not be regarded as a hypsometric instrument of great precision, yet with care it can be made to give results with sufficient accuracy for reconnoissance or exploration. For this purpose, it is unexcelled by any other instrument; and this is about the only use the engineering profession can make of it.

371. Methods of Observing. In §§ 348–61 it was shown that barometric leveling was liable to large errors owing to barometric gradient, and to errors in determining the temperature and humidity of the atmosphere. In the best work, the observations are made in such a way as to eliminate part of these errors. The several methods employed to accomplish this will now be considered.

372. Single Observations. The simplest, and also the most common, method consists in using only one barometer, which is carried from station to station. By taking one or more observations at each station, the errors of observation (§§ 364–67) are nearly eliminated; but the vastly greater errors due to gradient, temperature, and humidity (§§ 349–61) are undiminished. Results obtained by a single observation, or even by several in quick succession, are only the rudest kinds of approximations (§ 354), and the greater the horizontal and vertical distances between the two stations the greater the error.

Distant stations are sometimes connected by inter-

mediate ones to eliminate changes of gradient, temperature, etc., during the time the exigencies of travel require the barometer to remain at the intermediate station. For example, to determine the difference of elevation between *A* and *C*, make an observation at *A* and proceed towards *C*, making an observation at *B*, an intermediate point, on arrival at that station and a second one on leaving it; and finally make an observation at *C*. The difference of elevation of *A* and *B* is determined from the first two readings, and that of *B* and *C* from the second two; and the difference of elevation of *A* and *C* is equal to the sum of the partial differences.

373. Simultaneous Observations. The errors due to *change* of gradient (§§ 349–56) are partially eliminated by making simultaneous observations at the two stations. If the phase and the amplitude of the variation were the same at both stations, which probably seldom or never occurs, simultaneous observations would give results independent of changes in the gradient.

Errors due to gradient are still further reduced by making a number of simultaneous observations and using the mean. This may eliminate the variable element, but fails to take account of permanent gradient (see § 369).

374. Observations at Selected Times. It is often recommended that the observations be made at certain hours of the day, at which time the *diurnal* gradient is supposed to be zero. These times can be determined only by observation, and will vary greatly with the state of the atmosphere, the season, the locality, the elevation, etc. The U. S. Coast Survey recommend the following times.* They were probably deduced for the middle Atlantic coast. The hours refer to the middle of the

* Report for 1876, p. 349.

month, and other times are to be determined by interpolation.

January				1	P.M.
February	10	A.M.	and	4	"
March	8	"	"	6	"
April	7 30	"	"	7	"
May	7	"	"	7	"
June	6.30	"	"	9.30	"
July	6.30	"	"	9.30	"
August	7	"	"	7.30	"
September	8	"	"	6	"
October	10	"	"	3.30	"
November	10.30	"	"	2.30	"
December	at no time.				

Obviously, a single observation, even if taken in accordance with a table similar to the above, could be considered only a rough approximation owing to the liability of error due to non-periodic gradient (§ 354). For a long series of observations, the results would probably be more accurate if the observations were made in accordance with some such scheme; but the determination of the best times at which to make the observations would necessitate an extensive preliminary series of observations at short intervals. Such a method is clearly inapplicable, since the time and expense required to obtain approximate results by the barometer would be nearly or quite as great as that required to determine an equal number of more accurate results by the stadia (Chapter X) or by spirit leveling (Chapter XI).

375. Notice that none of the preceding methods eliminate the error due to the fact that the mean of the observed temperatures does not represent the mean temperature of the air column (§§ 357–59).

376. Williamson's Method.* This method is specially adapted to reconnoissances and topographical surveys.

* Williamson's On the Use of the Barometer, pp. 39–42, 150–59.

A centrally located station, called a base station, is chosen, at which the barometer and thermometers are read at stated hours each day for several days. In the meantime, itinerary observers make observations at the points whose elevations are to be determined, taking pains to have each observation correspond in time with one of the observations at the base station. In the progress of the itinerary survey, a series of observations, similar to those at the base station, are made as frequently as practicable at semi-permanent camps. The object of the series at the base station and at the semi-permanent camps is to ascertain the nature of the diurnal variation of pressure and temperature.

The barometric readings at the base stations, after being corrected for temperature of the instruments (§ 389), are plotted upon ruled paper so as to exhibit their curve, and all readings shown by inspection to be influenced by abrupt and violent atmospheric disturbances, such as thunder storms, are discarded, their places being filled by interpolated values. From the corrected observations at the base stations, a correction is deduced, which, being applied to the several barometric readings, reduces them to the daily mean. Applying this correction eliminates at least part of the effect of the diurnal gradient (§ 352).

Instead of determining the temperature of the air column from the temperature at the time of observing, the mean temperature of the day is used. This can be quite accurately determined at the base stations, but is only approximately known at the other stations. Notice, however, that the mean of the observed temperatures will not be the mean temperature of the stratum of air included between the two stations (§§ 357–59).

The difference of altitude can be computed from the reduced barometric readings and the mean daily temperature, by using any statical formula (Art. 4). Wil-

liamson himself used his translation of Plantamour's formula (§ 401).

377. Whitney's Method. From observations made in connection with the Geological Survey of California, a series of corrections were deduced for reducing the barometric readings made at different hours of the day of the different days of the different months, and for different altitudes, to the daily mean for the year.

These corrections can only be used in the neighborhood in which the obseıvations on which they were based were made. Similar tables made for different climates differ materially from each other. For tables constructed upon this principle for the climates of Germany, Philadelphia, and Greenwich, respectively, see Smithsonian Miscellaneous Collection, Vol. I (3d Edition), Art. III, Group D, pp. 80–82, 86, and 93–94.

378. Plantamour's Method. In the hypsometric survey of Switzerland, Plantamour made simultaneous observations of the barometer, thermometer, and psychrometer at Geneva, St. Bernard, and at the station whose height was to be determined. The approximate difference of altitude between the new station and Geneva, and between it and St. Bernard, and also between Geneva and St. Bernard, were computed by Plantamour's formula (§ 401). The computed difference of elevation between Geneva and St. Bernard compared with the difference of altitude determined by the spirit level, gave a correction to be applied to the computed difference for the new station. The ingenious details of the computation are too complex to be described here.

Marshall employed, in the geographical surveys in the Rocky Mountains, a method somewhat similar to the above.*

* U. S. Geological Surveys West of the 100th Meridian, Vol. II, pp. 522, 523.

379. Rühlmann's Method. This method differs from Plantamour's (§ 378) chiefly in that Rühlmann makes use of the pressure and the moisture factors observed at two stations whose difference in level is known, to compute the temperature of the intervening air column. The temperature thus derived he afterwards applies in the computation of the unknown difference in level of two other stations.

This method is applicable only to a detailed topographical survey, and requires such an elaborate series of observations at base stations that the time and expense are nearly, or quite, as great as that required to determine the results with the stadia (Chap. X) or the spirit level (Chap. XI).

380. Gilbert's Method.* This method is practically a combination of Plantamour's and Rühlmann's, and requires simultaneous observations of the barometer at three stations, the vertical distance between two of which is known, but does not require observations of the temperature and humidity. From the known difference between two of the stations and the observations at each, the actual density of the air, which is dependent upon the pressure, temperature, and humidity, may be determined by reversing the ordinary barometric formula. The true density is then used to compute the difference of elevation between either of these stations and the third. For the formula employed in this method, see § 407.

This method has not proved as satisfactory in practice as was hoped when it was proposed. It is simply another attempt to secure accurate results by a modification of a general method which is inaccurate in its essential features.

381. Conclusion as to Methods. Unfortunately all methods of eliminating gradient and temperature

* Report of U. S. Geological Survey for 1880–81, pp. 405–561.

errors involve considerable time and expense, and even then do not thoroughly accomplish the desired end, which shows that when great accuracy is desired the barometer should be dispensed with altogether, and the difference of elevation be determined by some other means.

ART. 4. BAROMETRIC FORMULAS.

382. Barometric leveling formulas may be divided into two classes; viz., (A) those that assume the atmosphere to be in statical equilibrium, and (B) those that take account of the fact that the air is more or less in motion. The first will be designated statical formulas, and the latter dynamical. The former are by far much more frequently used.

A. *Statical Formulas.*

383. ASSUMPTIONS. All the barometric leveling formulas in common use are dependent upon the assumption that the air is in a state of statical equilibrium.* If this state were possible, we might suppose the whole atmosphere to be arranged in a system of horizontal layers, each of which is denser than the one above it and rarer than the one below it, each being uniform throughout in temperature and humidity.

But the air is never in a state of statical equilibrium, but is perpetually undergoing local changes of pressure, temperature, and humidity. For example, during the day the sun imparts a certain amount of heat to the whole atmosphere, but a much higher temperature is given to the ground and by it communicated to the contiguous layer of air. At night the atmosphere loses heat by radiation to space, but the ground loses it more

* There are only three not thus included (see §§ 406–8), and practically they are not used.

rapidly and imparts its low temperature to the lowest stratum of air. The lower strata, therefore, have exceptional warmth by day and exceptional coolness by night. Again, if the air is moist, during the day it intercepts a greater quantity of solar heat than if it were dry, so that a less quantity reaches the ground; but at night the moisture checks radiation from the ground. The power of the earth's surface to receive or part with heat varies with its character; naked rocks and cultivated fields, bare earth and grass, forest and snow are affected very differently by the heat rays of the sun, and exert equally diverse influences on the adjacent air, so that one tract of land is often in a condition to heat the air while an adjacent tract is cooling it. Then, too, the sun's heat is unequally distributed through the year; outside the tropics there is a progressive accumulation of heat through summer and a progressive loss through winter. The ocean undergoes less change of temperature than the land, and its rate of change is slower, so that there is frequent, and indeed almost continuous, contrast of condition between it and the contiguous land. As a result of all these influences, together with others that might be enumerated, the equilibrium of the air is constantly disturbed, and the winds, which tend to readjust it, are set in motion.

The temperature of the air is continually modified by external influences; the static order of densities is broken and currents are set in motion; and the inequalities of temperature also conspire to produce inequalities of moisture. Every element of equilibrium is thus set aside, and the air is rendered heterogeneous in density, temperature, and humidity.

384. All of the common barometric formulas are dependent upon two assumptions; viz., (1) that a difference of pressure is due only to a difference of eleva-

tion; and (2) that the mean of the temperatures of the air at the two stations represents the mean temperature of the level stratum of air between them. A few of the formulas involve also a third assumption; viz., that the mean humidity of the air at the two stations is the same as the humidity of the layer between them.

A consideration of the facts stated in § 383 and discussed more fully in §§ 348–61, will show that any formula dependent upon the preceding assumptions can be, at best, only approximate. In §§ 371–81 were explained several methods of reducing the errors due to the above assumption; but notice that errors of gradient, temperature, and humidity were eliminated by the methods of making the observations and computing the results, rather than by the use of any particular formula.

385. Fundamental Relations. Let A and B, Fig. 85, represent the two stations, and assume that it is required to determine the vertical distance between them. A and B are not necessarily, and not even usually, in the same vertical line. Let C represent any point in AB, and D a point an infinitesimal distance below C.

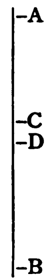

Fig. 85.

Let a_0 = the weight of a cubic inch of dry air at the sea level, in latitude 45°, at 32° F., when the barometer stands at 29.92 inches.

a = the weight of a cubic inch of air under the pressure, temperature, etc., existing between C and D.

c = the co-efficient of expansion of air.

H_0 = the height of the barometric column at the sea level in latitude 45°, = 29.92 inches.

H' = the height, in inches, of the barometer at the upper point, reduced to 32° F.

H_1 = the height, in inches, of the barometer at the lower point, reduced to 32° F.

m_0 = the weight of a cubic inch of mercury at the sea level, in latitude 45°, when the barometer reads 29.92 inches.

m' = the weight of a unit volume of mercury at the upper station.

m_1 = the weight of a unit volume of mercury at the lower station.

ϕ = the mean latitude of the two stations.

P = the pressure per square inch, at D.

dP = the difference in pressure between C and D, *i.e.*, dP is the differential pressure.

P_0 = the pressure per square inch, at the sea level, in latitude 45°.

P' = the pressure at the upper station.

P_1 = the pressure at the lower station.

R = the radius of the earth.

T = the mean temperature of the layer of air between A and B.

T' = the temperature of the air at the upper station.

T_1 = the temperature of the air at the lower station.

X = the difference in elevation.

Z = the elevation of the lower station above the sea level.

386. It is clear that the increase of pressure from C to D is equal to the weight of a column of air having a unit section and the height CD. Expressing this in the language of the calculus, we have

$$-dP = a\,dx. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

By Marriotte's law, $a : a_0 :: P : P_0$, from which

$$a = a_0 \frac{P}{P_0} . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Equation (2) gives the weight of a unit of air at 32° F., and for any other temperature T,

$$a = a_0 \frac{P}{P_0} \frac{1}{1 + cT} . \quad . \quad . \quad . \quad (3)$$

Equation (3) gives the weight of a unit of air at latitude 45°, and for any latitude ϕ,

$$a = a_0 \frac{P}{P_0} \frac{1}{1 + cT} \frac{1}{1 + 0.002,60 \cos 2\phi} . \quad . \quad (4)$$

Equation (4) gives the value of a at the sea level, and for any other altitude $(Z + X)$,

$$a = a_0 \frac{P}{P_0} \frac{1}{1 + cT} \frac{1}{1 + 0.002,60 \cos 2\varphi} \frac{R^2}{(R + Z + X)^2} . \quad (5)$$

Combining equations (1) and (5), integrating, and transposing,

$$X = \frac{P_0}{a_0} \text{ N.log} \frac{P_1}{P'} \frac{1}{1 + cT}$$

$$\frac{1}{1 + 0.002,60 \cos 2\phi} \frac{(R + Z)(R + Z + X)}{R^2} . \quad . \quad (6)$$

$$P_0 = H_0 m_0 = 29.92 \, m_0 . \quad . \quad . \quad . \quad (7)$$

$$\frac{P_1}{P'} = \frac{H_1 m_1}{H' m'} . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

The weight of a body varies inversely as the square of the distance from the center of the earth, and therefore

$$\frac{m_1}{m'} = \frac{(R+Z+X)^2}{(R+Z)^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

Combining equations (6), (7), (8), and (9), we get

$$X = 29.92\,\frac{m_0}{a_0}\,\text{N.log}\left[\frac{H_1}{H'}\,\frac{(R+Z+X)^2}{(R+Z)^2}\right]\left\{\begin{array}{l}(1+c\,T)\\[4pt](1+0.002,60\cos 2\phi)\\[4pt]\dfrac{(R+Z)(R+Z+X)}{R^2}\end{array}\right. \qquad (10)$$

The quantities at the right of the brace are three factors of the second member of the equation.

Substituting, in equation (10), the sum of the logarithms for the logarithm of the product, passing to common logarithms, and *expressing* X, Z, and R, *in feet*,

$$X\text{ ft.} = 5.744\,.\,.\,\frac{m_0}{a_0}\log\frac{H_1}{H'}\left\{\begin{array}{l}(1+c\,T)\\[4pt](1+0.002,60\cos 2\phi)\\[4pt]\left(1+\dfrac{2\log\left(1+\dfrac{X}{R+Z}\right)}{\log\dfrac{H_1}{H'}}\right)\dfrac{(R+Z)(R+Z+X)}{R^2}\end{array}\right. \qquad (11)$$

Notice that the preceding equation involves no approximations.

387. Equation (11) includes the principal relations involved in determining differences of height with the barometer. The formula to be used in practice has

been given differently by different investigators, according to the values chosen for the constants, to the individual preference for one form over another, and to the degree of refinement desired.

388. The Constant. The value of the term 5.744 $\ldots\frac{m_0}{a_0}$, generally known as the barometric co-efficient, will depend upon the values for the densities of air and mercury which are used. Boit and Arago found $\frac{m_0}{a_0}$ = 10,467,* which makes the barometer co-efficient 60,096.3 ft. (18,317 meters). Regnault's values,* which are the most recent and probably the most accurate, give 60,384 feet (18,404.8 meters).

Raymond in 1803 found * the valuè of the barometric co-efficient by determining the value it should have to make the results by the formula agree with those furnished by trigonometrical leveling. The value obtained in this way is 60,158.6 ft. (18,336 meters). Even under the most favorable circumstances, the observations, eight in all,† were too few to determine such a co-efficient with sufficient accuracy, owing to the variations in the barometric pressure (§§ 349–56), the temperature (§§ 357–59), the humidity (§ 360), and the refraction (§ 318). The method employed by Raymond is the least accurate of any, although his co-efficient is more frequently used than any of the others.

One of the author's students determined the barometric co-efficient by reversing Laplace's formula,—equation (13), page 339,—and using the barometric readings made by the U. S. Weather Bureau, together with the difference of level of the stations. The results are not very satisfactory owing to the uncer-

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* Smithsonian Miscellaneous Collections, Vol. I, Guyot's Meteorological Tables, Group D, page 9.

† Report U. S. Coast and Geodetic Survey, 1881, p. 235.

tainty in the values for the differences of level. The co-efficient obtained from twelve pairs of stations, using the mean of tri-daily observations for three to twelve years, is 60,156 feet.*

For a discussion showing that the errors due to gradient and temperature are very much greater than the errors due to the differences between the several barometric constants, see Williamson's On the Use of the Barometer, pages 221–33.

389. Temperature of the Barometer.—Before inserting the barometric readings in equation (11), page 332, they must be reduced to the corresponding heights they would have at a common temperature. This correction may be made in either of two ways; viz., (1) one barometer may be reduced to the temperature of the other, or (2) both may be reduced to any other temperature assumed as a standard.

1. The co-efficient of expansion of mercury is 0.000,100,1 for 1° F.; and that of brass, of which the scales are generally made, is 0.000,010,4. The difference—the relative co-efficient of expansion of mercury—is 0.000,089,7. For the centigrade scale this difference is 0.000,161,41. If h' represents the height of the barometer at the upper station reduced to the temperature of the lower, and t' and t_1 the temperature of the barometers at the upper and lower stations respectively,

$$h' = H'[1 - d(t' - t_1)], \quad . \quad . \quad . \quad . \quad (12)$$

in which d stands for one of the above differences, according to the kind of thermometer used. When this method is employed, a term is inserted in the formula to correct for the difference in temperature of the barometers. For an example see § 400.

* Bachelor's Thesis of Edward E. Ellison, Class of '88, University of Illinois.

2. To reduce both readings to a common temperature, as for example the freezing point of water, apply equation (12) to both readings, considering t_1 to represent the temperature of melting ice (32° F. or 0° C.) and t' the reading of the attached thermometer. Numerous tables have been computed for facilitating this reduction; for example, Smithsonian Miscellaneous Collection, Vol. I, Guyot's Collections (3d ed.), Group C, pp. 61–127; *ibid.*, Group D, pp. 30, 46, 53, etc.; Lee's Tables (3d ed.), pp. 152–59; Williamson's On the Use of the Barometer, pp. 1–64 of the appendix.

390. Temperature of the Air. The term $(1 + c\,T)$ is frequently called the temperature term. The name is not fortunate, since some barometric leveling formulas have also a term to correct for the difference of temperature of the barometers (§ 389).

The co-efficient c is equal to 0.003,75 per degree centigrade. It is usually approximated at 0.004, which makes some allowance for the greater expansive power of the watery vapor contained in the atmosphere.

The mean temperature of the layer of air between the two stations is usually assumed to be equal to the mean of its temperature at the two stations. Under this assumption, which however may be greatly in error (see §§ 357–58), $T = \frac{1}{2}(T_1 + T')$, and the temperature term becomes

$$1 + \frac{2(T' + T_1)}{1{,}000} \text{ for centigrade degrees,}$$

and

$$1 + \frac{T' + T_1 - 64}{900} \text{ for Fahrenheit degrees.}$$

391. Latitude Term. The term $(1 + 0.002{,}60 \cos 2\phi)$ of equation (11), page 332, is known as the latitude

term. A few formul as still contain an older and less accurate co-efficient of $\cos 2\phi$ than the above. However, this correction is unimportant, for even at the equator or the poles, where this term is a maximum, it is only 0.002,60 of the difference of elevation. Furthermore, in comparison with the possible errors due to gradient, temperature, and observation (see §§ 349–67), the error caused by the omission of the latitude term is wholly inappreciable.

392. Altitude Term. The last term of equation (11), page 332, takes account of the variation of gravity with the altitude. This term is always relatively small, and may be omitted without materially affecting the result. Furthermore, owing to the matters considered in §§ 349–67, as well as the fact that the barometer at best can be read only to a thousandth of an inch (which corresponds to 1 foot of altitude), the appearance of extreme accuracy by retaining the altitude and latitude terms can be regarded only as a mathematical illusion, inapplicable to ordinary practice.

393. Humidity Term. In deducing equation (11), page 332, it was assumed that the atmosphere was composed exclusively of dry air; but it is really a mixture of air (oxygen and nitrogen), watery vapor, and carbonic acid. The carbonic acid is very small and nearly constant, and hence need not be considered here; but the watery vapor is both large and variable. If dry air and aqueous vapor had even nearly the same density under the same conditions, the presence of the latter would not affect the problem; but as watery vapor is only five eighths as dense as dry air, the weight of a unit of volume of the atmosphere will depend upon the amount of vapor which it contains. Accordingly a humidity term has been introduced into some barometric formulas.

The introduction of a humidity term requires that

the hygrometric state of the air column shall be known, and therefore an observation with a hygrometer must be made at each station. For this purpose the wet bulb hygrometer, or psychrometer, is generally preferred, because of its greater accuracy and convenience. It consists of a pair of accurate thermometers, one of which is exposed to the air in the usual manner, while the other is exposed with a moistened bulb. The evaporation of moisture from the surface of the wetted bulb has a cooling effect, and causes that thermometer to indicate a lower temperature than the other.

Knowing the readings of the wet and dry bulb thermometers, the barometric pressure due to the aqueous vapor in the air may be determined from tables,* which are the results of experiment.

"In a very general sense, in temperate climates near the sea level, the amount of vapor in the atmosphere is from 0.2 to 0.4 of an inch, or about one hundredth of the total barometric pressure."

394. The correction for humidity may be applied in either of two ways; viz., (1) the observed heights of the barometer may be corrected for the pressure of the aqueous vapor before substituting them in the formula; or (2) the observed heights may be used uncorrected, and the resulting altitude be multiplied by a factor—the humidity term of the barometric formula—to correct for the effect of the aqueous vapor in the atmosphere. For formulas containing a humidity term, see §§ 401–402. Laplace slightly increased the co-efficient of expansion of air (§ 396) to partially compensate for the greater expansive power of the aqueous vapor. This method of correcting for humidity is incorrect in principle, and

* Williamson's On the Use of the Barometer, Table C of the Appendix; Smithsonian Miscellaneous Collections, Vol. I, Guyot's Tables, Group B, pp. 46–72, and pp. 102–6; Lee's Tables (3d ed.), pp. 172–76.

may at times give rise to considerable error, particularly in case of an extremely dry or an extremely moist atmosphere, or when the temperature of the air is at or near 32° F.

395. "It is doubtful whether any considerable increase of accuracy is obtained by including a separate correction for the aqueous vapor. The laws of the distribution and transmission of moisture through the atmosphere are too little known, and its amount, especially in mountain regions, is too variable and depends too much upon local winds and local condensation, to allow a reasonable hope of obtaining the mean humidity of the layer of air between the two stations by means of hygrometrical observations taken at each of them. The observations for humidity are made in the stratum of air next to the surface of the earth, which probably contains the greatest amount of moisture, and which is therefore least representative of the layer of air between the two stations. At any rate, the gain, if there is any, is not sufficient to compensate for the extra trouble in making the observations and the undesirable complication of the formula." *

At best the determination of heights by a barometer is only an approximate method (§§ 368–69), and no additional instruments, nor any refinements of computation, can make it a comparatively accurate method of leveling; and therefore it is not thought wise to consider this phase of the subject further.

396. TYPICAL FORMULAS. Laplace's Formula. Laplace was the first to give a rational formula for determining heights by the barometer, and his formula has served as a basis for several others. It is, for Fahrenheit degrees,

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* Professor A. Guyot, in Smithsonian Miscellaneous Collections, Vol. I, Art. III, Group D, p. 33.

$$X \text{ ft.} = 60158.6 \log \frac{H_1}{H'} \begin{cases} \left(1 + \dfrac{T' + T_1 - 64}{900}\right) \\ (1 + 0.002{,}60 \cos 2\phi). \\ \left(1 + \dfrac{X + 52{,}252}{20{,}886{,}860} + \dfrac{Z}{10{,}443{,}430}\right). \end{cases} \qquad (13)$$

X in the last term is the value of the preceding part of the formula.

The last term of equation (13) is derived from the corresponding term of equation (11), page 332, as follows: Place $\log\left(1 + \dfrac{X}{R+Z}\right) = 0.43 \ldots \dfrac{X}{R+Z}$. Find $\log \dfrac{H_1}{H'}$ by omitting the terms in equation (11) after the brace and solving, calling $5.744 \dfrac{m_0}{a_0} = 60{,}158$. Substitute these results in the last term of equation (11), perform the operations indicated, and omit the terms containing R^2 in the denominator. See § 392.

The true co-efficient of expansion of air is 0.003,75 per 1° C., but Laplace increased it to 0.004 "to allow for the greater expansive force of the aqueous vapor in the atmosphere." * See §§ 394 and 395.

The second term in the last line of equation (13) is usually called "the correction for the variation of gravity on the mercury," and the third term in the same line "the correction for the variation of gravity on the air"; but an examination of the method by which the last term was deduced will show that this explanation of these terms is erroneous.

Numerous tables have been prepared to facilitate the application of the above formula; for example, see

* Prof. A. Guyot in Smithsonian Miscellaneous Collections, Vol. I, Art. III, Group D, p. 9.

Smithsonian Miscellaneous Collections, Vol. I, Art. III, Group D, Guyot's Table, pp. 35–48; *ibid.*, Loomis's Table, pp. 49–53; Loomis's Practical Astronomy, pp. 390–91.

Tables are useful where a great number of observations are to be reduced; but they generally contain an unnecessary number of figures, and hold forth a show of extreme accuracy which the nature of the observations can not justify.

397. Babinet's Formula. Babinet's formula* for Fahrenheit degrees is

$$X \text{ ft.} = 60{,}334 \log \frac{H_1}{H'}\left(1 + \frac{T' + T_1 - 64}{900}\right). \quad (14)$$

Notice that this formula has no term for the variation of gravity. It is sometimes claimed † that the barometric co-efficient is Laplace's (§ 396) increased to include the correction due to the variation of gravity with the altitude; but owing to the nature of the case this procedure is entirely indefensible

Searle's Field Engineering (p. 4 and pp. 307–9) contains tables to facilitate the application of this formula.

398. *Babinet's Approximate Formula.* If H' and H_1 do not greatly differ it can readily be shown that

$$\log \frac{H_1}{H'} = 2\frac{H_1 - H'}{H_1 + H'}M, \quad . \quad . \quad . \quad . \quad (15)$$

in which M is the modulus of the common system of logarithms. Making this substitution in equation (14) gives Babinet's approximate formula,

$$X \text{ ft.} = 52{,}406\frac{H_1 - H'}{H_1 + H'}\left(1 + \frac{(T' + T_1 - 64)}{900}\right). \quad (16)$$

* Guyot's Tables, Group D, p. 68.

† Prof. A. Guyot in Smithsonian Miscellaneous Collection, Vol. I, Art. III, Group D, p. 9.

The mathematical approximation involved in equation (15) is inappreciable for elevations less than 3,000 feet.

399. The following formula* is essentially the same as Babinet's approximate formula—equation (16)—except the value of the barometric co-efficient and the omission of the temperature term. Notice that the barometric constant of equation (17) is larger than that of equation (16), which was derived from a relatively large value, *i.e.*, that of equation (14). A similar formula is frequently given with slightly different constants; for example, see Lee's Tables (3d ed.), p. 151.

$$X \text{ ft.} = 54{,}500\frac{H_1 - H'}{H_1 + H'} \pm \frac{X}{200} \pm 10. \quad . \quad . \quad . \quad . \quad (17)$$

The last two terms of equation (17) are supposed to show the degree of reliance to be placed upon the result. Formulas (16) and (17) are much used with "compensated" aneroid barometers (see §§ 342–43).

400. Bailey's Formula. All the preceding formulas require that the barometric reading be reduced to 32° F. before being inserted in the formula; but in Bailey's formula, equation (18),† the readings are used without any reduction (see § 389).

$$X \text{ ft.} = 60{,}346 \log \frac{h_1}{h'} \frac{1}{1 + 0.0001(t_1 - t')} \left\{ \begin{array}{l} \left(1 + \dfrac{T' + T_1 - 64}{900}\right) \\ (1 + 0.002{,}695 \cos 2\phi) \end{array} \right\}, \quad (18)$$

* U. S. Coast Survey Report, 1876, pp. 352–53.
† Guyot's Tables, Group D, p. 69.

in which h_1 and h' are the observed readings of the barometers at the two stations, and t_1 and t' the temperatures of the barometer in Fahrenheit degrees.

Tables for the application of Bailey's formula are given in Lee's Tables (2d ed.), pp. 84, 85, and in Smithsonian Miscellaneous Collections, Vol. I, Art. III, Group D, pp. 70, 71.

401. Plantamour's Formula. None of the preceding formulas takes account of the humidity of the atmosphere, except by the erroneous method of increasing the co-efficient of expansion of air (§ 396). Bessel was the first to propose a separate humidity term. Plantamour's formula,* which differs from the one proposed by Bessel only in the form and the value of the constants, is

$$X \text{ (in meters)} = \log H_1 - \log H' + V + \frac{1}{1 - W\dfrac{s+s'}{\sqrt{H_1 H'}}} + q; \quad (19)$$

in which

$$V = \frac{398.25}{397.25 - cT}\,\frac{m_0}{a_0}\,(1 + cT),$$

$$W = \frac{0.348{,}07}{397.25 - cT}\,10^{0.030{,}197{,}5T - 0.000{,}080{,}170T^2},$$

$$q = 1 - 0.002{,}625{,}7 \cos\phi,$$

s and s' = the fraction of saturation of the layer of air between the two stations.

Tables for the application of this formula are given in Smithsonian Miscellaneous Collections, Vol. I, Guyot's Tables, Group D, pp. 78, 79.

* Guyot's Tables, Group D, p. 75.

402. Williamson's Formula. Williamson's formula,* equation (20), is a translation of Plantamour's formula into Laplace's form.

$$X \text{ ft.} = 60{,}384 \log \frac{H_1}{H'} \left\{ \begin{aligned} &\left(1 + \frac{T_1 + T' - 64}{982.2647}\right) \\ &(1 - 0.002{,}625{,}7 \cos 2\phi) \\ &\left(1 + \frac{X + 52{,}252}{20{,}886{,}860} + \frac{Z}{10{,}443{,}430}\right) \\ &(1 + M) \end{aligned} \right. \qquad (20)$$

in which $(1 + M)$ is the humidity term (§§ 393–95).

Williamson's On the Use of the Barometer—pages 111–55 of the Appendix—contains elaborate tables for the application of this formula. The same tables are also given in Lee's Tables (3d ed.), pp. 152–71.

403. Altitude Scales of Aneroids. The dials of many aneroid barometers have a scale engraved upon them, by which the elevation is read directly. The scale may be graduated according to any of the preceding formulas, but apparently equation (17), page 341, is most frequently employed for this purpose. Sometimes the zero of the altitude scale is placed to correspond with a pressure of 30 inches of the mercurial barometer, as though the scale of elevations could be employed to determine absolute elevations. But, obviously, the altitude scale can not give absolute elevations with any considerable accuracy, owing to the variations in pressure, temperature, and humidity (§§ 349–61); and consequently it should be used only to find *differences* of elevations, in which case the difference of level is obtained by subtracting

* Williamson's On the Use of the Barometer, p. 102 of the Appendix.

the reading in feet at the lower station from that at the upper. To prevent a misuse of the aneroid in this respect, the zero of the altitude scale is sometimes placed to correspond with 31 inches of pressure, which gives such large numbers as to suggest, particularly when finding small elevations, the proper use of the instrument.

Sometimes the altitude scale is engraved upon a movable ring encircling the dial, so that the scale may be set to agree with the known altitude of any station (see § 343, paragraph 7); and then, as the instrument is carried about, the pointer will indicate with a fair degree of approximation, for some hours, the altitude of stations at which it is read. This device is convenient, and in the hands of an intelligent observer who requires rapid work and desires only approximate results, it is a valuable modification; but such a use of the movable scale may at times involve large errors, as it is based on the assumption that differences of pressure correspond at all heights with the same differences of elevation.

404. The altitude scale can be correct only for some particular temperature of the air; and consequently, if the most accurate results are desired, a correction for the temperature of the air (§ 390) must be applied. With some instruments this correction is made by moving the altitude scale. In this case the scale is graduated for some particular temperature, its zero is adjusted by reading the instrument at some station of known elevation, and this position of the zero is marked by a line on the dial numbered to correspond with the normal temperature; then points are determined—to quote the words of the inventor of this method of applying this correction,—"partly by trial and partly by computations," at which the zero of the altitude scale should be set for different temperatures of the air. With this form of instrument, the scale must be set for the mean

temperature of the air, before any readings are taken; and it must not be shifted during the progress of the work.

The method of applying the correction for the temperature of the air by shifting the altitude scale, is not theoretically correct; and the accuracy of the results depend entirely upon the accuracy of the scale by which the altitude scale is shifted. The best plan is to dispense entirely with altitude scales, whether fixed or movable, and calculate the heights.

405. Conclusion. The preceding do not comprise all the formulas which have been proposed for barometric leveling, but include the more common ones, and illustrate all the principles involved—except those discussed in §§ 406–9. Some of the omitted formulas are approximate, some have empirically determined pressure co-efficients, etc. Owing to the matters discussed in §§ 349–67 it is comparatively unimportant which of the generally recognized barometric formulas is used.

B. *Dynamical Formulas.*

406. Ferrel's Formula. Ferrel has deduced a formula* which is claimed to be independent of the defects of all formulas founded upon an assumed statical condition of the atmosphere. Although the formula is very carefully and ingeniously worked out, it is probably of little use for ordinary hypsometrical work, since it requires observations to be made for a long time over a considerable area, to get the data by which to compute corrections for gradient, temperature, and humidity. Without the data for making these reductions, this formula is essentially the same and has essentially the

* Report of U. S. Coast and Geodetic Survey, 1881, pp. 225–68, the formula itself being given on page 235.

same defects as the formulas depending upon an assumed statical condition of the atmosphere.

407. Gilbert's Formula. This formula, unlike all those in division *A* (§§ 383–405), does not require the observation of either the temperature or humidity of the air; but requires observations of the barometer at three stations, the difference of elevation of two of which is known and the elevation of the third of which is to be determined. The formula is

$$X \text{ ft.} = B\frac{\log l - \log n}{\log l - \log u} + \frac{A(B-A)}{490{,}000}, \quad . \quad . \quad (21)$$

*

in which B is the difference in elevation of the two stations, l is the reading of the barometer at the lower station, reduced to 32° F., u the same for the upper station, and n the same for the station whose elevation is to be determined, and A is the value of the first term of the second member. This formula is deduced on the assumption that the three stations are in the same vertical, and also that the station whose altitude is to be determined is between the other two. The less nearly these conditions are fulfilled, the more nearly this formula is liable to the same errors as statical formulas. The denominator of the last term was determined by experiment, and is therefore liable to error. See § 380.

408. Weilenmann's Formula. The Report of the U. S. Coast and Geodetic Survey for 1876, pages 388–90, contains a comparatively brief discussion of a barometric leveling formula based upon the dynamical theory of heat. In the *one* case to which this formula is applied, it gives errors one half less than the best of the ordinary barometric formulas; but as the errors in the

* G. K. Gilbert in U. S. Geological Survey Report for 1880–81, p. 448.

single example to which it is applied, due to diurnal variations of pressure, amount to one per cent, and as it is much more complicated than the ordinary formulas, it will not be considered further.

409. Conclusion. Owing to the possibility of errors due to gradient, temperature, and humidity (§§ 348–67), there is but little difference in accuracy between statical and dynamical formulas. See § 381.

APPENDIX I.

ELIMINATION OF LOCAL ATTRACTION IN SURVEYING WITH THE MAGNETIC COMPASS.

1. It is very common to hear the remark that the magnetic compass can not be used because of local attraction. It is well known that there are many localities in which the needle is deflected by the attraction of iron, etc.; but the object of this article is to show that no matter how much the local deflection, the common magnetic compass can be made to give as accurate results as though there was no disturbing influence.

Since the conditions for mine surveying differ slightly from those for land surveying, this subject will be considered under the heads of Mine Surveys and Land Surveys.

Art. 1. Mine Surveys.

2. Generally a survey of a mine is made to determine the relative position of the underground workings and the boundary lines of surface property. Consequently it is necessary to find in the mine (1) a point directly under a known point on the surface, and (2) the direction of a line corresponding to a known line on the surface. The first may be found by means of a plumb-line, or its equivalent; and the second by the use of two plumb-lines, or by a transit—usually the former. For the present it will be assumed that the true direction of the initial underground line is known.

For the sake of illustration, assume that a survey is

to be made, starting from some point, say *A*, on a line whose direction is, say, S. 78° 10′ E. Set the compass at *A*, and also at each corner or station of the survey, successively, and read the bearing of the two lines meeting in each station. Call the sight made in the direction in which the survey progresses a fore-sight, and that made in an opposite direction a back-sight. Keep one end of the box next to the eye on fore-sights and the other end on back-sights. But if one sight of the compass consists of a slit and the other of a hair, the same end must necessarily be kept next to the eye; therefore read the letters from one end of the needle for fore-sights, and from the opposite end for back-sights, but the degrees from the same end all the time. When all the bearings have been taken, the record will be similar to the first three columns of the following table, with the exception of the small figures written above each bearing, which will be explained presently.

TABLE I.

Station.	Back-sight.	Fore-sight.	Correction.
A	S. $\overset{78}{78^\circ}\ \overset{10}{00'}$ E.	N. $\overset{16}{15^\circ}\ \overset{05}{55'}$ W.	10′ B.
Q	N. $\overset{16}{16^\circ}\ \overset{05}{45'}$ W.	N. $\overset{80}{81^\circ}\ \overset{30}{10'}$ W.	40′ F.
S	N. $\overset{80}{80^\circ}\ \overset{30}{05'}$ W.	S. $\overset{46}{46^\circ}\ \overset{10}{35'}$ W.	25′ B.
U	S. $\overset{46}{46^\circ}\ \overset{10}{35'}$ W.	S. $\overset{83}{82^\circ}\ \overset{00}{35'}$ E.	25′ B.
V	S. $\overset{83}{82^\circ}\ \overset{00}{25'}$ E.	S. $\overset{78}{77^\circ}\ \overset{00}{25'}$ E.	35′ B.

As explained above, the true bearing of the initial or reference line is S. 78° 10′ E., and hence, if there had been no local attraction at *A*, the back-sight at that station, *i.e.*, the bearing of the reference line, would have been S. 78° 10′ E. But as it read S. 78° 0′ E., there is a local attraction of 10′, and therefore the bearings

taken at *A* are in error 10′. The needle read S. 78° 0′ E., but if there had been no local attraction it would have read S. 78° 10′ E. The true reading is therefore 10′ farther from the south toward the east than the observed reading, and hence, to get the true bearing the needle should be moved 10′ in a direction from the south towards the east. The fore-sight at *A* is N. 15° 55′ W.; but to get correct bearings at this station, we must, in imagination, move the needle 10′ in a direction from the south towards the east, which is the same direction as from the north towards the west, and hence, the correct bearing of the line starting from *A* is N. 15° 55′ W., plus 10′, which gives N. 16° 5′ W.

For simplicity of explanation, let us call a direction from the south towards the west *forward*, and the opposite direction *backward*, these terms being assigned to agree with the direction of the motion of the hands of a watch. It makes no difference in this matter whether we consider N., S., E., and W. in their true relations, or in their reversed position as given by the face of the compass. We will assume them to be in their true relations, and briefly say that the correction at *A* is 10′ backward. This is the correction which is to be applied to the fore-sight at *A*, and is written in the column headed "correction." The true bearing of the initial line is written over the back-sight taken at *A*, and the corrected fore-sight is written above the fore-sight observed at *Q*.

At *Q* the back-sight, *i.e.*, the bearing of the line from *A* towards *Q*, is N. 16° 45′ W.; but as the true bearing is N. 16° 5′ W., as found for the corrected fore-sight at *A*, the error at *Q* due to local attraction is 40′. The correction at *Q* is therefore 40′ forward, and the corrected fore-sight is 80° 30′. The corrections and corrected bearings for the other stations are found in a similar manner.

3. As is easily seen, when the true bearing of the initial line is known, the above method absolutely eliminates all local attraction. On the other hand, if the direction of the initial line is not known, this method will give a fairly good determination of it. To find the true bearing of all of the lines, including that of the first one, set the compass at each station, and read the back-sights and fore-sights at each as in the preceding case. The record will then be as in Table II, except for the small figures above the bearings.

TABLE II.

Station.	Back-sight.	Fore sight.	Correction.
A	S. $\overset{78}{77}^\circ\ \overset{10}{50}'$ E.	N. $\overset{16}{15}^\circ\ \overset{05}{45}'$ W.	20′ B.
Q	N. $\overset{16}{16}^\circ\ \overset{05}{35}'$ W.	N. $\overset{80}{81}^\circ\ \overset{30}{00}'$ W.	30′ F.
S	N. 80° 30′ W.	S. 46° 10′ W.	0
U	S. 46° 10′ W.	S. 83° 00′ E.	0
V	S. 83° 00′ E.	S. 78° 00′ E.	0
W	S. $\overset{78}{78}^\circ\ \overset{00}{10}'$ E.	N. $\overset{85}{85}^\circ\ \overset{35}{25}'$ E.	10′ F.

Notice that the fore-sight at *S* agrees with the back-sight at *U*, and also that the fore-sight at *U* agrees with the back-sight at *V*. This makes it extremely probable that there is no local attraction at these three stations, and that the correction at these stations is 0°. To find the correction at *Q*, write the back-sight at *S* above the fore-sight at *Q*, and take the difference between the observed and the corrected fore-sight. This difference is 30′, and is the correction at *Q*. A moment's reflection shows that this correction is "forward," as marked. The remaining bearing at *Q* and the two at bearings *A*

may be found as already explained, and similarly those at W; that is to say, the bearings in the upper part of the table are corrected by working up from S, and those in the lower part by working down from V.

4. It may happen that not even two successive stations are found at which the corresponding back-sight and fore-sight agree. When this occurs, assume, *for temporary purposes*, that the correction at the first station is zero, and correct all the bearings accordingly. The result will then be as in Table III.

TABLE III.

Station.	Back-sight.	Fore-sight.	Correction.
A	S. 78° 00′ E.	N. 75° 5′ E.	0
P	75 55 N. 75° 35′ E.	80 0 N. 80° 10′ E.	20′ F.
R	80 20 N. 80° 10′ E.	85 30 S. 85° 40′ E.	10′ F.
T	85 30 S. 85° 50′ E.	88 50 N. 88° 30′ E.	20′ F.
X	88 50 N. 88° 20′ E.	78 00 S. 78° 30′ E.	30′ F.
Y	78 00 S. 78° 20′ E.	85 20 N. 85° 00′ E.	20′ F.

Notice that there are three stations at which the correction is the same, *i.e.*, 20′ F. This indicates that the apparent local attraction at these stations is the same; it is therefore *probable* that the needle had its normal or natural position at these stations, and that the local attraction was confined wholly to the other stations. If this conjecture be correct (its validity will be examined farther presently), then the corrections at P, T, and Y are zero. By inserting this correction at these stations, and correcting the notes as previously explained, we get Table IV, which gives the *most probable* bearings of the several lines.

5. For the preceding illustration the conjecture is slightly defective, since the agreement of the corrections at only three stations might possibly be due to an acci-

TABLE IV.

Station.	Back-sight.	Fore-sight.	Correction.
A	78 20 S. 78° 00′ E.	75 35 N. 75° 55′ E.	20′ B.
P	N. 75° 35′ E.	N. 80° 00′ E.	0
R	00 N. 80° 10′ E.	85 50 S. 85° 40′ E.	10′ B.
T	S. 85° 50′ E.	N. 88° 30′ E.	0
X	88 30 N. 88° 20′ E.	78 20 S. 78° 30′ E.	10′ F.
Y	S. 78° 20′ E.	N. 85° 00′ E.	0

dental agreement of the local attraction at these stations. If this is the case, then it is impossible by this method to find the true bearings of the lines, unless proportionally more stations can be found at which the apparent local attractions agree. For example, if the next five stations give an apparent local attraction agreeing with that found in Table III for station *X*, then those stations must be regarded as having the normal position of the needle, and the bearings of the others must be corrected accordingly. On the other hand, if it is impossible to find a number of stations at which the apparent local attractions agree, it is impossible to determine the true bearings with the needle alone. What then?

If the true direction of the initial line has not been determined by plumbing the shaft, and if the apparent local attraction does not agree at a proportionally large number of stations, observe the back-sights and fore-sights as previously described, and correct the bearings with reference to the initial line as explained in Table III. Of course this method does not certainly deter-

mine the true bearings, but it finds correctly the relative directions of the several lines, and all the computations and plats will be perfectly correct, except that they will be turned around an amount equal to the difference between the assumed and the true bearing of the initial line. If the survey is made to determine the position of the various parts of the mine relative to the land lines above, then in many cases, particularly in the coal mines of Illinois, this method would give fairly good results, since the error would probably be slight, while the value of the vein would be nearly uniform. Farther, if at any subsequent time the true direction of the initial line should be determined, then the correct bearings of all the lines of the underground survey could be determined at once by applying, as a "correction," the difference between the true and the assumed direction of the first line.

6. Finally, it may happen that the apparent local attractions as found by the trial balance, Table III, preponderate in one direction. For example, in Table III it will be noticed that all the corrections are "forward" except that at *A*. As a general rule local attraction is as likely to be in one direction as another, and hence, *unless the local conditions furnish some good reason to the contrary*, the corrections found by the trial balance should be about as much in one direction as the other, *i.e.*, the sum of the "forward" corrections should be nearly equal to the sum of the "backward" corrections. This is not so in Table III, which is an actual survey. The local conditions gave no indications that the local attraction was more in one direction than another, and we may reasonably assume that the corrections are all one way in the table because the true correction at *A* is backward an amount about equal to the average of the corrections at the other stations, *i.e.*, about 18′. If this correction were applied to the original bearings in

Table III, the resulting directions would be nearly the true ones; but if the correction at any station is considerably larger than any of the others, it is evident that the local attraction at this station is considerably more than the average, and hence this method of correcting the bearings should not be employed,—at least the excessive correction should not be included in finding the average.

In Art. 2 a modification of the preceding methods will be considered, which, under certain conditions, may be of great importance in making mine surveys.

ART. 2. LAND SURVEYS.

7. Land surveys differ from mine surveys in that the former close, *i.e.*, return to the point of beginning, while the latter, as a rule, do not. When the survey closes, the above method of observing and correcting the bearings possesses, in addition to finding the true bearings as explained above, two important advantages: 1, it affords under every condition the means of checking the accuracy of the bearings; 2, it enables the true area to be determined, even if the true bearing of the sides can not be found. There are then two cases, I, when only the area is wanted; II, when the area and also the true bearings are required.

CASE I. *When only the area is wanted.*

8. The angles are read as described in Art. 1, the results being as in Table V (page 357).

Since the assumption is that only the area is required, it is immaterial where we commence to correct the angles; and therefore we may assume that the bearings taken at any station, say Z, are correct, and that the correction at Z is zero. Beginning then at Z, we may correct the bearings by the method already ex-

plained. It is immaterial whether the angles are corrected in the order in which they were surveyed, or the opposite.

TABLE V.

Stations.	Back-sight.	Fore-sight.	Correction.
Z	S. $21^\circ\ 00'$ W.	N. $6^\circ\ 55'$ W.	0
Y	N. $\overset{6}{7}{}^\circ\ \overset{55}{05}{}'$ W.	N. $\overset{55}{55}{}^\circ\ \overset{25}{35}{}'$ W.	10′ F.
X	N. $\overset{55}{55}{}^\circ\ \overset{25}{35}{}'$ W.	S. $\overset{36}{36}{}^\circ\ \overset{30}{20}{}'$ W.	10′ F.
W	S. $\overset{36}{36}{}^\circ\ \overset{30}{50}{}'$ W.	S. $\overset{2}{2}{}^\circ\ \overset{20}{00}{}'$ E.	20′ B.
U	S. $\overset{2}{2}{}^\circ\ \overset{20}{40}{}'$ E.	S. $\overset{43}{43}{}^\circ\ \overset{00}{20}{}'$ E.	20′ F.
T	S. $\overset{43}{43}{}^\circ\ \overset{00}{05}{}'$ E.	S. $\overset{69}{69}{}^\circ\ \overset{25}{30}{}'$ E.	5′ F.
S	S. $\overset{69}{69}{}^\circ\ \overset{25}{30}{}'$ E.	N. $\overset{21}{21}{}^\circ\ \overset{5}{00}{}'$ E.	5′ F.

It will be noticed in the example cited that the back-sight at *Z* differs from the corrected fore-sight at *S*. If the angles had been correctly read, these two would have agreed exactly. The difference between the first corrected back-sight and the last corrected fore-sight shows the error of reading the angles. This difference should not exceed 5′ or 10′ (see § 50, page 52). (The example in Table V is an actual survey made by one of the author's students.)

CASE II. *When the area and also the true bearings are required.*

9. This case requires one of two things: either that several successive back-sights shall agree with their corresponding fore-sights before either have been corrected, or that there shall be a true meridian which can be connected with a corner of the field by lines whose

bearings are to be found in the same manner as are those of the boundary lines.

10. In the first instance it may safely be assumed that, since a number of the fore-sights and back-sights agree, the observed bearings are the true ones. Having some of the correct bearings, if there are any stations at which the back-sights do not agree with the corresponding fore-sights, they may be corrected as in Table III. If the declination is not set off on the compass, it may be applied as a correction to those stations at which the back-sight and fore-sight corresponded as above, and the correction may then be carried on to the remaining stations.

11. In the second instance, the true bearings of the several lines can be found in succession, beginning at the meridian. The example in Table VI will illustrate

TABLE VI.

Stations.	Back-sight.	Fore-sight.	Correction.
Z		N. $\overset{73}{69}^\circ\ \overset{10}{20}'$ E.	3° 50′ F.
Q	N. $\overset{73}{69}^\circ\ \overset{10}{15}'$ E.		3° 55′ F.
Q	N. $\overset{14}{10}^\circ\ \overset{40}{45}'$ E.	S. $\overset{93}{89}^\circ\ \overset{40}{45}'$ W.	3° 55′ F.
T	S. $\overset{93}{89}^\circ\ \overset{40}{35}'$ W.	S. $\overset{43}{39}^\circ\ \overset{05}{00}'$ W.	4° 05′ F.
U	S. $\overset{43}{39}^\circ\ \overset{05}{10}'$ W.	N. $\overset{14}{10}^\circ\ \overset{35}{40}'$ E.	3° 55′ F.

this case. *Z* is not a corner of the field, but a point on a true meridian, the line *ZQ* being run to determine the declination at *Q*, a corner of the field. All the remaining stations are corners of the field.

Notice that the first two lines of the above record are preliminary to the survey of the field, and are required

only to find the declination or correction at *Q*. Having found the correction at *Q*, the bearings are corrected as in the previous case. The back-sight from *Q* to *U* differs 5′ from the fore-sight from *U* to *Q*, which shows that there was an error or inaccuracy of 5′ in reading the angles.

12. Undoubtedly the method of connecting with a meridian is more exact than the previous one, although the former is more convenient and shorter, and is the one which will generally be used in practice. However, if the bearings show local attraction at a number of stations, the first method of Case II can not be applied, while the second will give strictly correct results whatever the number of stations at which local attraction exists.

Notice that when the survey closes, the method explained above absolutely eliminates all local attraction, and also that this method is frequently applicable in mine surveying. Finally, the back-sight should always be taken, even though it is probable that no local attraction exists, since the back-sights afford a very valuable means of checking the readings of the needle.

APPENDIX II.

FINDING AREA BY TRAVERSING WITH THE TRANSIT.

1. Taking the Field Notes. Assume that not only the area is desired, but also the angles which the several sides make with the meridian. Place the instrument over the first station, say *A* in Fig. 86, and make the vernier read zero. Direct the telescope along the meridian, clamp the instrument, take a back-sight

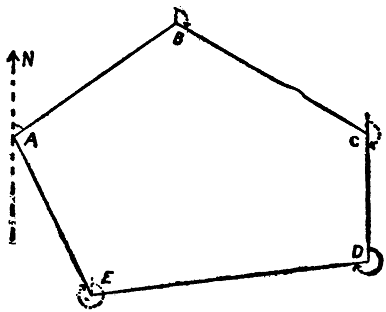

Fig. 86.

upon the last station, and proceed around the field according to the process described in § 137. On getting to the last station and looking back to the first, the fore-sight should be the same as the back-sight from the first station. Fig. 86 and Table I (page 361) mutually explain each other.

The angles in the column headed fore-sights are the azimuths or angles which the several courses make with

the reference line, in this case the meridian, and are marked in Fig. 86. If only the area is desired, it is immaterial whether the reference line be a meridian or not.

TABLE I.

FIELD NOTES FOR TRAVERSE SURVEYING.

Sta	Back-sight.	Fore-sight.	Dist.	Remarks.
A	320° 19′	63° 48′		Vernier *A* read, zero being set on true meridian.
B	63° 48′	123° 21′		Azimuths counted from north toward east.
C	123° 21′	180° 00′		
D	180° 00′	260° 35′		
E	260° 35′	320° 19′		

It makes little or no difference which way the surveyor goes around the field.

2. Computing the Area. In computing the area by the common or Rittenhouse method, there is no check upon the work after the latitudes and departures have been balanced. The principal object of the method explained below is to overcome this defect.

With the following explanation of the nomenclature, the method can readily be understood by an inspection of Tables II and III, pages 362 and 363.

A_1, A_2, A_3, etc., are the azimuths of the several sides, respectively.

C_1, C_2, C_3, etc., are the lengths of the several sides, respectively.

x_1, x_2, x_3, etc., are the latitudes of the several sides, respectively.

y_1, y_2, y_3, etc., are the departures of the several sides, respectively.

X_1, X_2, X_3, etc., are the total latitudes of the several corners; *i.e.*, the total distance which any corner is north or south of some other corner which is assumed as an initial one for the purposes of computation.

TABLE II.

Symbolic Representation of the Method of Computing Areas in Traverse Surveying.

Azimuth.	Course.	Latitude.	Departure.	Total Latitude.	Double Area.	
$A.$	$C.$	$x = C \cos A.$	$y = C \sin A.$	$X = \Sigma x.$	1st Method.	2d Method.
A_1	C_1	$x_1 = C_1 \cos A_1$	$y_1 = C_1 \sin A_1$	$X_1 = x_1$	$X_1(y_1 + y_2)$	$y_1 X_1$
A_2	C_2	$x_2 = C_2 \cos A_2$	$y_2 = C_2 \sin A_2$	$X_2 = X_1 + x_2$	$X_2(y_2 + y_3)$	$y_2(X_1 + X_2)$
A_3	C_3	$x_3 = C_3 \cos A_3$	$y_3 = C_3 \sin A_3$	$X_3 = X_2 + x_3$	$X_3(y_3 + y_4)$	$y_3(X_2 + X_3)$
A_4	C_4	$x_4 = C_4 \cos A_4$	$y_4 = C_4 \sin A_4$	$X_4 = X_3 + x_4$	$X_4(y_4 + y_5)$	$y_4(X_3 + X_4)$
etc., to	etc., to	etc., to	etc., to	etc., to	etc., to	etc., to
A_n	C_n	$x_n = C_n \cos A_n$	$y_n = C_n \sin A_n$	$X_n = 0$	$X_{n-1}(y_{n-1} + y_n)$	$y_n X_{n-1}$

Total Departure.	Double Area.	
$Y = \Sigma y.$	3d Method.	4th Method.
$Y_1 = y_1$	$- Y_1(x_1 + x_2)$	$- x_1 Y_1$
$Y_2 = Y_1 + y_2$	$- Y_2(x_2 + x_3)$	$- x_2(Y_1 + Y_2)$
$Y_3 = Y_2 + y_3$	$- Y_3(x_3 + x_4)$	$- x_3(Y_2 + Y_3)$
$Y_4 = Y_3 + y_4$	$- Y_4(x_4 + x_5)$	$- x_4(Y_3 + Y_4)$
etc., to	etc., to	etc., to
$Y_n = 0$	$- Y_{n-1}(x_{n-1} + x_n)$	$- x_n Y_{n-1}$

TABLE III.

Example of the Method of Computing Areas in Traverse Surveying.

Field Notes.			Latitude.	Departure.	Area by 1st Method.			Area by 2d Method.		
Sta.	Azimuth.	Dist.	+ S.	+ W.	T. L.	A. D.	Double Area	A. T. L.	D.	Double Area.
R	S. 283° 01 W.	241.83	− 54.45	+ 235.62	− 54.45	+ 62.88	− 3423.75	− 54.45	+ 235.62	− 12829.06
T	47° 59′	232.50	− 155.60	− 172.74	−210.05	−233.88	49127.20	− 264.50	− 172.74	45689.02
G	68° 35′	65.68	− 23.98	− 61.14	−234.03	− 51.04	11944.71	− 444.08	− 61.14	27153.07
Z	354° 26′	104.17	− 103.67	+ 10.10	−337.70	−141.61	47822.71	− 571.73	+ 10.10	− 5777.34
K	112° 24′	164.10	+ 62.55	− 151.72	−275.15	− 11.84	3257.50	− 612.85	− 151.72	92980.38
N	206° 57′	308.64	+ 275.15	+ 139.88	00.00		00.00	− 275.15	+ 139.88	− 38487.71
	Error of Closure = 1′.		Cor. per 100 ft. = 0.090.	Cor. per 100 ft. = 0.000	Total double area, in sq. ft. . .		108728.37	Total double area, in sq. ft.		108728.36
					Single area, in acres, . . .		1.2480298	Single area, in acres .		1.2480298

Y_1, Y_2, Y_3, etc., are the total departures of the several corners.

Σ represents the sum of the quantities to which it is prefixed.

n is the total number of sides.

T. L., in the heading of Table III, stands for *total latitude*, and *A. T. L.* for *adjacent total latitude*, that is for X_1, $X_1 + X_2$, $X_2 + X_3$, etc.

A. D., in Table III, represents the sums $y_1 + y_2$, $y_2 + y_3$, etc., of Table II, which are called *adjacent departures.*

In computing the latitudes and departures, attention must be given to the signs of the trigonometrical functions. The latitudes and departures are to be balanced as in the method of ordinary land surveying In applying this method, it will be necessary to prepare a column in which to write the adjacent departures—$y_1 + y_2$, $y_2 + y_3$, etc.,—and another in which to write the sums of the adjacent total latitudes—$X_1 + X_2$, $X_2 + X_3$, etc. These columns, and two similar ones for the third and fourth methods, were omitted from Table II for convenience in printing.

If only two computations of the area are to be made (two will generally give a sufficient check), it is shorter to use either the first and second method or the third and fourth, than to pair them differently. For simply a numerical check, the signs of the area may be disregarded; but if the signs in the table are conformed to, the areas will agree in sign as well as in amount.

Notice that in the first method, each partial area is obtained by multiplying the total latitude of any corner by the sum of the departures of the adjacent sides, or briefly, each partial area is the product of the total latitude and the sum of the adjacent departures; and note also that in the second method each partial area is the product of the departure and the sum of the adjacent total latitudes. Note further, that the third

and fourth methods are the same as the first and second excepting the substitution of latitudes for departures. The fourth method is the usual one of double meridian distances, except that the former deals with the co-ordinates of the corners of the field instead of the middle of the sides.

The example shown in Table III is a problem solved by one of the author's students in the ordinary class work. For reasons not necessary to explain, the computations are carried to an unusual, and ordinarily an indefensible, number of decimal places.

APPENDIX III.

PROBABLE ERROR.

1. All quantities determined by observation are subject to error, and any one who deals with such quantities should recognize the certainty of error in his data. This limitation is frequently disregarded—as, for example, when, in a report which recently came under the author's eye, the cost per cubic yard of concrete was figured out to the thousandth of a cent. Other examples could be cited in which the result, although less ridiculous, involved vastly greater consequences. The object of this discussion is to present some of the elementary principles involved in the comparisons of results derived from observation.

All observations are liable to three classes of error; viz., mistakes, constant errors, and accidental errors. (1) Mistakes are errors due to inexperience, to lack of care, to mental confusion, etc.; as, for example, reading 28° instead of 32°, or reading the wrong vernier, or counting from the wrong end of the tape. Frequently such errors may be corrected by comparison with other observations. (2) Constant errors are those produced by well understood causes—as, for example, the chain may be too long, or it may be used at a temperature differing from that at which it is of standard length, or there may be phase in the target sighted at, etc. Such errors can always be eliminated by the application of computed corrections, and, strictly speaking, are not errors at all. (3) Accidental errors are those still remaining after all evident mistakes and all constant

errors have been eliminated—as, for example, the errors in leveling due to a movement of the bubble after its inspection, to an imperfect bisection of the target, to an inclination of the rod, etc. Only the last class of errors will be considered here.

To many persons it seems strange, and even improper, to speak of the law of irregular and unknown errors; but it is nevertheless true that even accidental errors follow a rigorous mathematical law—the law of probability.

The whole theory of probable error depends upon the three following theorems, derived from experience:

1. Small errors are more frequent than large ones.
2. Positive and negative errors are equally frequent.
3. Very large errors do not occur.

2. The probable error is such a quantity that there is an even chance that the real error is greater or less than it. Or, in other words, if the errors of a series of observations were arranged in the order of their magnitude, the middle error in that series would be the probable error. Notice that the probable error would occur in the middle of the series, but would be less than the mean of the errors, because small errors are more likely to occur than large ones. Or, again, the probable error is taken by mathematicians as the limit within which it is as likely as not that the truth will fall. Thus, if 5.45 be the mean of a number of determinations, and 0.20 be the probable error, then the true result is as likely to lie between 5.25 ($= 5.45 - 0.20$) and 5.65 ($= 5.45 + 0.20$) as to lie outside of these limits. The probable error is usually represented by writing it after the number, but preceding it by the sign indicating *plus or minus*. For example: 5.45 ± 0.20 indicates that 5.45 is the mean of the observed quantity, and that 0.20 is the probable error of this value.

3. When a number of separate observations of any kind have been made with equal care, the mean or average of them all is the most probable value of the quantity sought, and is considered the true value.

Let n = the number of the observations.

d = the difference between any one observation and the mean of the observations. (The quantities represented by d are generally called residuals.)

E_1 = the probable error of a single observation.

f = the mean of the errors.

E_m = the probable error of the mean of all the observations.

q = 0.6745—a constant determined by computations according to the theory of probability.

Σ = a symbol signifying *sum of.*

Then

$$E_1 = q\sqrt{\frac{\Sigma d^2}{n-1}} \quad \ldots\ldots \quad (1)$$

$$E_m = \frac{E_1}{\sqrt{n}} \quad \ldots\ldots\ldots \quad (2)$$

$$E_1 = 0.8453f, \text{ approximately.} \quad . \quad (3)$$

Less labor is required in applying equation (3) than in using (1). If the number of observations were infinite, the two would give exactly the same results.

Mathematicians agree far better as to the form of the law of error, and also as to the method of computing the probable error, than they do as to the manner in which the law can be deduced. Therefore no demonstration will be attempted; but to prevent misapprehension it may be well to remark that the law as stated above has frequently been tested and found to agree

very closely with experience. It should be borne in mind that the method is grounded upon the hypothesis that a large number of observations have been taken. However, when only a limited number of observations have been made, the probable error, when computed according to the above formulas, is sufficiently exact for all purposes of comparison. It should not be forgotten that the probable error can be computed legitimately only after all constant errors have been corrected. Nor should it be forgotten that the probable error considers only errors that are as likely to occur on one side as on the other.

4. Tables I and II (pages 370 and 371) will illustrate the method of applying the preceding formulas. The observations were made by leveling up the instrument, sighting the target, and then reading the rod. The instrument was then disleveled, the target was moved, and another observation was made. The observations as recorded were consecutive, *i.e.*, no poor ones were thrown away. The observations were made by one of the author's students in ordinary class-work, and are fairly representative of what an inexperienced but very careful man can do under favorable conditions as to wind, light, etc., with an ordinary wye level.

Incidentally the tables give some data concerning the relative accuracy of two forms of level targets (§ 268), and also show the effect of increasing the length of sight. All the observations were made by the same man on the same day. For the distances in the table, the error of setting the target seems to vary about as the square root of the length of sight, but the error probably decreases with the distance for a time and then increases. Notice that the probable error of setting the quadrant target (the one generally used in practice) at 300 feet (about the ordinary distance) was 0.002 ft. Under the usual conditions this error would

TABLE I.

Error of Reading Level Target.

Rod 100 *Feet from Instrument.*

Quadrant Target.			Diamond Target.		
Reading in feet.	d in thousandths.	d^2.	Reading in feet.	d in thousandths.	d^2.
3.169	0	0	3.172	1	1
3.170	1	1	3.174	1	1
3.170	1	1	3.174	1	1
3.170	1	1	3.172	1	1
3.169	0	0	3.173	0	0
3.168	1	1	3.173	0	0
3.171	2	4	3.173	0	0
3.168	1	1	3.173	0	0
3.169	0	0	3.175	2	4
3.168	1	1	3.174	1	1
3.169	0	0	3.173	0	0
3.170	1	1	3.174	1	1
3.169 mean	0.75 mean	11 sum	3 173 mean	0.67 mean	10 sum

Quadrant Target:

Prob. error of single obs.

$$= 0.67 \sqrt{\frac{11}{11}} = 0.00067 \text{ ft.}$$

Approx. prob. error of single obs.

$$= 0.84 \times 0.75 = 0.00063 \text{ ft.}$$

Prob. error of mean

$$= \frac{0.67}{\sqrt{12}} = 0.00019 \text{ ft.}$$

Diamond Target:

Prob. error of single obs.

$$= 0.67 \sqrt{\frac{10}{11}} = 0.00064 \text{ ft.}$$

Approx. prob. error of single obs.

$$= 0.84 \times 0.67 = 0.00056 \text{ ft.}$$

Prob. error of mean

$$= \frac{0.64}{\sqrt{12}} = 0.00018 \text{ ft.}$$

probably be considerably more than this—possibly two or three times as great.

5. To resume the discussion of the general principles of the probable error, notice that according to equation (2) the probable error of the mean of a number of observations varies inversely as the square root of their number. For example, if a compensating error

TABLE II.

ERROR OF READING LEVEL TARGET.

Rod 300 *Feet from Instrument.*

Quadrant Target.			Diamond Target.		
Reading in feet.	d in thousandths.	d^2.	Reading in feet.	d in thousandths.	d^2.
4.843	5	25	4.848	2	4
4.835	3	9	4.851	1	1
4.836	2	4	4.851	1	1
4.837	1	1	4.852	2	4
4.834	4	16	4.849	1	1
4.834	4	16	4.849	1	1
4.837	1	1	4.851	1	1
4.839	1	1	4.852	2	4
4.838	0	0	4.848	2	4
4.835	3	9	4.850	0	0
4.842	4	16	4.846	4	16
4.841	3	9	4.852	2	4
4.838 mean	2.6 mean	107 sum	4.850 mean	1.6 mean	41 sum

Quadrant Target:

Prob. error of single obs.

$$= 0.67\sqrt{\frac{107}{11}} = 0.0021 \text{ ft.}$$

Approx. prob. error of single obs.

$$= 0.84 \times 2.6 = 0.0022 \text{ ft.}$$

Prob. error of mean

$$= \frac{21}{\sqrt{12}} = 0.0006 \text{ ft.}$$

Diamond Target:

Prob. error of single obs.

$$= 0.67\sqrt{\frac{41}{11}} = 0.0013 \text{ ft.}$$

Approx. prob. error of single obs.

$$= 0.84 \times 1.6 = 0.0013 \text{ ft.}$$

Prob. error of mean

$$= \frac{13}{\sqrt{12}} = 0.0004 \text{ ft.}$$

of 0.01 of a foot per chain is made in measuring, the error in a line 100 chains long would be only $0.01 \times \sqrt{100} = 0.1$ feet, and not $0.01 \times 100 = 1.0$ foot. Again, if the probable error of a single reading of a level target is 0.005 feet, then the error in determining a difference of elevation by a single setting of the instrument is $\sqrt{2} \times 0.005$ feet $= 0.007$ feet nearly, since

the determination requires two settings of the target. If sixteen such pairs of observations were taken to determine the difference in elevation of two remote points, then the probable error of the difference of level of the extreme points would be $0.007 \times \sqrt{16} = 0.028$ feet.

The following illustrations show slightly different forms of the above conclusion. If a represents the mean of a measured distance having a probable error x, and b represents another distance having a probable error y, then the probable error of the sum of these distances is $\sqrt{x^2 + y^2}$. Or, stating this algebraically we have

$$(a \pm x) + (b \pm y) = a + b \pm \sqrt{x^2 + y^2}. \quad . \quad . \quad (4)$$

Again, if $c \pm x$ and $d \pm y$ represent the two sides of a rectangle, the area will be represented by

$$cd \pm \sqrt{c^2y^2 + d^2x^2}. \quad . \quad . \quad . \quad . \quad . \quad (5)$$

If z represents the probable error per unit (say per chain), then the area is represented by

$$cd \pm z\sqrt{cd(c + d)}. \quad . \quad . \quad . \quad . \quad . \quad (6)$$

To apply the last formula, assume that a lot 20×100 ft. is laid out with a chain with such a degree of accuracy that the probable error per chain is 0.01 ft., then

$$\begin{aligned} cd \pm z\sqrt{cd(c + d)} &= 2{,}000 \pm 0.01\sqrt{2{,}000(20 + 100)} \\ &= (2{,}000 \pm 4.90) \text{ sq. ft.} \end{aligned}$$

There are various ways in which the preceding principles may be made use of in practical work, some of which will readily suggest themselves, but it is not wise to discuss them here.

APPENDIX IV.

PROBLEMS IN TESTING, ADJUSTING, AND USING ENGINEERS' SURVEYING INSTRUMENTS.

THE following problems are solved by the author's students in connection with the study of the preceding text. Some of them are solved several times—with different instruments and under different conditions as to distance, weather, experience, etc. A report of each problem is made upon a sheet of standard co-ordinate paper.

PROB. 1. ERROR OF MEASURING WITH A STEEL TAPE.

1. Measure a distance of about 1,000 feet with a steel tape. Do not use a spring balance or a thermometer, but take every other precaution to secure extreme accuracy. Make at least four measurements, each man being fore chain-man at least once in each direction. Compare the tape with the standard, both before and after taking the measurement. Report the true horizontal distance between the ends of the line.

2. Compute the probable error of a single measurement and also of the mean.

3. Estimate the values of the errors due to the several sources mentioned in § 19, *i.e.*, assign values to a, b, c, etc., in equation (2). page 26; and compare the estimated probable error of a single measurement with the corresponding observed probable error.

4. Level the line and compute the correction for inclination. How does the estimated quantity agree with the true value?

Prob. 2. Error of Measuring with two Steel Rules.

1. To determine the degree of accuracy with which distances can be laid off by means of short rules, use two 2-foot steel rules and lay off 50 feet on the floor. The rules may be used for either end or line measurement; *i.e.*, the rules may be placed end to end, or they may be lapped and the coincidence of lines observed. Make three measurements of the distance by each method.

Report which method is considered the better, and state the reasons therefor.

Prob. 3. Angles with a Tape.

1. With a steel tape, measure four angles around a point such that their sum = 360°, no two of the angles being of the same size.

2. Measure the three angles of a triangle having sides about 250 to 300 feet long.

3. Report the sums in each case, and also state the radius used, the form of target (flag pole or chaining pin), weather, etc.

Prob. 4. Adjustment of the Magnetic Compass.

1. Test to see whether the center of graduation is in the line of sight.

2. Test to see if the zero of the vernier is in the line of sight.

3. Adjust the levels perpendicular to the vertical axis.

4. Place the sights (*a*) in the same plane, and (*b*) perpendicular to the plate.

5. Place the point of suspension of the needle (*a*) in

the vertical plane of the ends, and (*b*) in the horizontal plane of the ends.

6. Sharpen the pivot and place it in the center of the graduation.

7. If the instrument is provided with a means of setting off a perpendicular, test it.

8. Test the magnetism of the needle.

PROB. 5. ERROR OF SIGHTING A MAGNETIC COMPASS.

1. Set up the magnetic compass and clamp the vertical axis. At 100 feet from the instrument, set a flag pole in the line of the slits, and mark its position on a board laid on the ground for that purpose. Then slightly change the position of the flag pole, and line it in again. Make at least ten sightings on the pole. Measure the position of the marks from one end of the board, and compute the linear distance corresponding to the probable error of sighting. Reduce the linear error to angular error.

2. Repeat the above process at 300 and also at 600 feet.

PROB. 6. ANGLES WITH A MAGNETIC COMPASS.

1. With a compass measure four angles around a point, such that their sum = 360°. Make the observations as follows: Sight upon the first line and read the needle; sight the second line and read again. The difference of these readings is the first angle. Move the vernier a little to eliminate personal bias, and turn the compass slightly to eliminate any sticking of the needle; then measure the second angle as before. Do similarly for the other angles.

Try to have the conditions as to length of sight, targets, etc., as nearly as possible like those of Prob. 3.

2. Measure the three angles of a triangle having sides 250 to 300 feet long.

3. Report the sums in each case; and also state the form of target, length of sight, and condition of wind, sun, etc.

PROB. 7. ELIMINATION OF LOCAL ATTRACTION.

1. With a magnetic compass observe the back-sights and fore-sights of the several sides of a field of at least five sides. Read the bearings to the nearest five minutes.

2. Correct the field notes to eliminate local attraction, and to determine the angular error of closure.

PROB. 8. TRUE MAGNETIC BEARINGS.

1. Determine the true bearings of the several sides of a field with a magnetic compass by connecting one corner of the field with a true meridian. See § 11, Appendix II.

2. Find the true bearings and also the angular error of closure.

PROB. 9. TESTING A TELESCOPE.

1. Test (*a*) spherical aberration, (*b*) chromatic aberration, (*c*) defining power, and (*d*) flatness of field.

2. Measure the magnifying power. Use two methods and make three determinations by each.

3. Measure the angular width of the field of view.

4. Compare two telescopes (*a*) for illumination, and (*b*) for definition. Are the illuminating powers of the two telescopes in the direct ratio of the areas of real apertures and in the inverse ratio of the squares of the magnifying powers?

PROB. 10. STRETCHING SPIDER WEBS.

1. Attach three spider webs to a diaphragm, parallel to each other and $\frac{1}{50}$th of an inch apart. Be sure that all these are in the same plane.

2. Fasten a fourth web perpendicular to the others and in the same plane.

3. After the webs are dry, breathe upon them or hold them in a gentle current of steam. If they become slack, they were not sufficiently stretched before being fastened.

PROB. 11. MAKING A VERNIER.

1. Graduate a scale of equal parts on a piece of Bristol board, using a right-line pen and India ink, and make a vernier to read the scale. The unit of the scale and vernier will be assigned. The accuracy of the graduation is the most important feature of this problem.

PROB. 12. ADJUSTMENT OF A TRANSIT.

1. Test the eccentricity of the graduation.

2. Adjust the plate levels perpendicular to the vertical axis.

3. Adjust the line of collimation perpendicular to the horizontal axis.

4. Adjust the horizontal axis perpendicular to the vertical axis.

5. Test the motion of the objective in a vertical plane.

6. Adjust the level under the telescope (if there is one) parallel to the line of collimation, by the two-peg method.

7. Adjust the zero of the vertical circle to read zero when the line of sight is horizontal.

PROB. 13. ERROR OF SIGHTING A TRANSIT.

1. Set up the transit and clamp the vertical axis. At 100 feet from the instrument set a flag pole in the line of sight of the telescope, and mark the position of the pole on a board laid on the ground for that purpose. Slightly change the position of the flag pole, and line it in again. Make at least ten sightings on the pole. Measure the position of the marks on the board, from one end of it, and compute the linear distance corresponding to the probable error of sighting. Reduce the linear error to angular error.

2. Repeat the above process at 300 and also at 600 feet.

PROB. 14. ANGLES WITH TRANSIT.

1. With a transit and the other conditions about as in Probs. 3 and 6, measure the four angles around a point. Make the observations as follows: Sight upon one target and read the vernier, then turn to the second target and read again. Next turn the instrument a trifle, with the lower movement, to eliminate personal bias, and sight upon the second target and read the vernier; then turn to the third target and read again. In a similar manner measure all four of the angles.

2. Measure the three angles of a triangle. Measure both the interior and exterior angle at each station as a check.

3. Report the sum of the four and also of the three angles, and state the length of sights, the kind of targets, the least count of the vernier, the condition of the weather, etc.

PROB. 15. ANGLES WITH A TRANSIT BY REPETITION.

1. With the conditions as nearly as possible the same as in Prob. 14, measure the three angles of a triangle by repetition. Repeat each angle three times. Measure both the interior and exterior angles as a check.

2. Report the sum of the three angles, and the details of the work.

PROB. 16. TRAVERSING WITH A TRANSIT.

1. With a transit, determine the azimuths of the several sides of a field, using one of the sides as the reference line.

What will be the effect of an error of collimation?

PROB. 17. TRAVERSING WITH A TRANSIT.

1. With a transit, determine the azimuths of the several sides of a field, using a true meridian as the reference line.

PROB. 18. AREA WITH A TRANSIT.

1. Determine the area of a field having at least five sides, using a transit and a 100-foot steel tape.

2. Compute the area by two methods (see Tables II and III, Appendix III).

3. Report the field notes and the plat upon one sheet of regulation co-ordinate paper, and the computations upon another. State the area in acres and decimals thereof.

4. Try to secure uniform accuracy throughout. How many places of logarithms should be used? As worked, what is the weakest link?

PROB. 19. MERIDIAN WITH SOLAR TRANSIT.

1. Determine the direction of the zero line of the surveying spiral, with a solar transit. Make three observations in the forenoon.

2. Make an equal number of observations in the afternoon.

3. What conclusions can be drawn from a comparison of the forenoon and afternoon observations? What conclusion can be drawn from a comparison of the mean of all the observations and the true direction of the meridian?

PROB. 20. TESTING A HOME-MADE PLANE TABLE.

1. Test the straightness of the edge of the alidade.

2. Test the sights to see (*a*) if they are in a plane, (*b*) if they are perpendicular to the board, and (*c*) if their plane passes through the edge of the alidade.

3. Mark the point at which the centre of the bubble should stand when the top of the board is level.

PROB. 21. ANGLES WITH A PLANE TABLE.

1. Lay down four angles on paper, such that their sum $= 360°$, no two being of the same size. Determine the value of each by measuring the chords with a scale. The difference between the sum of the observed values and 360° shows the error of scaling off. Compute the probable error of scaling off a single angle. Compare this error with that of paragraph 1, Prob. 3.

2. With the conditions as nearly as possible like those in Probs. 3 and 6, measure the three angles of a triangle. Compute the probable error of a single angle. Compare this error with those of Probs. 3 and 6.

3. Report the results, and state the length of radius, kind of scale used, etc., and also the form of target, the condition of the weather, etc.

PROB. 22. ANGLES WITH A PLANE TABLE BY REPETITION.

1. Set out on the ground an approximately equi-angular triangle, having the length of sides, targets, etc., about as in Prob. 3, 6, and 14.

2. Measure each angle with the plane table by "repeating" it six times. Scale off the difference between six times the angle and 360°. Make this measurement with two different radii or two different scales, or both, to reduce the error of measuring this angle. Subtract this difference from 360°, and divide the result by six to get the observed value of the angle of the triangle.

3. Report the sum of the three angles of the triangle, the length of radius and kind of scale used in scaling off the angle, the length of sight, the kind of targets, the weather, etc.

PROB. 23. AREA WITH A PLANE TABLE BY RADIATION.

1. Find the area of a field by the method of radiation.

2. Present a reduced plat of the field, and state the scale of the plat shown and also the scale of that from which the area was determined. Show the directions of the cardinal points. State the area in acres and decimals.

PROB. 24. AREA WITH A PLANE TABLE BY TRAVERSING.

1. Find the area of a field by traversing.

2. Make a report as in paragraph 2 of Prob. 23.

PROB. 25. AREA WITH A PLANE TABLE BY RADIO-PROGRESSION.

1. Find the area of a field by radio-progression.

2. Make a report as in paragraph 2 of Prob. 23.

PROB. 26. THREE-POINT PROBLEM.

1. Knowing the lengths of the three sides of a triangle located upon the ground, and having the position of a fourth point given upon the ground, set the plane table over the latter and determine its position on the map by a mechanical solution.

2. Check the above work by a graphical solution.

3. Present a reduced plat of the work of the second method, giving the scale of the plat shown and stating that of the plat used in the field. State the difference between the results by the two methods.

PROB. 27. TWO-POINT PROBLEM.

1. Knowing the distance between two "inaccessible" points and having a third point given upon the ground, determine the position of the latter upon the paper by solving the two-point problem with the plane table.

2. Measure the distance from each of the "inaccessible" points to the point over which the instrument is set, and plat the true position of this point.

3. Present a reduced plat of the work, stating the scales used in the field and in making the reduced plat.

PROB. 28. STADIA CONSTANTS.

1. Establish eight or ten points approximately in line at irregular intervals on nearly level ground.

2. Set the instrument near the end of the line, and with the stadia determine the distance from the instrument to each point.

3. With a chain or steel tape measure the distance from the instrument to each point. Measure with the stadia first to avoid the possibility of any personal bias in reading the stadia rod.

4. Measure c and f.

5. Compute k in the formula $D = k\,s + c + f$, for each distance.

6. Find the mean value of k, and compute the probable error (in linear units on the ground) of a single observation, and also the probable error of the mean.

PROB. 29. HORIZONTAL DISTANCES WITH THE STADIA.

1. Locate a number of points approximately in line at irregular intervals.

2. Use the same instrument and rod as in Prob. 28, and determine the horizontal distances with the stadia.

3. Rod-man and instrument-man change places, and re-determine the distances.

4. Measure the distances with a steel tape.

5. Report the two series of stadia-determined distances, and also the true distances. Does the error of observation vary with the distance? How? Why?

PROB. 30. VERTICAL DISTANCES WITH THE STADIA.

1. Use the instrument and rod employed in Prob. 28, and determine the elevation of each point of the surveying spiral, with reference to the central one.

2. Rod-man and instrument-man change places, and re-determine the elevations.

3. Report the two series of results in tabular form.

PROB. 31. ERROR OF SETTING A LEVEL TARGET.

1. Take at least ten readings upon a rod at 100 feet. Preserve at least ten consecutive results.

2. Take ten or more readings at 300 feet. Preserve ten consecutive results.

3. Compute the probable error for a single reading for each distance.

4. In the report state the magnifying power of the telescope, the radius of curvature of the level vial, and the kind of weather.

PROB. 32. ADJUSTMENT OF THE WYE LEVEL.

1. Find (*a*) the radius of curvature of the level vial, (*b*) the value in arc of one division of the scale, and (*c*) the value in arc of one inch of the scale.

2. Adjust the line of the bottoms of the rings perpendicular to the vertical axis.

3. Adjust the level tube parallel to the bottoms of the rings.

4. Make the line of collimation to coincide with the axis of the rings. In making this adjustment use a near point, and then test the adjustment for a remote point. If it is in adjustment for the latter, the telescope slide is correct in all particulars.

5. Test the accuracy of the adjustments and the equality of the rings, by a check level.

PROB. 33. ADJUSTMENT OF THE DUMPY LEVEL.

1. Find (*a*) the radius of the curvature of the level vial, (*b*) the value in arc of one division of the scale, and (*c*) the value in arc of one inch of the scale.

2. Adjust the level tube perpendicular to the vertical axis.

3. Adjust the line of sight parallel to the level tube, by the two-peg method.

4. Repeat the adjustment for a check.

PROB. 34. DIFFERENTIAL LEVELING.

1. Level a circuit of about a half mile, returning to the point of beginning.

2. State the error per mile, assuming it to vary as the square root of the distance. State also the time given to the field work.

Prob. 35. Profile Leveling.

1. Determine the relative elevations of the several points of the surveying spiral and close upon the point of beginning.

2. Draw a profile on profile paper, assuming the distance between successive stations to be 100 feet.

3. Submit the *original* field notes with the profile.

Prob. 36. Precise Leveling.

1. Level a circuit of about a half mile, returning to the point of departure. Use the method of double leveling with one rod—see III, Fig. 78, page 268.

2. State the error per mile, assuming it to vary as the square root of the distance. State also the time given to field work.

Prob. 37. Error of Reading Mercurial Barometer.

1. Read the attached thermometer. Try to have the conditions such that the temperature of the barometer will not change during the observations.

2. Read the barometer, and preserve at least ten consecutive results. Alter the vernier and also the adjustment of the mercury and the ivory point, after each observation, to eliminate unconscious bias.

3. Read the attached thermometer again, as a check against a change of temperature of the instrument.

4. Compute the probable error of a single reading.

Prob. 38. Leveling with a Mercurial Barometer.

1. Determine the elevation of the clock tower above the observatory, by reading the barometer and the attached and detached thermometers at each place. Take three readings at the observatory, three in the tower, and then three more in the observatory.

2. Reduce all the readings to 32° F. What do the

three readings at each point show as to the accuracy of barometric leveling? What do the means of the two series at the observatory show as to gradient error?

3. Compute the difference of elevation by at least three standard formulas, using the mean of the six readings at the observatory for the reading at the lower station, and the mean of those in the tower for the reading at the upper station.

4. After having performed the above operations, ask the instructor for the true difference of level. What does the difference between the true and the observed result show as to the accuracy of barometric leveling? Are the conditions of this problem favorable or unfavorable as a test of the accuracy of barometric leveling?

Prob. 39. Leveling with a Mercurial Barometer.

Repeat Prob. 38, using two points having a considerable *horizontal* distance between them.

Prob. 40. Leveling with an Aneroid Barometer.

1. Determine the difference of elevation between the floor of the observatory and each floor of University Hall, and also that of the clock tower. Read the aneroid —both the scale of inches and the scale of elevations— and the detached thermometer at the observatory, and also at each floor on the way up, and repeat at each floor on the way down and at the observatory.

2. Compute the elevation of each floor above the observatory, for each of the two readings, by at least two formulas.

3. Compare the computed differences with the differences obtained from the scale of elevations. What do the differences show?

4. Are the conditions of this problem favorable or unfavorable as a test of the accuracy of the aneroid?

INDEX.

LEV—NOT

OBJ—ROD

TRA—VER

VER—WYE